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Effective contests

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ABSTRACT

We find that two-stage contests could be ineffective, namely, there is a higher chance of low-ability players participating (and winning) than high-ability players. However, imposing a fee on the winner can guarantee that the contest will be effective.

1. Introduction

In many contests, players first have to decide whether or not to enter a contest. Afterwards, if there is more than one entrant, they must compete to determine the winner. For example, these can occur in promotion contests, open calls for research, or political contests.

In this paper, we model this situation as a two-stage game where there is an entry stage and a contest stage. The cost of participating in the contest has two components: first, the entry costs and second, the expenditure used in the contest. We model the former as privately known and fixed and the latter as publically known and variable. Also, each player has a publically known ability, either high or low.

The timing of the model is as follows. In entry stage, the players engage each other. They indicate their interest in entering and every player learns the abilities of his potential opponents. Then, given his private cost of entry, he decides whether or not to participate in the contest stage. The players who decide to participate pay their entry costs. After this stage, all players incur their entry costs and learn who has entered. In the second stage, the players compete against each other in what we model as an asymmetric all-pay auction under complete information.[1] Each player chooses expenditure (effort) and the player with the highest expenditure wins the contest. Ability factors into the cost of expenditure. Those with higher ability have an easier time competing. Independent of success, all players bear the costs.

We find that our model has cutoff equilibria, where any player with an entry cost higher than the cutoff for his type (ability) will decide to stay out of the contest and any player with an entry cost lower than the cutoff for his type will decide to participate in the contest. We show that given these equilibrium entry decisions the contest may be ineffective; namely, the chance that a high-ability player will participate may be lower than the chance that a low-ability player will participate. Consequently, there may be a higher chance that the winner of the contest will be the low-ability player. We show that a designer can overcome this problem and guarantee that the contest will be effective by imposing a requirement (task or fee) to be paid by the winner of the contest.

2. The model

Consider n players competing in a contest for one prize. The players have the same value for winning the position (contest) which

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[1] In the economic literature, all-pay auctions are studied under complete information where the players' valuations for the object are common knowledge (see, for example, Hillman and Riley, 1989; Baye et al., 1993, 1996; Che and Gale, 1998; Kaplan et al., 2003) or under incomplete information where each player's valuation for the object is private information to that player and only the distribution of the players' valuations is common knowledge (see, for example, Amann and Leininger, 1996; Krishna and Morgan, 1997; Moldovanu and Sela, 2001, 2006).

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is normalized to be 1. Player i's ability, $\alpha_i \geq 0$, is common knowledge. Assume that there are n_1 players with high ability of α_1 and n_2 players with a low ability of $\alpha_2 < \alpha_1$.[2] Participating in the contest generates a (sunk) cost c_i/α_i for player i, where c_i is the entry cost which is private information and is drawn independently from the cumulative distribution function F which is on the interval $[\underline{c}, \bar{c}]$ where $0 \leq \underline{c} < \min\ \alpha_i$. We assume that F is continuously differentiable with $F(\underline{c}) = 0$ and is common knowledge.[3] In the first stage, all the players are engaged, they learn the valuations of their opponents and each one decides whether to stay out or participate in the second stage of the contest. The players who decide to participate pay their entry costs. Then, in the second stage, these players see who else has decided to participate and compete in an all-pay auction under complete information such that the player with the highest expenditure x_i wins the contest, while all the players pay their cost of expenditure, which is $\frac{x_i}{a_i}$ (higher ability players have an easier time putting forth an effort). Thus, if player i decides to participate at the second stage of the contest, pays his entry cost c_i, spends an expenditure of x_i and wins the contest, then his payoff is given by $1 - \frac{(x_i + c_i)}{\alpha_i}$. On the other hand, if he does not win the contest his payoff is given by $-\frac{(x_i + c_i)}{\alpha_i}$.

3. Equilibrium

In our model there are frequently trivial equilibria strategies in which one of the players decides to always participate independent of his entry cost, and all the other players decide to stay out of the contest in the second stage. In order to prevent such equilibrium strategies (when n_1, $n_2 > 1$) we assume that players of the same type (same α) follow the same strategy. We say that an equilibrium is type-symmetric if all players of the same type follow the same strategy.

In the second stage the players compete in the all-pay auction where the players' abilities are common knowledge. If there is only one entrant in the second stage, he will bid zero and win. If there is more than one entrant, there are three cases that need to be examined. Let us denote e_i for the number of entrants of type i. By Baye et al. (1996), we have the following type-symmetric equilibrium in the second stage:

Case 1. There are two or more entrants with low abilities (type 2) only.

Then, these players randomize on the interval $[0,\alpha_2]$ according to their expenditure cumulative distribution functions $F_2(x)$, which is given by the indifference condition:

$$\alpha_2 F_2^{e_2 - 1}(x) - x = 0. \tag{1}$$

Thus, each player's expenditure is distributed according to $F_2(x) = \left(\frac{x}{\alpha_2}\right)^{\frac{1}{e_2 - 1}}$. Total expenditure is $e_2 \int_0^{\alpha_2} x dF_2(x) = \alpha_2$ and the expected payoff of every player is $u_2 = 0$.

Case 2. There are $e_1 \geq 2$ entrants with high abilities (type 1) and any number of entrants with low abilities.

In this case all the players of type 2 stay out and the players of type 1 enter in the second stage. These players randomize on the interval $[0,\alpha_1]$ according to their expenditure cumulative distribution functions $F_1(x)$, which is given by the indifference condition:

$$\alpha_1 F_1^{e_1 - 1}(x) - x = 0. \tag{2}$$

Thus, players' expenditure is distributed according to $F_1(x) = \left(\frac{x}{\alpha_1}\right)^{\frac{1}{e1 - 1}}$. The total expected expenditure is $e_1 \int_0^{\alpha_1} x dF_1(x) = \alpha_1$ and the expected payoff of every player is $u_1 = 0$.

Case 3. There is only one entrant with high ability and $e_2 \geq 1$ entrants with low abilities.

Then, the players randomize on the interval $[0,\alpha_2]$ according to their expenditure cumulative distribution functions, $F_1(x)$ and $F_2(x)$, which are given by the indifference conditions:

$$\begin{aligned} &\alpha_1 F_2^{e_2}(x) - x = \alpha_1 - \alpha_2, \\ &\alpha_2 F_1(x) - x = 0. \end{aligned} \tag{3}$$

Thus, type 1's expenditure is distributed according to $F_1(x) = \frac{x}{\alpha_1}$, while type 2's expenditure is distributed according to $F_2(x) = \left(\frac{x + \alpha_1 - \alpha_2}{\alpha_1}\right)^{\frac{1}{e_2}}$. The total expected expenditure is $\int_0^{\alpha_2} x dF_1(x) + e_2 \int_0^{\alpha_2} x dF_2(x) = \frac{\alpha_2 + 3e_2\alpha_2 + 2e_2^2(\alpha_1 - \alpha_2)\left(\left(1 - \frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{e_2}} - 1\right)}{2(e_2 + 1)}$, and the respective expected payoffs are $u_1 = \alpha_1 - \alpha_2$ and $u_2 = 0$.

Now, given the analysis of the players' behavior in the second stage of the contest, we can analyze their entry decisions in the first stage. In the first stage, n_1 players with ability of α_1 and n_2 players with ability of α_2 are engaged and each of them decides whether to participate or not, and those who decide to participate pay their private entry costs. Denote by $d_i(c)$ the entry decision (the probability of entering) if one has entry cost c and ability $\alpha_i > 0$.

Proposition 1. *The entry decision (the probability of entering) of a player with cost c_i and ability $\alpha_i > 0$ in the first stage is*

$$d_i(c) = \begin{cases} 1 \text{ if } c \leq c_i^*, \\ 0 \text{ if } c > c_i^* \end{cases}$$

where the equilibrium cutoffs c_i^*, $i = 1,2$ are given by[4]

$$c_1^* = (\alpha_1 - \alpha_2)(1 - F(c_1^*))^{n_1 - 1} + \alpha_2 (1 - F(c_2^*))^{n_2} (1 - F(c_1^*))^{n_1 - 1}, \tag{4}$$

$$c_2^* = \alpha_2 (1 - F(c_1^*))^{n_1} (1 - F(c_2^*))^{n_2 - 1}. \tag{5}$$

In the symmetric case where $\alpha = \alpha_1 = \alpha_2$ and n is the total number of players, the symmetric entry decision is given by

$$d_i(c) = \begin{cases} 1 \text{ if } c \leq c^*, \\ 0 \text{ if } c > c^* \end{cases}$$

where the equilibrium cutoff $c^* > 0$ is the solution of[5]

$$c^* = \alpha (1 - F(c^*))^{n-1}. \tag{6}$$

Proof. See Appendix A. □

The entry decision described by Proposition 1 is such that any player with ability α_i and an entry cost higher than the equilibrium cutoff c_i^* will stay out of the contest and any player with ability α_i and an entry cost lower than the equilibrium cutoff c_i^* will participate in the second stage of the contest.

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[2] For simplicity, we assume two types of abilities. Our results can be generalized to the case with any number of types.

[3] To avoid a trivial solution assume that $F(\alpha_2) > 0$ (there is a chance that player i has a cost lower than α_2).

[4] Obviously, this equilibrium is for n_1, $n_2 \geq 1$. If $n_1 \geq 2$, $n_2 \geq 2$ and $\underline{c} = 0$, then any type-symmetric equilibrium must be interior. If $n_1 = 1$ or $n_2 = 1$ the type-symmetric equilibrium can be non-interior with $c_1^* \geq \bar{c}$, $c_2^* \leq \underline{c}$ or $c_2^* \geq \bar{c}$, $\underline{c} < c_1^* < \bar{c}$ (and for $\underline{c} > v1 - v2$, non-interior with $c_2^* \geq \bar{c}$, $c_1^* \leq \underline{c}$). A cutoff $c_i > \bar{c}$ implies that everyone of type i would enter and a cutoff $c_i < \underline{c}$ implies that everyone of type i stays out.

[5] For the symmetric case, any symmetric equilibrium is interior.

4. Effectiveness

Given the equilibrium strategies, a player with ability α_2 has a positive payoff only if he is the only entrant. Thus, the payoff of a player with ability α_2 and entry cost $c \leq c_2^*$ is

$$\alpha_2(1-F(c_1^*))^{n_1}(1-F(c_2^*))^{n_2-1}-c = c_2^*-c.$$

Similarly, a player with ability α_1 will profit α_1 when he is in the second stage of the contest alone and will profit the difference $\alpha_1-\alpha_2$ when he is in the second stage with only players with abilities of α_2. Thus, the payoff of a player with ability α_1 and entry cost $c \leq c_1^*$ is

$$(\alpha_1-\alpha_2)(1-F(c_1^*))^{n_1-1} + \alpha_2(1-F(c_2^*))^{n_2}(1-F(c_1^*))^{n_1-1}-c = c_1^*-c.$$

Therefore, the expected payoff of a player with ability α_i, $i=1,2$, is

$$\int_0^{c_i^*}(c_i^*-c)dF(c). \quad (7)$$

Definition 1. We say that a contest is effective if the chance of participation of candidates is increasing in their abilities. That is, a contest is effective if $\alpha_i > \alpha_j$ implies $c_i^* > c_j^*$.

In our model the contest is effective iff $c_1^* > c_2^*$. Below we show by examples that a player with high ability and a relatively low entry cost may decide to stay out of the contest whereas a player with low ability and a relatively high entry cost may decide to participate in the contest. In other words, the contest is ineffective.

Example 1. Consider a contest where $n_1=2$, $n_2=1$, $\alpha_1=2.25$, $\alpha_2=2$ and F is a uniform distribution on [0,1].

By Eqs. (4) and (5) the equilibrium interior cutoffs are given by[6]:

$$c_1^* = (2.25-2)(1-c_1^*) + 2(1-c_2^*)(1-c_1^*),$$
$$c_2^* = 2(1-c_1^*)^2.$$

There are two solutions to this system of equations: 1. $c_1^*=0.34255$ and $c_2^*=0.8644$ 2. $c_1^*=0.62993$ and $c_2^*=0.2739$. Note that in the first solution the contest is ineffective. The equilibrium cutoff of the player with the low ability α_2 is higher than the equilibrium cutoff of the players with the higher ability α_1. This result implies that the expected payoff of the player with the low ability α_2 is larger than the expected payoff of his opponents with the higher abilities α_1. This then implies that the player with the low ability is more likely to be selected than the player with the high ability.

The intuition for why this is possible is that a player's willingness to enter depends upon his expected surplus of being in the contest. This surplus depends not only upon the player's ability but who else decides to enter the contest. Hence, if high-ability players are less likely to enter the contest, then it is indeed possible for low-ability players to be more willing to enter since they are more likely to be alone and reap all the profits.

The contest designer can guarantee that the contest will be effective by the following method.

Proposition 2. *If the winner of the contest pays a constant fee equal to $t=\max(0,2\alpha_2-\alpha_1)$ then $c_1^* > c_2^*$ such that the contest is effective.*[7]

[6] Note that for simplicity of exposition, in our examples, we will write the equilibrium cutoff Eqs. (4) and (5), assuming there is an interior solution and then see if this is indeed the case.

[7] Note that the cost of a fee equal to t for candidate i is t/a_i.

Proof. By Eqs. (4) and (5) if $\alpha_1 > 2\alpha_2$ we have,

$$c_1^* = (\alpha_1-\alpha_2)(1-F(c_1^*))^{n_1-1} + \alpha_2(1-F(c_2^*))^{n_2}(1-F(c_1^*))^{n_1-1},$$
$$> \alpha_2(1-F(c_1^*))^{n_1-1}(1+(1-F(c_2^*))^{n_2}) > \alpha_2(1-F(c_1^*))^{n_1}(1-F(c_2^*))^{n_2-1} = c_2^*.$$

If $\alpha_1 < 2\alpha_2$, by imposing a fee of $t=2\alpha_2-\alpha_1$ the players have actually new abilities given by

$$\tilde{\alpha}_i = \alpha_i - t = \alpha_i-(2\alpha_2-\alpha_1), \; i=1,2.$$

Since $\tilde{\alpha}_1 = 2\tilde{\alpha}_2$ and $\tilde{\alpha}_i > 0$ for all i, the result is obtained. □

Hence by imposing a constant fee of $t=\max(0,2\alpha_2-\alpha_1)$ the designer can guarantee that the chance of participation of the high-ability players will be larger than those of the low-ability players. However, it is important to note that by imposing a constant fee, the designer lowers expected participation in the contest. Thus, before imposing a constant fee, the designer should trade-off having the benefit of higher participation versus desirability of an effective contest.

Appendix A. Proof of Proposition 1

Given the equilibrium in the second stage, a player with a low valuation α_2 will profit only when he is in the second stage of the contest alone. The probability of this is $(1-F(c_1^*))^{n_1}(1-F(c_2^*))^{n_2-1}$ which implies Eq. (5). On the other hand, a player with a high valuation α_1 will profit α_1 when he is in the second stage of the contest alone and will profit the difference $\alpha_1-\alpha_2$ when he is in the second stage with only players with valuations of α_2. These happen with probability $(1-F(c_2^*))^{n_2}(1-F(c_1^*))^{n_1-1}$ and $(1-F(c_1^*))^{n_1-1}(1-(1-F(c_2^*))^{n_2})$ which implies Eq. (4). The solution to these two equations form an equilibrium since a player's expected utility with private cost c is of the form $g(c_1^*,c_2^*)-c$, that is, strictly decreasing in his cost. The cutoff corresponds to when a player is indifferent to entry. For all costs strictly lower than this cutoff, a player has strictly positive utility to entering. For all costs strictly higher than this cutoff, a player has strictly negative expected utility to entering. This also explains why there cannot be mixed strategies involving cutoffs. Given the strategies of all other players, there can only be at most one value of c where a player is indifferent. Two different cutoffs would imply that a player is sometimes entering and sometimes not entering for two different values of c.

The existence of the equilibrium is derived by Brower's Fixed Point Theorem. The RHS of Eqs. (4) and (5) form a bounded function from $[0,\alpha_1]\times[0,\alpha_2]$ to $[0,\alpha_1]\times[0,\alpha_2]$ that is continuous since F is continuous. Therefore, a fixed point must exist. (Note that if cutoff c_i^* of the fixed point is above $\bar{c}$, then it would imply that everyone with value α_i enters. Likewise, if cutoff c_i^* of the fixed point is below $\underline{c}$, then it would imply that everyone with value α_i stays out.)

In the following we show that if $n_1, n_2 \geq 2$, and $\underline{c}=0$, then any fixed point is interior, that is $F(c_1^*)$, $F(c_2^*)$ are from (0,1).[8] The RHS of Eqs. (4) and (5) are decreasing in c_1^* and c_2^*. If $F(c_1^*)=0$, then the RHS of Eq. (4) is greater than or equal to $\alpha_1-\alpha_2>0$ — a contradiction. If $F(c_1^*)=1$, then the RHS of Eq. (4) is zero — also a contradiction. Hence, $0<F(c_1^*)<1$. A similar argument shows that $0<F(c_2^*)<1$ as well. The symmetric case can be shown in a similar, but simpler manner. □

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[8] We assume that $F'(a)>0$.

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