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Endogenous Cycles and Growth with Indivisible Technological Developments*

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When large, discrete technological improvements require the accumulation of research or infrastructural investment over time, growth paths display cyclical patterns even in the absence of any shocks. Particularly interesting equilibrium features of these cycles include declines in output and asymmetries in the cyclic patterns displayed during expansions and recessions. *Journal of Economic Literature* Classification Numbers: E22, E32, O41.

1. INTRODUCTION

Are upswings and downswings in economic activity simply random deviations around a steady-growth trend, or does the engine of growth have an inherent cyclic component that contributes to economic fluctuations?

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Resolving this question is important for two reasons. First, if cyclical movements in output and investment have important endogenous components, then the forces that generate them ought to be taken into account in attempts to design and evaluate stabilization policies. Second, certain features of cyclical movements of economic activity—such as asymmetric responses to upswings and downswings and the determinants of cycle length—can only be explained by a model that endogenizes the deterministic part of these cycles and the patterns of economic activity within them.

This paper proposes a model of large and costly technological changes that endogenously generate deterministic cycles and long run growth. The level of technology in our model increases in discrete increments, called innovations, which increase the productivity of capital. To achieve the next innovation, an economy must accumulate a sufficient amount of resources diverted from consumption and physical investment.[1] When an innovation occurs, increased returns to capital encourage the economy to devote more resources to physical investment and thus less to the arrival of the next innovation. As the marginal product of capital using the existing technology declines, the economy again devotes more resources towards the realization of the next innovation.[2] In this way the economy progresses in cycles of constantly changing consumption and investments of both types, where the length of these cycles is linked to its long-term growth.

One can think of these innovations as large infrastructural improvements or as ‘‘knowledge’’ generated from costly research. Our emphasis on the large indivisible nature of investment in infrastructural or research projects reflects the lumpy nature of the increased productivity attained by them.[3] This lumpiness implies that a costly process must be completed before the greatest part of a project’s benefits in output can be realized. The length of that process depends on the intensity of the research or infrastructural investment.[4] Projects that require a large investment of

[1] Fan (1995) has also endogenized the growth rate by explicitly deriving the time between successive innovations as a solution to an optimal R & D investment problem but fixes investment and consumption growth at constant rates within each innovation cycle.

[2] Most existing growth models with deliberate R & D assume away the trade-off between capital accumulation and technological progress by considering labor as the only input to the R & D process [e.g., Grossman and Helpman (1991), Segerstrom (1991), Aghion and Howitt (1992), Cheng and Dinopolous (1992a, b), Young (1993), Parente (1994), and Fan (1995)]. Bental and Peled (1996) model the allocation of an accumulable resource between R & D and production, but separate this decision from the saving decision by households.

[3] The input A may be interpreted as a wide variety of factors, public or private. Human capital, public works, and private structures are among other suggested interpretations of this input. All are plausible to the extent that they require a prolonged investment process and have a large impact.

[4] Fan (1995) and Kleinknecht (1987) among others attempt to document that the rate at which major technological changes and projects are completed is influenced by the intensity of investments in such projects.

resources over endogenously determined periods in order to discretely increase the productivity of capital can be exemplified in both R & D and infrastructural contexts. Space research and satellite programs and major medical research are examples of prolonged and costly R & D whose benefits come primarily at the conclusion of the project. Infrastructural examples include the installation of wide systems of communication (telegraph, telephone, internet), of transportation (shipping canals, interregional highways, railroads, mass transit), or of electricity transmission that once complete may increase the productivity of many economic sectors.[5]

We do not deny the existence of exogenous and random shocks that can generate fluctuations in economic activity. We offer our model as a controlled study of the implications of the single assumption that endogenously determined technological improvements come in large discrete units. From this we hope to distinguish equilibrium implications of our endogenous deterministic cycles from those that would be predicted by models of exogenous stochastic technological improvements. We find that our model is content with the observations of a number of other studies, observations that might seem puzzling in economies in which all fluctuations are generated by exogenous stochastic shocks.

One puzzle raised by the assumption of exogenous technology shocks featured in other models lies in the interpretation of output declines as the result of negative technology shocks. Oil price rises and bad weather may account for some negative shocks to output, but are the rest of the observed downward shocks the result of losses in knowledge or technology?[6] Our model generates output declines even though technology in our model cannot decline or be made obsolete.

Our model of endogenous cycles also predicts particular co-movements among investment in major technological developments or in major infrastructural projects, on one hand, and investment in physical capital, consumption and output, on the other hand. Specifically, our model predicts that expansions in research expenditures precede expansions in physical capital as observed at the industry level by Lach and Rob (1992). It predicts that investment in major technological changes and infrastructural projects be weakest when output and physical capital growth are strongest. This also accords with the empirical findings of Hartman and Wheeler (1979) and Kleinknecht (1987) in the context of long waves.

Finally, the model predicts sharp increases in output as the economy exits recessions, but only gradual slips into recessions, in line with evidence

[5] See a review of theories and some evidence in van Duijn (1983), Kleinknecht (1987) and Greenwood and Yorukoglu (1996).

[6] See, for example, Summers's (1986) criticism of real business cycle models for their assumption of negative technological shocks.

on such asymmetries reported by Emery and Koenig (1992), Sichel (1993), and Balke and Wynne (1995) but inconsistent with exogenous stochastic technology shocks.

Because our social planning solution of the model can be decentralized as a competitive equilibrium, the demonstrated optimality of the resulting cyclic allocation stands in contrast to those of earlier models of research-driven growth, in which externalities or noncompetitive equilibria are central to the growth process. In particular, ‘‘creative destruction’’ is not a feature of the model [as in Aghion and Howitt (1992)]; new technological advances are not assumed to reduce the productivity or profitability of past advances. Nor is imperfect competition essential for creating incentives for investing in R & D [as in Grossman and Helpman (1991)]. The model thus demonstrates that the cyclical nature of growth paths is not necessarily the result of some inefficient equilibrium. Therefore, while the model’s mechanism of growth may be called Schumpeterian (1939) in that technological progress takes place in large discrete steps, intertwining the business cycle and economic growth, the model is not an exposition or defense of all of Schumpeter’s many ideas.

What we call development or infrastructure buildup costs bear some relation to costs of adopting new technologies, which have been recently examined in the context of growth and fluctuations by Jovanovic (1996), Hornstein and Krusell (1966), and Greenwood and Yorukoglu (1996). These studies take as exogenous the arrival and possibly the size of new technologies, and examine the cyclic implications for productivity of firm-specific costs of adoption. We focus instead on the implications for growth and cycles of costs of developing new technology and infrastructure, emphasizing the capital theoretic nature of these activities as alternative forms of investments in enhanced productive capacity.

Our model thereby contributes to the analysis of fluctuations induced by the cyclical nature of R & D activity, recently studied by Jovanovic and Rob (1990), who examined the choice between innovation and imitative refinements, Helpman and Trajtenberg (1994), who examined the interaction between general and specific purpose technological developments, and Bental and Peled (1996), who examined the cyclic implications of the complementarily between the size of the capital stock and the intensity of research activities.

In the next section we introduce the model, and present it as a recursive planning problem. In Section 3 we study the cyclical behavior of economic aggregates that characterize the optimal solution to the planning problem. In Section 4 we demonstrate the magnitude of these cyclical properties by numerically computing stationary solutions to completely specified economies. Section 5 highlights the cyclical properties of the planner’s problem and relates them to some empirical findings. Section 6 shows the equivalence of the planner's solution with the competitive equilibrium under a particular decentralization of the economy. Section 7 concludes by listing some historical cases of big infrastructural or research projects that give some perspective to our theoretical analysis, and summarize our findings. Existence proofs and computational methods are offered in Appendix A and Appendix B, respectively.

2. INNOVATIONS-DRIVEN CYCLES

2.1. *The Environment*

There exists a single infinitely lived representative agent living in continuous time. His preferences are given by the time-discounted utility function $\int_{t=0}^{\infty} e^{-\rho t} u[C(t)]\,dt$, where $C(t)$ denotes his consumption at t and ρ is a positive constant. The function $u(\cdot)$ is twice continuously differentiable, increasing, and concave, with $u'(C) \to \infty$ as $C \to 0$. We will study in particular the constant relative risk aversion function $u(C) = [1/(1-\sigma)]C^{1-\sigma}$ for $\sigma > 0$, $\sigma \neq 1$, and $u(C) = \ln(C)$ for $\sigma = 1$.

Production takes place at a continuum of plants on the unit interval. Production at each plant at time t is a function of $A(t)$, the technology available at t, and $K(t)$, the capital available at that plant at t. Note that with a unitary measure of plants, $K(t)$ is the total amount of capital at time t. Technology is a nonrival (but excludable) input, affecting the output of all plants. The production function takes the Cobb–Douglas, constant returns to scale form, $A(t)^{\alpha}K(t)^{1-\alpha}$.[7] Output from production at instant t can be used as consumption, $C(t)$, investment in new capital, $\dot{K}(t)$, or investment in the development of a new technology, $D(t)$. The initial capital stock per plant is positive and given as K_0. A constant fraction δ of the capital stock depreciates at each instant, $0 < \delta < 1$, implying the following time-t feasibility constraint in output:

$$NA(t)^{\alpha}K(t)^{1-\alpha} - \delta NK(t) = C(t) + D(t) + N\dot{K}(t). \qquad (2.1)$$

Technology advances in a series of discrete steps called innovations. The jth technology available to the economy, A_j, $j = 1, 2, 3, \ldots,$ is as effective in production as γ times the previous technology, or

$$A_j = \gamma A_{j-1}, \qquad \gamma > 1, \qquad j = 1, 2, 3, \ldots . \qquad (2.2)$$

[7] The essential features of the production function are that capital and technology are complements in production and that each has a diminishing marginal product. The Cobb–Douglas production function with a fixed number of plants is adopted for tractability in determining a solution.

This implies that the jth addition to the level of technology ($j > 0$) is given by

$$a_j \equiv A_j - A_{j-1} = (\gamma - 1) A_{j-1}. \qquad (2.3)$$

The initial level of technology, $A_0 = a_0$, is positive and given. Notice that the jth technology is essentially the sum of the initial technology and the j existing innovations

$$A_j = \sum_{i=0}^{j} a_i. \qquad (2.4)$$

Technology is advanced through the accumulation of research. Specifically, when enough research is accumulated under the current technology, the next technology in line becomes available.

The points in time in which the next technology in line becomes available are endogenous in that a prespecified amount of cumulative research is needed to attain the next technology in line, but the time taken to achieve this accumulation is endogenous. Let t_j be the first moment in time at which technology A_j (innovation a_j) becomes available, and let $T_j \equiv t_{j+1} - t_j$ be the length of the interval during which technology A_j is being utilized and technology A_{j+1} is being developed (the jth cycle), $j = 0, 1, 2, \ldots$. We set $t_0 = 0$ and $A_0 = 1$, implying $A_j = \gamma^j$.

The input of D units of goods when technology A_j is available produces $h_j(D) = [1/(1 - \phi)](D/A_j)^{1-\phi}$ units of research for $\phi \in (0, 1)$. Let $D(t)$ denote investment in research input at time t, and let $H(t)$ denote the units of research accumulated by time t from the moment at which the current technology was made available.

Then the accumulation of research is given by

$$H(t) = \int_{s=t_j}^{t} h_j[D(s)]\, ds, \qquad t \in [t_j, t_{j+1}), \qquad j = 0, 1, 2, \ldots . \qquad (2.5)$$

When the economy using the jth technology accumulates H^* units of research, the new innovation a_{j+1} becomes available, increasing the level of technology to A_{j+1}. The cumulative research is reset to zero upon attaining each innovation. Notice that the research effort required to achieve a technological breakthrough increases at the same rate, γ, as the magnitude of the breakthrough.

2.2. *The Planner's Problem*

Let us first examine the social optimum, defined to be the maximum feasible utility of the representative agent, as a solution of a planner's problem. We examine the planner's problem both for its expositional ease and to underscore that the generated cycles are not the result of any market failure or inappropriately assigned property rights.

Although time is continuous in the model, technological advances take place in discrete steps. Accordingly, the analysis of the social optimum involves a combination of discrete and continuous time maximization. The social planner's problem involves the choice of time paths for consumption, research effort and capital stocks, as well as the points in time in which the new technologies become available, given the initial capital stock K_0, and the initial technology A_0:

$$W(K_0, A_0) = \max_{C(\cdot), D(\cdot), K(\cdot), \{t_j | j=1,2,\dots\}} \int_{t=0}^{\infty} e^{-\rho t} u[C(t)]\, dt \qquad (2.6)$$

subject to (2.1), (2.5) and

$$\begin{aligned} A(t) &= A_j, \qquad t \in [t_j, t_{j+1}), \qquad t_0 = 0, \qquad j = 0,1,2,\dots, \\ A_j &= \gamma A_{j-1}, \qquad \gamma > 1, \qquad j = 1,2,\dots, \qquad (2.7) \\ H(t_{j+1}) &= H^*, \qquad j = 0,1,2,\dots . \end{aligned}$$

$K(0) = K_0 > 0$, and $A_0 > 0$ are given.

Within each technology cycle, $[t_j, t_{j+1})$, the allocation of resources solves an optimal control problem. The end points of these cycles, when the required research for the new technology is accomplished, are then chosen along with the levels of the physical capital stock to solve a discrete dynamic programming problem. It will be useful to repose the planner's problem recursively in terms of these cycles. To this end we index variables by the time within the cycle, so that $X_j(\tau) = X(t_{j-1} + \tau)$ for any variable X. Accordingly, $K_j(\tau)$, $C_j(\tau)$ and $D_j(\tau)$ are, respectively, the capital stock per plant, the consumption and the research effort τ units of time into the jth cycle, and $\hat{K}_j = K_j(0)$ and $\hat{K}_{j+1} = K_j(T_j)$ are the levels of the capital stock per plant at the beginning and end of the jth cycle. The ability to augment the capital stock only by accumulation implies $K_j(T_j) = K_{j+1}(0)$.

Define $V(T, K_0, K_T, A)$ as the maximal within-cycle discounted utility, given the cycle total length T, beginning and end of cycle capital stocks, K_0 and K_T, respectively, when the available technology is A:

$$V[T, K_0, K_T, A] = \max_{\{C(\tau), D(\tau), K(\tau) | \tau \in [0,T]\}} \left\{ \int_{\tau=0}^{T} e^{-\rho\tau} \frac{[C(\tau)]^{1-\sigma}}{1-\sigma}\, d\tau \right\} \qquad (2.8)$$

subject to:

$$\dot{K}(\tau) = A^{\alpha}K(\tau)^{1-\alpha} - \delta K(\tau) - C(\tau) - D(\tau), \qquad 0 \le \tau \le T, \quad (2.9)$$

$$H(\tau) = \int_{s=0}^{\tau} \frac{1}{1-\phi}\left(\frac{D(s)}{A}\right)^{1-\phi} ds, \qquad 0 \le \tau \le T, \quad (2.10)$$

$$H(T) = H^*, \quad (2.11)$$

$$K(0) = K_0, \quad (2.12)$$

$$K(T) = K_T, \quad (2.13)$$

K_0, K_T, and T are given.

The original planner's problem can now be written as

$$W(K_0, A_0) = \max_{\{T_j \hat{K}_{j+1}\}_{j=0}^{\infty}} \left\{ \sum_{j=0}^{\infty} e^{-\rho t_j} V\left[T_j, \hat{K}_j, \hat{K}_{j+1}, A_j\right] \right\}, \quad (2.14)$$

where T_j is the length of the jth cycle, and $\hat{K}_j$, $\hat{K}_{j+1}$ are, respectively, the capital stocks at the beginning and the end of the jth cycle, and where

$$t_j = \sum_{m=0}^{j-1} T_m, \qquad j = 1, 2, 3, \ldots, \qquad t_0 = 0$$
$$A_j = A_0 \gamma^j, \qquad j = 0, 1, 2, \ldots, \quad (2.15)$$

K_0 given. Thus, the planner's problem has a simple recursive structure which can be summarized by

$$W(K_0, A_0) = \max_{T, K_T} \left\{ V(T, K_0, K_T, A_0) + e^{-\rho T} W(K_T, \gamma A_0) \right\}, \quad (2.16)$$

where $V(\cdot)$ is defined in (2.8).

The Hamiltonian function associated with the within-cycle problem for the jth cycle version of (2.8) is

$$\begin{aligned} &\Gamma\left[\tau, T_j, C_j(\tau), D_j(\tau), K_j(\tau), H_j(\tau)\right] \\ &\quad = e^{-\rho\tau} u\left[C_j(\tau)\right] \\ &\qquad + \mu_{j1}(\tau)\left[A_j^{\alpha} K_j(\tau)^{1-\alpha} - \delta K_j(\tau) - C_j(\tau) - D_j(\tau)\right] \\ &\qquad + \mu_{j2}(\tau) h_j\left[D_j(\tau)\right] \end{aligned} \quad (2.17)$$

for which $C_j(\tau)$ and $D_j(\tau)$ are the control variables, $K_j(\tau)$ and $H_j(\tau)$ are the state variables. The first order conditions are

$$\frac{\partial \Gamma}{\partial C_j(\tau)} = e^{-\rho\tau} u'\left[C_j(\tau)\right] - \mu_{j1}(\tau) = 0, \tag{2.18}$$

$$\frac{\partial \Gamma}{\partial D_j(\tau)} = -\mu_{j1}(\tau) + \mu_{j2}(\tau) h'_j\left[D_j(\tau)\right] = 0, \tag{2.19}$$

$$\dot{\mu}_{j1}(\tau) = -\frac{\partial \Gamma}{\partial K_j(\tau)} = -\mu_{j1}(\tau)\left\{(1-\alpha) A_j^{\alpha} K_j(\tau)^{-\alpha} - \delta\right\}, \tag{2.20}$$

$$\dot{\mu}_{j2}(\tau) = -\frac{\partial \Gamma}{\partial H_j(\tau)} = 0. \tag{2.21}$$

Conditions (2.18)–(2.21) are the usual control and costate equations.

Notice that if $K_j(T_j)$ is itself a choice variable in the jth within-cycle problem, then the associated optimality condition for that variable must be derived from (2.16), for the "salvage" value $W(K_T, \gamma A)$ is the only reason to have positive capital stock at the end of the current cycle. The optimality condition associated with an endogenous $K_j(T_j)$ is then

$$\mu_{j1}(T_j) = e^{-\rho T_j} \frac{\partial W}{\partial K}\left[K(T_j), \gamma A_j\right]. \tag{2.22}$$

Likewise, if the cycle length T_j is to be chosen optimally, then from (2.16) this variable must satisfy the following condition:[8]

$$\begin{aligned} 0 = {} & e^{-\rho T_j} u\left[C_j(T_j)\right] + \mu_{j1}(T_j)\dot{K}_j(T_j) \\ & + \mu_{j2}(T_j)\dot{H}_j(T_j) - \rho e^{-\rho T_j} W\left[K_{j+1}(0), \gamma A_j\right]. \end{aligned} \tag{2.23}$$

2.3. *Rescaling*

It is convenient at this point to rescale the level of three choice variables in the jth cycle by the technology in use during that cycle. For any $\tau \in [0, T_j]$, let

$$\begin{aligned} c_j(\tau) &\equiv C_j(\tau)/A_j, \\ d_j(\tau) &\equiv D_j(\tau)/A_j, \\ k_j(\tau) &\equiv K_j(\tau)/A_j. \end{aligned} \tag{2.24}$$

[8] See Kamien and Schwartz (1981), p. 147, Eqs. (19) and (28), in particular.

The resource constraint, (2.9), becomes

$$\dot{k}_j(\tau) = k_j(\tau)^{1-\alpha} - \delta k_j(\tau) - c_j(\tau) - d_j(\tau), \qquad (2.25)$$

and the within-cycle value function from (2.8) is redefined as

$$v(T, k_0, k_T) \equiv V(T, K_0, K_T, A)/A^{1-\sigma},$$

where

$$k_0 \equiv K_0/A,$$

$$k_T \equiv K_T/A.$$

The technology-scaled within-cycle problem is fully described in Appendix A. The Bellman equation of the planner's problem, (2.16), can then be written as

$$W(K_0, A_0) = w(k_0) = \max_{T, k_1}\left\{v(T, k_0, \gamma k_T) + e^{-\rho T}\gamma^{1-\sigma}w(k_T)\right\}. \qquad (2.26)$$

Of particular interest will be the case of stationary cycles in which all cycles are of equal length, $T_j = T^*$ all j, and the technology-scaled variables are cycle independent, so that $c_j(\tau) = c^*(\tau)$, $d_j(\tau)^* = d^*(\tau)$, and $k_j(\tau) = k^*(\tau)$ for any $\tau \in [0, T^*]$. (In Appendix A we sketch a proof for the existence of such a stationary solution to the planner's problem.)

Along such a solution, the optimal trajectories of these variables are simply their trajectories along the previous cycle multiplied by the technology improvement factor γ. In particular,

$$k^*(T) = K_j(T)/A_j = K_{j+1}(0)/A_j = \frac{A_{j+1}}{A_j}\frac{K_{j+1}(0)}{A_{j+1}} = \gamma k^*(0).$$

It follows that if the economy starts out with the "right" level of capital stock, $k_0^* = k^*(0)$, the recursive representation of the planner's problem, (2.26), implies

$$w(k_0^*) = v(T^*, k_0^*, \gamma k_0^*) + e^{-\rho T^*}\gamma^{1-\sigma}w(k_0^*)$$

so that

$$w(k_0^*) = \frac{\int_{\tau=0}^{T^*} e^{-\rho\tau}\left(c^*(\tau)^{1-\sigma}\right)/(1-\sigma)\,d\tau}{1 - e^{-\rho T^*}\gamma^{1-\sigma}}. \qquad (2.27)$$

Notice that for the existence and boundedness of the optimal value function we require

$$e^{-\rho T^*}\gamma^{1-\sigma} < 1. \tag{2.28}$$

3. CHARACTERISTICS OF ENDOGENOUS CYCLES

Introducing our assumed functional forms and using (2.18) to eliminate $\mu_{j1}(\tau)$ from (2.20), we find, after some rearrangement of terms, a standard Ramsey rule for the growth rate of consumption within a cycle:

$$\frac{\dot{c}(\tau)}{c(\tau)} = \frac{1}{\sigma}\left[(1-\alpha)k(\tau)^{-\alpha} - \delta - \rho\right]. \tag{3.1}$$

PROPERTY 1. *Consumption is always continuous.*

Proof. The continuity of consumption within a cycle is directly implied by (3.1) together with the continuity of the capital stock. For continuity of consumption *across* cycles, we use the following argument. From (2.18) for $\tau = T_j$ and (2.22) we have

$$\frac{\partial W}{\partial K}\left[K_j(T_j), \gamma A_j\right] = u'\left[C_j(T_j)\right]. \tag{3.2}$$

On the other hand $W[K_j(T_j), \gamma A_j] \equiv W[K_{j+1}(0), A_{j+1}]$ also measures the marginal value of an additional unit of resources per plant at the beginning of the $j+1$ cycle. Along the optimal path, this marginal value should be equally obtained by putting that unit to any of its three possible uses: consumption, production or research. Consequently, we also have

$$\frac{\partial W}{\partial K}\left[K_j(T_j), \gamma A_j\right] \equiv \frac{\partial W}{\partial K}\left[K_{j+1}(0), A_{j+1}\right] = u'\left[C_{j+1}(0)\right] \tag{3.3}$$

which proves, given strict concavity of u, that $C_j(T_j) = C_{j+1}(0)$. *Q.E.D.*

A more intuitive explanation for the continuity of consumption along the optimal path is based on the following "variational" argument. Consider a jump discontinuity in consumption at t_j, such that $C_j(T_j) < C_{j+1}(0)$. Consider a short enough time interval of length Δ, such that $C_j(T_j - \epsilon) < C_{j+1}(\epsilon')$, $\forall \epsilon,\ \epsilon' \in (0, \Delta)$. Due to concavity of $u(\cdot)$, discounted utility over the interval $(t_j - \Delta, t_j + \Delta)$ will increase if we can raise $C(\cdot)$ on $(t_j - \Delta, t_j)$ at the expense of $C(\cdot)$ on $(t_j, t_j + \Delta)$. Such a change is possible by reallocating resources between capital accumulation and consumption on

$(t_j - \Delta, t_j)$, without altering the resources allocated to knowledge accumulation before and after t_j. Simply lower $K(\cdot)$ on $(t_j - \Delta, t_j)$, and use the resources freed up for consumption. Because $D(\cdot)$ has not been changed, t_j will not change either. However, the initial capital stock when A_j becomes available is now lower than it was originally, requiring a reduction in the initial level of $C_{j+1}(0)$. Moreover, the lower capital stock at t_j implies a higher marginal product of capital with the new technology, and allows for a higher than before rate of growth in consumption, according to (3.1), without altering $D(\cdot)$. Hence, the consumption path is lower on $(t_j, t_j + \Delta)$ than it was, but its growth is higher so as to coincide with the original consumption path at $t_j + \Delta$.

Although consumption is continuous, its growth rate is discontinuous at the time of an innovation. From (3.1) we see that the growth rate of consumption is the difference between the marginal product of capital and the constants δ and ρ. An innovation discretely shifts up the marginal product of capital schedule, implying that the growth rate of consumption immediately after an innovation exceeds the rate immediately before the innovation.

Using (2.18) to eliminate $\mu_{j1}(\tau)$ from (2.19), we find

$$\mu_{j2}(\tau)^{-1/\phi} d(\tau) = e^{\rho\tau/\sigma} c(\tau)^{\sigma/\phi}. \tag{3.4}$$

Noting from (2.21) that μ_{j2} is independent of τ, the last equation implies

$$d(\tau) = (\mu_2)^{1/\phi} e^{\rho\tau/\sigma} c(\tau)^{\sigma/\phi} \tag{3.5}$$

and

$$\frac{\dot{d}(\tau)}{d(\tau)} = \frac{\rho}{\phi} + \frac{\sigma}{\phi}\frac{\dot{c}(\tau)}{c(\tau)}, \tag{3.6}$$

or, with (3.1):

$$\frac{\dot{d}(\tau)}{d(\tau)} = \frac{1}{\phi}\left[(1-\alpha)k(\tau)^{-\alpha} - \delta\right]. \tag{3.7}$$

The differences in the within-cycle growth rates of research and consumption come from two sources: (i) differences in ϕ and σ, the relative curvatures of the functions representing utility and knowledge production, and (ii) differences in the rate of time discount applied to those two functions. Notice that if the curvatures are the same for both functions ($\sigma = \phi$), research grows faster than consumption over the cycle (by the constant ρ/σ) because no rate of time discount is applied to knowledge, in contrast to the time-discounted utility from consumption. This property will hold in the more general case in which knowledge or infrastructure accumulation is also subject to discounting, but at a lower rate than utility from consumption.[9]

While consumption is everywhere continuous, research effort decreases discontinuously at the time of an innovation; i.e.,

$$\begin{aligned} \gamma D_j(0) &= D_{j+1}(0) < D_j(T_j), \qquad \text{or} \\ \gamma d(0) &< d(T). \end{aligned} \tag{3.8}$$

These implications constitute Property 2 of the stationary social optimum.

PROPERTY 2. *Research effort declined discontinuously at the time of innovations.*

Proof. Suppose instead that research effort satisfies $\gamma d(0) \geq d(T)$. From (3.5), then

$$\gamma(\mu_2)^{1/\phi} c(0)^{\sigma/\phi} \geq (\mu_2)^{1/\phi} e^{\rho T/\phi} c(T)^{\sigma/\phi}. \tag{3.9}$$

Because consumption is continuous, $c(T) = c(0)$, implying that (3.9) is satisfied only if

$$1 \geq e^{\rho T} \gamma^{\sigma - \phi}. \tag{3.10}$$

Because $\phi < 1$,

$$e^{\rho T} \gamma^{\sigma - \phi} > e^{\rho T} \gamma^{\sigma - 1} \tag{3.11}$$

But recall from (2.28) that $e^{\rho T} \gamma^{\sigma - 1}$ must be greater than 1 for a solution to be a maximum, contradicting (3.10) and proving Property 2 by contradiction. *Q.E.D.*

Given that research effort jumps down at the time of innovations, while output jumps up and consumption is continuous, it can only be that investment jumps up at the time of innovations.

PROPERTY 3. *Investment discontinuously increases at the time of innovations.*

[9] However, this observation also suggests that a preference specification under which utility at time t depends on discounted consumption aggregated over time might eliminate this difference in the growth rates of R & D investment and consumption. Such preferences have been used to describe services from durable goods. We thank John Heaton for this observation.

Proof. Continuity of consumption and capital stock paths, together with (2.24), imply that $c(T) = \gamma c(0)$ and $k(T) = \gamma k(0)$. From the feasibility condition (2.9) in a stationary social optimum for $\tau = 0$ we then get

$$\begin{aligned} \gamma \dot{k}(0) &= \gamma k(0)^{1-\alpha} - \gamma\delta k(0) - \gamma c(0) - \gamma d(0) \\ &> \gamma\left[\frac{1}{\gamma}k(T)\right]^{1-\alpha} - \delta k(T) - c(T) - d(T). \end{aligned} \tag{3.12}$$

The last inequality follows from Property 2 [that $d(0) < d(T)/\gamma$]. Moreover, since $\gamma^{-\alpha} < 1$,

$$\gamma \dot{k}(0) > k(T)^{1-\alpha} - \delta k(T) - c(T) - d(T) = \dot{k}(T). \tag{3.13}$$

Thus, unlike the capital stock and consumption, investment in physical capital discontinuously jumps up at innovation points in time. *Q.E.D.*

Finally, in a stationary equilibrium, the condition defining the optimal terminal time T, (2.23), may be written after some algebra as

$$\frac{c(T)}{1-\sigma} + \frac{d(T)}{1-\phi} + \dot{k}(T) = \rho\gamma^{1-\sigma} w[k(0)] c(T)^{\sigma}. \tag{3.14}$$

At this point we can summarize a stationary social optimum as a cycle length T and a triple $[c(\tau), d(\tau), k(\tau)]$ defined over the interval $[0, T]$, satisfying the endpoint conditions $c(T) = \gamma c(0)$, $k(T) = \gamma k(0)$, $\int_0^T [1/(1-\phi)]\, d(\tau)^{1-\phi}\, d\tau = H^*$, the laws of motion (3.1), (3.7), and (2.25), and where the terminal time T satisfies (3.14) and (2.27).

4. COMPUTING STATIONARY CYCLES

In order to demonstrate how the cyclical features of the solution depend on the value of environmental parameters, we proceed by computing numerical solutions of stationary optimal paths. Based on these solutions we can obtain illustrations of the magnitude and nature of the cyclical fluctuations along the optimal growth paths, as a function of key parameters of the economy.

We utilize the recursive technology-scaled version of the planner's problem, (2.26), rewritten as

$$w(k) = \max_{T, k'} \{v(T, k, \gamma k') + e^{-\rho T}\gamma^{1-\sigma} w(k')\}, \tag{4.1}$$

where k is the (technology scaled) stock of physical capital at the beginning of the cycle, and the only choice variables for the current cycle are T, the endogenous length of the cycle, and $\gamma k'$, the endogenous end-of-cycle stock of capital (in terms of the current cycle technology-scaled capital).

It can be shown that under some mild restrictions on parameter values there exists a unique solution to this functional equation which attains the supremum function of the representative agent's problem (see Appendix A). Let the optimal policies attaining that solution be $T = \psi_1(k)$, $k' = \psi_2(k)$. Further, there exists a stationary solution in terms of technology-scaled capital stock, k^*, and cycle length, T^*, such that $k^* = \psi_2(k^*)$ and $T^* = \psi_1(k^*)$.

In terms of the original "unscaled" variables, this stationary optimal path has the following structure, provided $K_0 = k^*$. Cycle lengths are all equal to T^*, so that $t_j = j \cdot T^*$, $j = 0, 1, 2, \dots$. The capital stock at the beginning of each cycle is γ times larger than it was at the beginning of the previous cycle, $K(t_j) = \gamma K(t_{j-1}) = \gamma^j k^*$, $j = 1, 2, 3, \dots$. The optimal consumption path is continuous across cycles, so that $C(t_j) = \gamma C(t_{j-1})$. Finally, the growth rate of output, given that $Y(t_j + \tau) = \gamma Y(t_{j+1} + \tau)$ for any j and $0 \leq \tau \leq T^*$, is

$$g = \frac{\ln \gamma}{T^*}. \tag{4.2}$$

Using these properties, we can solve for the stationary optimal path numerically, for a broad range of parameters (see Appendix B) for details on the computation method).

The parameters used for the illustrative solution are: $\alpha = 0.7$, $\sigma = 0.8$, $\phi = 0.56$, $\rho = 0.05$, $\gamma = 1.3$, $\delta = 0.10$, and $H^* = 10$. The resulting values for endogenous variables in a stationary optimum for these parameters are: $T^* = 9.0$, $k^* = k(0) = 1.687$, $c(0) = 0.669$, and $d(0) = 0.102$.[10] Figure 1 shows the time paths of endogenous variables over the first three consecutive stationary cycles along a stationary solution, while Fig. 2 shows the within-cycle growth rates of these variables and the share of gross investment out of total output.

Figures 1 and 2 depict the movements within a stationary cycle in which capital declines near the end of a cycle. Notice that the growth rates of research and consumption begin to rise as capital starts to decline near the end of a cycle [see Eqs. (3.1) and (3.7)]. Essentially the maintenance of the capital stock is neglected while resources are instead devoted to complet-

[10] The resulting value for the technology-scaled value function $w[k(0)]$ in (3.27) is 105.08, while the end of cycle values of the endogenous variables are $c(T) = 0.87$, $d(T) = 0.33$, and $k(T) = 2.19$. These values satisfy (4.14), the condition for the optimal terminal time.

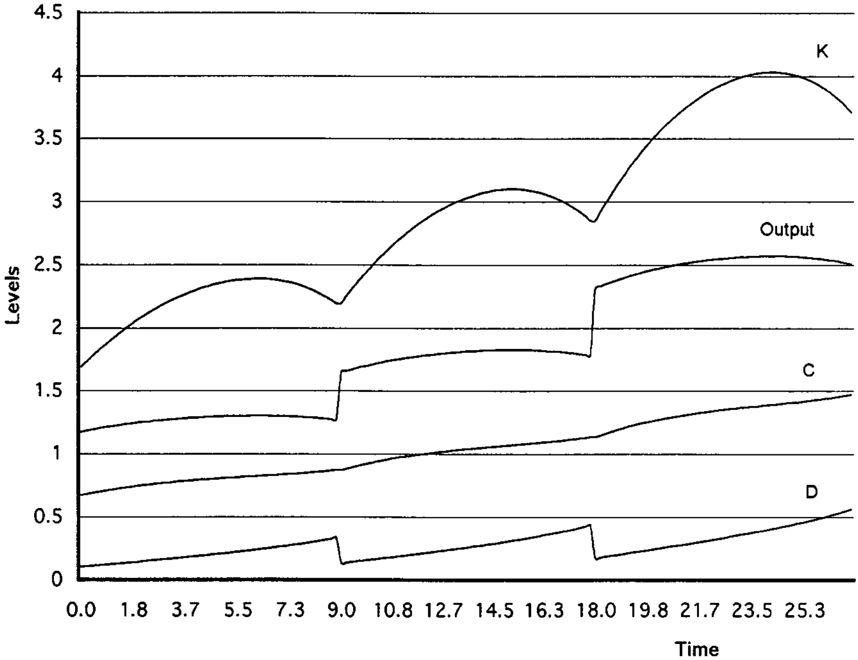

FIG. 1. Three consecutive cycles (levels).

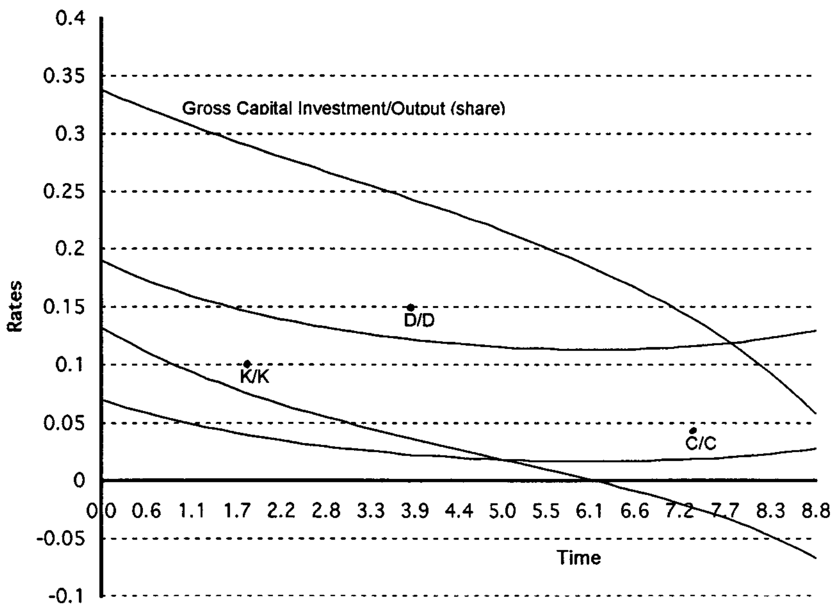

FIGURE 2

ing the upcoming innovation (and to raising consumption toward the level that will be chosen upon the arrival of the new innovation).

While reductions of output and capital are not features of the optimal planning problem for all parameter values, the potential for output declines as a part of a socially optimal cycle is worth some attention. It demonstrates that neither negative technology shocks nor creative destruction are necessary to explain output declines.

To evaluate the impact of parameter values on the optimal length of the cycle and capital accumulation patterns, we compute the stationary solutions for alternative sets of parameters, presented in Table I. Each entry in the table gives the elasticity of the endogenous variable with respect to changes in the row parameter. The variable $k_{\max}/k_{T^*}$ represents the ratio the highest capital stock within the cycle to the stock at the end of the cycle, while g, the growth rate, is given by (4.2).

For all parameters except γ, the elasticity of the length of a cycle, T^*, is of equal size but opposite sign of that of the growth rate, g [see (4.2)]. Changes in γ have a compound effect: growth increases directly as $\ln(\gamma)$ increases, but in addition T^* declines, thus further enhancing the growth rate. Parameters capturing the technological improvement (γ), the productivity of knowledge accumulation (ϕ), or the required cumulative R & D for the new technology (H^*), have the expected impact on the growth rate. A higher discount factor prolongs the optimal time over which the required knowledge for the next technology is accumulated, thus reducing the growth. A higher value for σ represents a lower marginal utility from high levels of consumption, which reduces the desire for growth.

The impact of parameter changes on the pattern of capital accumulation is less obvious. First, note that for the base case reported in the table, any parameter change has opposite sign effects on the growth rate, and the measure of capital's end-of-cycle decline (as captured by $k_{\max}/k_{T^*}$). However, these are not robust conclusions, as we have computed examples

TABLE I
Elasticities of Endogenous Variables

Parameter	g	$k_{\max}/k_{T^*}$
σ	−0.40	0.07
ϕ	2.52	−0.50
ρ	−0.73	0.11
H^*	−1.62	0.18
γ	6.09	−0.36

Note. Base parameters: $\alpha = 0.7$, $\sigma = 0.8$, $\phi = 0.56$, $\rho = 0.05$, $H^* = 10$, $\gamma = 1.3$, $\delta = 0.1$; endogenous variables: $T^* = 9.00$, $g = 0.029$, $k_{\max}/k_{T^*} = 1.088$.

where the effect of γ on capital's decline, for instance, depends on the magnitude of H^*: it increases the amount of capital's end-of-cycle decline for high values of H^*, and reduces it for low values of H^*. Higher discount rates increase the amount of capital's decline within the cycle, since the alternative of financing research effort by foregone consumption becomes less attractive.

The reported elasticities are not necessarily representative of what occurs at different parametric configurations. Yet, they illustrate the simplest form of trade-off between consumption, capital accumulation and investment in technology enhancement that have been ignored by and large in the growth literature. Adding any form of adoption or implementation costs associated with new technologies can considerably complicate these relationships, but is unlikely to wash them away.

5. CYCLICAL IMPLICATIONS

Properties 1–3, and their graphic illustrations in Figs. 1 and 2, allow us to characterize some features of the socially optimal stationary business cycle. Despite the discontinuous technological advances, consumption is continuous within and between cycles. An innovation causes an output increase and an upward shift in the marginal product of capital. The increased marginal product of capital causes investment to jump up. At the same time, research effort discontinuously declines because the marginal product of another innovation is low. As time passes within the cycle, the increased investment in physical capital lowers the marginal product of capital and thus investment in capital, leaving more resources for consumption and research effort. Notice that the local peak level of research effort (just before the innovation) immediately precedes the local peak of physical capital investment (just after the innovation), consistent with the finding of Lach and Rob (1992), who report that expansions in research expenditures precede expansions in physical capital at the industry level, business cycles frequencies. Hartman and Wheeler (1979) also report evidence indicating that infrastructural investment seems to peak at the trough of output cycles.[11] Similarly, the highest output growth rates occur just after the lowest, consistent with the finding of Balke and Wynne (1995).

The model's implication of the asymmetry of growth and related variables within the business cycle has been noted in a variety of empirical studies. Emery and Koenig (1992), Sichel (1993), and Balke and Wynne (1995) find that the economy bursts out of recessions into expansions but

[11] Lach and Rob (1992) report that expansions in research expenditures preceed expansions in physical capital at the industry level, business cycles frequencies.

only gradually slips back into recession.[12] They find that growth rates at the beginning of expansions exceed those at the end of expansions but that growth rates within recessions exhibit no significant changes. This asymmetry cannot be examined by the common approach of comparing the second moments of artificially generated data from calibrated business cycle models to those of real world data.[13] Balke and Wynne (1995) therefore go on to show that the artificially generated data from a calibrated business cycle model differs significantly from the asymmetric pattern they observe in the actual data. Our model of innovations, in contrast, generates a stylized form of the pattern found in the actual data: sharp expansions in output (at the time of the innovation) with subsequent gradual declines in output growth.

Recent research into the existence of long waves (45–60 years in length), suggests that at least for the last 100 years there have been statistically significant differences in the growth rates of various measures of economic activity over time that fit the time framework of Kondratieff long waves (Kleinknecht, 1987). Kleinknecht also compared lists of ''major'' or ''radical'' industrial innovations against lists of broader inventive activities of an incremental nature, made during the 19th and 20th centuries. These lists were compiled by various authors for the U.S. and U.K., using different (subjective) criteria. He concludes that: ''all samples show clear evidence of a slowdown of radical innovations in the course of the long wave upswing'' (p. 117). At the same time, broader measures of inventive activity such as general U.S. patenting, as well as general investment in capital, tend to move in tandem with output growth.

This model may also display absolute declines in output resulting from decreases in the capital stock. If they occur, such output declines will be observed at the end of a cycle, just before the next innovation, when research effort is at its highest. Essentially, the research push just before the innovation is financed in part from resources that would otherwise be devoted to replacing depreciated capital. It is important to recall that the model does not adopt Schumpeter's assumption of creative destruction; the innovation does not make any part of the capital stock obsolete. The output declines that we may nevertheless observe as implications of the

[12] Asymmetries in unemployment rates over the cycle have also been noted. Neftci (1984) suggested that unemployment rates are asymmetric in the sense that consecutive increases in that rate are more likely than consecutive declines. Sichel (1989) corrected a mistake that reversed Neftci's results, although his own subsequent work, Sichel (1993), as well as others', like Rothman (1991), use different methods that uncover similar asymmetries to those found by Neftci.

[13] This approach originated with Kydland and Prescott (1982). An extensive and useful recent survey may be found in Cooley (1995).

model stem instead from the model's central feature that physical capital and research effort must compete for limited resources.

6. DECENTRALIZATION OF THE SOCIAL OPTIMUM

The social planner problem was solved for its relative tractability. We show here that its solution is equivalent to the allocation that emerges from a competitive equilibrium. A secondary purpose of this section is to demonstrate that, unlike many earlier models of growth through research and development, the equilibrium implications of our model do not depend on any special assumptions of externalities or imperfect competition.

Let us define a competitive equilibrium with reference to three distinct groups of optimizing, price-taking agents: households, manufacturing plants, and research firms, each of measure 1.

Households create capital and knowledge. Knowledge is accumulated by households until converted into an innovation of $a_j = A_j - A_{j-1}$ units of technology by a research firm. In return for accumulated knowledge, a research firm gives each household a share in the ownership of the next innovation proportional to that household's contribution of knowledge. At the time of one innovation, research firms compete by announcing the date of the next innovation and the units of knowledge a household must contribute at that time. Households choose the research firm offering the date of the breakthrough that they prefer and the pace of their accumulation of knowledge. Households rent existing technological innovations and capital to manufacturing firms. Innovations generate positive rents because they can be withheld from any plant (are "excludable"). All firms maximize profits. Free entry ensures zero profits in equilibrium.

6.1. *Manufacturing Plants*

The competitive manufacturing plants rent capital and technology taking as given the respective rental rates π_t and θ_t. It follows that the demand of a representative manufacturing plant for these inputs will equate the marginal product of each input to its rental rate:

$$\pi_t = (1 - \alpha) A_t^{\alpha} K_t^{-\alpha}, \tag{6.1}$$

$$\theta_t = \alpha A_t^{\alpha - 1} K_t^{1 - \alpha}. \tag{6.2}$$

As usual for a constant returns-to-scale production function, there are zero profits when inputs are paid their marginal product. Notice that by assumption no innovation is essential for production. In particular, a technology unit associated with the jth innovation is as good as a technology unit associated with the mth innovation (although these innovations differ in the number of such technology units that they contain). Consequently, competition in the market for technology will emerge when there exist many innovations, and will drive the technology rental rate per unit to its marginal product.[14]

6.2. *Research Firms*

Competition among potential research firms will force the research firms to offer shares in the ownership of the next innovation equal to the fraction of the required knowledge (H^*) supplied by households.

The research firm will also announce a date T_j at which the innovation will be accomplished. (Because the research firm earns zero profits at any such date, it will choose the date that maximizes household utility, effectively making the choice of this date part of the household decisions.)

6.3. *Households*

Again it will be helpful to divide time into cycles. The representative household chooses consumption $C_j(\tau)$, research effort $D_j(\tau)$, capital per plant $K_j(\tau)$, and the time of the next innovation t_j taking as given the capital rental rate $p_j(\tau)$ and the innovation rental rate $\theta_j(\tau)$ paid by each plant so as to maximize its lifetime utility. The household's budget constraint at moment τ of the jth cycle may then be written as

$$\dot{K}_j(\tau) = \pi_j(\tau)K_j(\tau) - \delta K_j(\tau) + \theta_j(\tau)A_j - C_j(\tau) - D_j(\tau). \quad (6.3)$$

The budget constraint takes into account that all existing innovations that make up the current technology A_j are owned and rented out by the household. The stock of research is given as before by

$$H_j(\tau) = \int_{s=0}^{\tau} h_j[D_j(s)]\, ds. \quad (6.4)$$

[14] The indivisible nature of innovations might be seen to imply that the pricing of innovations at their marginal product is an approximation to the competitive equilibrium in which new technologies are rented in a spot market. The most recent innovation is a more-than-infinitesimal $(1 - 1/\gamma)$ fraction of the total technology. While the rates of arrival of innovations may be increased if recent technology is overpriced in a less than fully competitive equilibrium relative to the social planner solution, this will not alter the cyclic implications of the model. Moreover, oligopoly profits are likely to induce other forms of markets such as research firms contracting in advance with the firms who will rent the new technology. While such questions of industrial organization are interesting in their own right, we wish to keep the focus of this paper on the cyclic and growth properties of the model.

A representative household (when the number of households is normalized to 1) owns A_j units of technology in the jth cycle, and is subject to the cycle's endpoint conditions:

$$H_j(T_j) = H^*, \tag{6.5}$$

$$K_j(T_j) = K_{j+1}(0). \tag{6.6}$$

The Hamiltonian function associated with this maximization problem may be written as

$$\begin{aligned} &\Gamma\left[\tau, T_j, C_j(\tau), D_j(\tau), K_j(\tau), H_j(\tau)\right] \\ &\quad = e^{-\rho\tau} u\left[C_j(\tau)\right] \\ &\qquad + \mu_{j1}(\tau)\left\{K_j(\tau)\left[\pi_j(\tau) - \delta\right] - C_j(\tau) - D_j(\tau)\right\} \\ &\qquad + \mu_{j2}(\tau) h_j\left[D_j(\tau)\right], \end{aligned} \tag{6.7}$$

for which $C_j(\tau)$ and $D_j(\tau)$ are the control variables, $K_j(\tau)$ and $H_j(\tau)$ are the state variables and T_j is the optimally chosen terminal time. The first order conditions are

$$\frac{\partial \Gamma}{\partial C_j(\tau)} = e^{-\rho\tau} u'\left[C_j(\tau)\right] - \mu_{1j}(\tau) = 0, \tag{6.8}$$

$$\frac{\partial \Gamma}{\partial D_j(\tau)} = -\mu_{1j}(\tau) + \mu_{2j}(\tau) h'_j\left[D_j(\tau)\right] = 0, \tag{6.9}$$

$$\dot{\mu}_{1j}(\tau) = -\frac{\partial \Gamma}{\partial K_j(\tau)} = -\mu_{1j}(\tau)\left[\pi_j(\tau) - \delta\right], \tag{6.10}$$

$$\dot{\mu}_{2j}(\tau) = -\frac{\partial \Gamma}{\partial H_j(\tau)} = 0, \tag{6.11}$$

$$\mu_{1j}(T_j) = e^{-\rho T_j} W'_{j+1}\left[K_j(T_j)\right], \tag{6.12}$$

$$\begin{aligned} 0 &= e^{-\rho T_j} u\left[C_j(T_j)\right] + \mu_{j1}(T_j)\dot{K}_j(T_j) \\ &\quad + \mu_{j2}(T_j) h_j\left[D_j(T_j)\right] - \rho e^{-\rho T_j} W_{j+1}\left[K_j(T_j)\right]. \end{aligned} \tag{6.13}$$

6.4. *Equivalence*

Using (6.1) and (6.2) to substitute the marginal products $(1-\alpha)A_t^{\alpha}K_t^{-\alpha}$ and $\alpha A_t^{\alpha-1}K_t^{1-\alpha}$ for the input prices π_t and θ_t in (6.3) and (6.10) demonstrates that the equilibrium conditions (6.1)–(6.13) satisfy the conditions (2.18)–(2.23) defining the solution to the planner's problem.

7. CONCLUDING REMARKS

7.1. *Infrastructural Examples*

There are numerous examples of big research or infrastructural projects that are characterized by huge investments and relatively long development periods, where most of the benefits occur only after the project is complete. In some cases, an extended development or construction period is imposed by technological limitations. Yet, in most of them, this period is endogenous in the sense that higher investment can make it considerably shorter. The actual development period of big projects varies depending on their nature. Still, the underlying trade-off between faster development of these projects and higher investment in existing technologies remains an important feature that has been left out of most models of endogenous technological development. We briefly offer in this section just a few motivating examples of such innovations in the economic sectors of transportation, communication, medical science and energy.

Work on the first line of 3.75 miles of the Metropolitan Railway, London's original underground subway company, began in 1860, using "cut-and-cover" methods, and the line began operating in 1863. (The Paris Metro's first line of 6.25 miles also took 3 years to become operational.) The line was an immediate success, carrying 9.5 million passengers in its first year of operation and had more than a local economic effect. The availability of this form of mass transit system enabled London to cope effectively with its inherited urban structure which was otherwise unsuited for becoming a world center for commerce and industry.

The railroads system in the U.S. developed over time at a rate that reflected both its cost and perceived benefits. Actual construction of the rail network in the U.S. began in 1828. The first line became operational in 1830, and at the end of that year the country had a total of 23 miles of railroads in operation. Five years later the system had 1100 miles (average annual rate of 220 miles), and by 1848, the system had 6000 miles (average annual rate 330 miles), with virtually all of it in states along the Atlantic seaboard. With general prosperity and the news of the California gold strike, railroad construction rose to the average rate of 3220 miles per year through the 1850's. In the 1880's construction boomed to at an average of more than 11,260 miles per year. Expansion at a lower rate continued until 1910. It is difficult to quantitatively evaluate the impact of the railroad network on economic development in the U.S., although its enormous effect can hardly be doubted. Braudel (1992) compared the volume of goods carried inland in Germany in the late 18th century and in 1913, and comes up with a factor of 130, which he views as evidence of the way railways opened up the country (p. 350). (Of course, many additional factors surely contributed to that growth.)

Telegraph systems using wires proliferated in Europe and the United States in the 1840's. Subsequently extending that service overseas required additional investments and improvements. It took some 20 years from the first successful laying of a submarine cable between Dover and Calais in 1851 for most of the major cities of the world to be connected by telegraph.

The Genome project of sequencing and mapping the 3 billions base-pair human genome is an example of a huge research project conducted by an international consortium, involving billions of dollars and thousand of scientists. The rate at which this project proceeds is almost proportional to the amounts invested in it, since the sequencing requires tedious work and expensive equipment, but involves almost nothing except data generation. The immediate benefits from that project include the ability to cope with major diseases such as tuberculosis, meningitis, and cholera, and the development of new drugs, particularly against antibiotic-resistant strains.

The construction of the Aswan High Dam on the Nile river extended over the period 1960–1971, and its cost was about $1 billion, a whopping 4% of the Egyptian annual GDP at the time. (A third of the cost was borne by the Soviet Union.) When completed, the hydroelectric capacity of the dam (2100 megawatts) was capable of supplying 60% of the Egyptian annual electricity consumption, prevented for the first time in history the annual floods of the Nile, and enabled irrigation of hundreds of thousands of new acres. Dams of similar magnitudes elsewhere (e.g., Hoover on the Colorado river, Grand Coulee Dam on the Columbia river) were typically constructed over 5–10 years.

The search for alternative sources of energy, although active for decades, received a big push in the 1970's following the 1973 oil crisis. The plummeting of oil prices has subsequently reduced the resources allocated to that research program, but the rate at which oil reserves are depleted all over the world makes renewed big future investments in that research a certainty.

These examples share several features. First, they involve big benefits that cannot be exploited before most of the project is complete. In some cases, such as Alaska's Pipeline or the Aswan High Dam, there are no gains whatsoever before the full completion of the project. In other cases, such as the railroad network, some benefits can occur gradually with the scope of the project, but most of the external and indirect benefits require its completion. Second, the examples involve a relatively long development period of several years. Third, the precise length of the development period is directly affected by the amounts invested in the project and in its associated refinements, long after most of the scientific and technological uncertainties have been resolved.

It is often difficult to draw the line between what we would call implementation costs of a project, and what other authors call adoption costs [see Jovanovic (1996), Greenwood and Yorukoglu (1996)]. What we emphasize in this paper are the cyclical and growth implications of the optimal funding of big and indivisible projects whose largest payoff is at the completion of the project.

7.2. *A Summary*

Our model is based on two basic assumptions: (i) that a significant fraction of advances in productivity do not fall from the sky but result from expensive and cumulative efforts undertaken in the expectation of future returns; and (ii) that some of these advances discretely increase the productivity of a substantial fraction of the capital stock. These innovations to the production process may take the form of new knowledge or improvements in an economy's physical infrastructure. The goal of this paper is to work out the equilibrium implications of these assumptions for the composition, shape, and duration of economic fluctuations in an endogenously growing economy.

Despite our citation of certain business cycle facts consistent with our model, we wish to be agnostic about the frequency at which an innovation-driven cyclical pattern of growth might show itself. It may be, for example, that large economy-wide innovations in knowledge or infrastructure affect economic performance at a longer frequency than those studied by real business cycle models using data detrended by a Hodrick−Prescott filter.

To focus on the implications that result from our ''Schumpeterian'' assumption of large technological advances, we have set aside many other plausible sources of economic fluctuations. Introducing shocks to government expenditures, oil prices, or the size of technological improvements will generate richer patterns of economic fluctuations but would cloud the role of endogenous discrete technological improvements. Costs or delays in adopting innovations or obsolescence of vintage technologies will likewise influence the pattern of the cycle. Sorting out the relative importance of alternative sources and channels of fluctuations at a variety of frequencies will require empirical methods beyond the scope of this stylized version of our model. What our model illustrates is the potential of inherently cyclical economic activity over time, stemming solely from changes in the relative profitability of alternative forms of investment, even absent other sources of fluctuations.

APPENDIX A: EXISTENCE OF STATIONARY SOLUTIONS

The Bellman equation associated with the social planner problem in (3.26) is

$$w(k) = \max_{T,k'} \{ v(T,k,\gamma k') + e^{-\rho T}\gamma^{1-\sigma} w(k') \}, \tag{A.1}$$

where the cycle-specific discounted utility is defined by

$$v(T,k_0,k_T) = \max_{c(\cdot),\,d(\cdot),\,k(\cdot)} \left\{ \int_{t=0}^{T} e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma}\, dt \right\} \tag{A.2}$$

subject to

$$\begin{aligned} \dot{k}(t) &= k(t)^{1-\alpha} - \delta k(t) - c(t) - d(t), \\ \dot{H}(t) &= d(t)^{1-\phi}/(1-\phi), \\ k(0) &= k_0, \\ k(T) &= k_T, \\ H(0) &= 0, \\ H(T) &= H^*, \end{aligned} \tag{A.3}$$

where the initial technology A_0 normalized to 1, and $t_0 = 0$.

We show that under certain mild conditions, the right-hand side of (A.1) is a contraction mapping in the space of relevant bounded functions so that it has a fixed point $w(\cdot)$.

Let $\mathscr{K} \equiv [k_{\min}, k_{\max}]$ and let $f(\cdot)$ and $g(\cdot)$ be any two continuous function from $\mathscr{K}$ to $\mathscr{K}$.[15] Let $v(T,k_0,k_T)$ be a given continuous function for $T > 0$, $k_0, k_T \in \mathscr{K}$, strictly increasing and concave in T and k_0, and strictly decreasing in k_T, with $v(0,k_0,k_T) = 0$ for any k_0, k_T. (Utility over a cycle increases at a decreasing rate the longer is the interval over which H^* needs to be accumulated and the larger is the initial capital stock, while it decreases with the required end-of-cycle capital stock.)

[15] Note that in the ‘‘technology scaled’’ version of the problem, (A.1)–(A.3), the existence of a maximal sustainable capital stock can be established in the standard manner.

Define the operator φ on such functions by

$$\varphi \circ f(k) = \max_{T, k'} \{ v(T, k, \gamma k') + e^{-\rho T} \gamma^{1-\sigma} f(k') \} \tag{A.4}$$

subject to

$$k' \in \mathscr{K}, \quad T \geq 0. \tag{A.5}$$

We show that φ is a contraction mapping by showing that it satisfies two sufficient conditions (Blackwell's Conditions):

(a) Monotonicity

Suppose $f \geq g$ [i.e., $f(k) \geq g(k)$, $\forall k \in \mathscr{K}$]. Let (T_f, k_f) and (T_g, k_g) attain $\varphi \circ f$ and $\varphi \circ g$, respectively, for some arbitrary given $k \in \mathscr{K}$. Then,

$$\begin{aligned} \varphi \circ g(k) &= v(T_g, k, \gamma k_g) + e^{-\rho T_g} \gamma^{1-\sigma} g(k_g) \\ &\leq v(T_g, k, \gamma k_g) + e^{-\rho T_g} \gamma^{1-\sigma} f(k_g) \quad \text{(since } g \leq f\text{)} \\ &\leq v(T_f, k, \gamma k_f) + e^{-\rho T_f} \gamma^{1-\sigma} f(k_f) \quad \text{(by definition)} \\ &= \varphi \circ f(k). \end{aligned} \tag{A.6}$$

(b) Discounting

Here we have to show that there exists $\beta \in (0,1)$ such that for any constant $x \geq 0$,

$$\begin{aligned} &\max_{T, k'} \{ v(T, k, \gamma k') + e^{-\rho T} \gamma^{1-\sigma} [f(k') + x] \} \\ &\quad \leq \max_{T, k'} \{ v(T, k, \gamma k') + e^{-\rho T} \gamma^{1-\sigma} f(k') \} + \beta x. \end{aligned} \tag{A.7}$$

Let $T_{\min}$ be the time it takes to accumulate H^* from $k_0 = k_{\max}$, devoting all resources to that purpose, so that c and $\dot{k}$ are both zero. Clearly, any optimally chosen T will exceed $T_{\min}$. Therefore, setting $\beta = e^{-\rho T_{\min}} \gamma^{1-\sigma}$ will satisfy this condition for any $x \geq 0$. However, we also require that $T_{\min} > (1 - \sigma)/\rho \ln \gamma$ so that $e^{-\rho T_{\min}} \gamma^{1-\sigma} < 1$. This, in turn, restricts H^* from below, given k_0, σ, γ, and ρ.

This proves the existence of a function $w : \mathscr{K} \to \mathscr{K}$ which satisfies (A.1).

Let $k' = \psi_1(k)$ and $T = \psi_2(k)$ be the optimal policy attaining w. Since $\psi_1(\cdot) : \mathscr{K} \to \mathscr{K}$, and is continuous, it has a fixed point k^* in $\mathscr{K}$, so that $k^* = \psi_1(k^*)$, $T^* = \psi_2(k^*)$, and

$$w(k^*) = v(T^*, k^*, \gamma k^*) + e^{-\rho T^*} \gamma^{1-\sigma} w(k^*). \tag{A.8}$$

Q.E.D.

APPENDIX B: NUMERICAL SOLUTION TO THE "WITHIN CYCLE" PROBLEM

The first order necessary and sufficient conditions for solving the "within-cycle" problem, (A.2), together with the resource constraint, (3.9), and the definition of knowledge accumulation, (3.10), imply the following system of differential equations in (c, d, k, H) as functions of elapsed time during the cycle, $t \in [0, T]$:

$$\frac{\dot{c}}{c} = ((1-\alpha)k^{-\alpha} - \delta - \rho)/\sigma, \tag{B.1}$$

$$\frac{\dot{d}}{d} = \frac{\sigma}{\phi}\frac{\dot{c}}{c} - \frac{\rho}{\phi}, \tag{B.2}$$

$$\dot{H} = d^{1-\phi}/(1-\phi), \tag{B.3}$$

$$\dot{k} = k^{1-\alpha} - \delta k - c - d, \tag{B.4}$$

together with the given endpoint values:

$$H(0) = 0, \tag{B.5}$$

$$H(T) = H^*, \tag{B.6}$$

$$k(0) = k_0, \tag{B.7}$$

$$k(T) = k_T, \tag{B.8}$$

$$T \text{ given}. \tag{B.9}$$

This is a "two point boundary problem," where some of the initial values are missing, and are replaced by given end point values. We use a standard routine [a modified version of the code provided in Press et al. (1992)] to solve this problem numerically. The solution includes the optimal consumption path, $c(\tau)$, $0 \le \tau \le T$, the end values of which to be denoted by $c(0; k_0, k_T, T)$, and $c(T; k_0, k_T, T)$, and the indirect utility $v(T, k_0, k_T) = \int_{\tau=0}^{T} e^{-\rho\tau}(c(\tau)^{1-\sigma})/(1-\sigma)\, d\tau$.[16]

The solution to the social planner problem requires us also to solve for the optimal cycle length. In addition, on a stationary solution, the capital stock and consumption grow by the factor γ across consecutive cycles. We therefore proceed as follows to solve for the stationary optimal solution. For a given k_0 we set $k_T = \gamma k_0$, and then solve the problem over a fine grid for T, the cycle length. With the values of $v(T, k_0, \gamma k_0)$ computed

[16] The "shooting" method for solving a "two point boundary problem" is extremely sensitive to initial conditions and parameter values. Consequently, the process is rather tedious and can be only partially automated.

over this grid, we use the first order condition for optimally choosing T in (A.1) to select the "optimal" cycle length for these end values of the capital stock. This condition is

$$0 = \frac{\partial v}{\partial T}(T, k_0, k_T) - \rho e^{-\rho T}\gamma^{1-\sigma}w(k_T). \tag{B.10}$$

Instead of the unknown function $w(\cdot)$ we use (3.27), assuming that we are on the optimal path. The partial derivative in (B.10) is evaluated numerically, and we denote the value of the "optimal" T which satisfies this equation by $T(k_0)$. We now let k_0 vary over a fine grid, and look for a value k^* at which the optimal consumption path satisfies

$$c[T(k^*); k^*, \gamma k^*, T(k^*)] = \gamma \cdot c[0; k^*, \gamma k^*, T(k^*)]. \tag{B.11}$$

This procedure gives us the stationary values $k_0 = k^*$, $k_{T^*} = \gamma k^*$, and T^* for the cycle-free version of the social planner problem. Using (3.24) we can then compute the paths of endogenous variables for all cycles.

As a check on the correctness of this procedure for identifying the stationary solution for the cycle-free problem, we ran the "two point boundary problem" solver routine on a problem with the end of cycle values as initial conditions. Getting the same cycle length, together with end of cycle values for k and c which were γ times larger than the beginning of the cycle, verified our procedure.

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