Redeem or Revalue?
Some Public-Debt Calculus

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Abstract

This paper studies the fiscal-monetary response to a sharp increase in the level of the public debt. To that end, we employ a general equilibrium model with distortionary income tax, distortionary financing, and endogenous capital accumulation. The model is calibrated to the US and EU economies. A main result is that in both economies the QE is superior, welfare-wise, to other policy prescriptions to the problem of explosive debt. A major difference between the EU and the US is that a Taylor rule of tight monetary and fiscal policy could reduce the US public debt, but given the fundamental properties of the EU economy, this policy cannot achieve this goal in Europe.

JEL Codes: E44; E47; E58; E63; H30; H63;

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1 Introduction

Levels of public debt in developed countries increased considerably in recent years. The ratio of government debt to GDP in the US rose from 64.8% in 2007 to 104.3% in 2015 (US Bureau of Public Debt). During the same time period, this ratio increased in the Euro area from 64.9% to 93.5% (Eurostat). The corresponding numbers for Japan are 162.4% in 2007 and 229.2% in 2015 (Ministry of Finance Japan). The current long economic stagnation makes GDP growth a less viable way to reduce the debt ratio. This leaves three channels that can potentially reduce debt levels. Fiscal actions that will raise tax revenue and reduce public spending, outright default, and buyback.

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This paper addresses the question: how should governments control their debt levels once it takes an explosive path? Instead of focusing on either the role of fiscal policy or the role of monetary policy in separation, this paper examines jointly the roles of monetary and fiscal policies in the context of public debt management. Our key contribution is to study the issues of debt control in a realistic environment which includes distortionary taxes, distortionary financing, and endogenous capital accumulation. In this setup, we provide policy prescriptions for the US and the EU economies for reducing their public debts. The prescriptions that we obtain for these two economies are quite different.

There are recent empirical investigations of possible ways to reduce public debt. Naturally, the emphasis is on inflation. Most papers conclude that the erosion effect of inflation is mild. Hilscher et. al. (2014) study U.S. data, and finds that given the current structure of debt maturity, the likelihood of a major impact of inflation on public debt is very low. Extending the maturity structure can reverse this result. Abbas et. al. (2013) consider 26 episodes of large debt reversals in advanced economies since the 1980s. They conclude that inflation was not an important factor in these cases. Fiscal policy, on the other hand, played an important role. Studying U.S. debt reductions from 1941 to 2009, Hall and Sargent (2010) derive a similar result. Finally, the same conclusion was made by Giannitsarou and Scott (2008), using VAR methodology on G7 countries from 1960 to 2005.

There are empirical studies with a different result regarding inflation and debt. Akitoby et. al. (2014) study G7 countries and conclude that raising inflation to 6% for five years would reduce the debt-to-GDP ratio by about 11%-14%. Reinhart and Sbrancia (2011) investigate the combined effect on public debt of (anticipated or unanticipated) inflation and financial repression in the form of tight regulation of the financial sector. Looking at historic episodes of the Bretton Woods era in advanced economies, they conclude that the combining inflation and financial repression had a major eroding effect on government debt.

Theoretical studies of possible ways to erode public debt include the work of Krause and Moyen (2013) who employ a new Keynesian DSGE model with a given maturity structure and uncertain inflation targets. The authors conclude that permanent adjustments to inflation targets, will have significant effect on the level of public debt. Aizenman and Marion (2011) consider an infinite-horizon economy characterized by maturity of debt, share of debt held by foreigners, and share of debt indexed to inflation. Their model predicts that a 6% inflation could reduce the U.S. debt-to-GDP ratio by 20% within four years. In an earlier paper in the spirit of Barro and Gordon (1983), Missale and Blanchard (1994) observe that credible commitment to low inflation implies that high levels of public debt yield shorter maturity.

Another theoretical approach is the Fiscal Theory of Price Level [for a comprehensive review see Leeper and Leith (2016)] which relaxes the assumption that inflation is determined solely by the central bank. Instead both fiscal and monetary authorities play a role. In the works subsequent to Leeper (1991), coordination between monetary and fiscal policy might yield high inflation. Under this view, the fiscal theory and the quantity theory are parts of a more general theory of price-level determination in which monetary and fiscal policies always interact with private-sector behavior to produce the equilibrium aggregate level of prices. Within a certain parametric family of monetary and fiscal rules, the seemingly distinct perspectives arise from different regions of the policy parameter space. Some "families" of monetary and fiscal rules, can potentially solve debt problems with relatively low costs in terms of social welfare measurements. One of our goals is to point out to such policy prescriptions.

The structure of the paper is as follows: Section 2 describes an economy with liquidity constrained consumers, and a government that imposes distortionary income taxes. In section 3 we calibrate the model to the US and the EU economies in order to obtain all the monetary-fiscal regimes that can bring about a unique rational-expectations equilibrium. In section 4 we obtain prescriptions to reduce the levels of public debt from 100% to 60% in the EU and the US economies within the regimes that are feasible to each economy. Section 5 concludes.

2 A Model with Public Debt and Distortionary Financing

2.1 The Households Sector

The economy is closed and populated by a continuum of identical infinitely long-lived households, with measure one. The representative household enjoys consumption and inelastically supplies its labor endowment, so its lifetime utility is given by

$$U_t = \int_t^{\infty} e^{-\rho s} u(c_s) ds \tag{1}$$

where $\rho > 0$ denotes the rate of time preference, c_s denotes consumption per capita, and $u(\cdot)$ is twice differentiable, strictly increasing, strictly concave, and satisfies the usual Inada conditions. Production takes place in a competitive sector via a constant-returns-to-scale production technology $f(k_t)$ where k_t denotes per-capita capital which depreciates at a rate δ. Finally, $f(k_t)$ is concave and twice differentiable.

Money enters the economy via a liquidity constraint on all transactions. Let m_t denote the per-capita stock of money denominated in the consumption good, and let ν denote money velocity. Then, a requirement that $\int_t^{t+\frac{1}{\nu}} [c(s) + I(s)]\, ds \leq m_t$ formalizes the liquidity constraint where I_t denotes per-capita investment and $\frac{1}{\nu}$ can be interpreted as the length of the period for which the representative household holds liquidity. A first-order approximation to the liquidity constraint gives the usual expression

$$c_t + I_t \leq \nu m_t \tag{2}$$

We assume that the government has access only to distortionary taxation and that deficits in the government's budget are financed via bond creation. As a consequence, the representative household's budget constraint becomes

$$c_t + I_t + \dot{b}_t + \dot{m}_t = (R_t - \pi_t) b_t - \pi_t m_t + (1 - \tau_t) f(k_t) + T_t \tag{3}$$

where $\tau_t \in [0, 1]$ is a flat income tax rate, b_t is a real measure of the stock of non-indexed government bonds, R_t is the nominal rate of interest, π_t is the rate of inflation, and T_t is a real lump-sum transfer. Capital accumulates according to

$$\dot{k}_t = I_t - \delta k_t. \tag{4}$$

Altogether, the household maximizes her lifetime utility given by (1) subject to the constraints (2)-(4), with a borrowing constraint such that $lim_{t\to\infty} a_t^H e^{-\int_0^t [R_s - \pi_s] ds} \geq 0$ where $a_t^H \equiv b_t + m_t$ is the real measure of the household's financial wealth. Each household chooses the time path $[(c_t, I_t, m_t)]_{t=0}^{+\infty}$ so as to maximize her lifetime utility, taking as given the initial stock of capital k_0, the initial stock of financial wealth a_0^H, and the time path $[(\tau_t, T_t, R_t, \pi_t)]_{t=0}^{+\infty}$ which is exogenous from the household's viewpoint. The necessary conditions for an interior maximum are

$$u'(c_t) = \lambda_t(1 + \frac{1}{\nu} R_t) \tag{5a}$$

$$\mu_t = u'(c_t) \tag{5b}$$

$$\zeta_t = \frac{1}{\nu} R_t \lambda_t \tag{5c}$$

$$\zeta_t(\nu m_t - c_t - I_t) = 0; \zeta_t \geq 0 \tag{5d}$$

where λ_t, μ_t are time-dependent co-state variables interpreted as the marginal valuations of financial wealth and capital, respectively, and ζ_t is a time-dependent Lagrange multiplier associated with the liquidity constraint.

Restricting attention to positive nominal interest rates, equations (5c)-(5d) imply that ζ_t is positive, which in turn implies that the liquidity constraint is binding. Second, and after substituting $m_t = \frac{1}{\nu}(c_t + I_t)$ and $a_t^H = b_t + m_t$ into equation (3), the state and co-state variables must evolve according to

$$\dot{\lambda}_t = \lambda_t [\rho + \pi_t - R_t] \tag{6}$$

$$\dot{\mu}_t = -\lambda_t(1 - \tau_t) f'(k_t) + (\rho + \delta) \mu_t \tag{7}$$

$$\dot{k}_t = I_t - \delta k_t \tag{8}$$

$$\dot{a}_t^H = (R_t - \pi_t) a_t^H + (1 - \tau_t) f(k_t) + T_t - (c_t + I_t) \left(1 + \frac{1}{\nu} R_t\right) \tag{9}$$

Solving equation (9) yields that the household's intertemporal budget constraint is of the form

$$
\begin{aligned}
0 \;\leq\; & lim_{t\to\infty} a_t^H e^{-\int\limits_0^t [R_s-\pi_s]ds} = \\
& a_0^H + \int\limits_0^\infty e^{-\int\limits_0^t [R_s-\pi_s]ds} \left[(1-\tau_t)f(k_t) + T_t - (c_t+I_t)\left(1+\frac{1}{\nu}R_t\right)\right] dt
\end{aligned}
$$

and the condition that her intertemporal budget constraint holds with equality yields the transversality condition

$$
lim_{t\to\infty} a_t^H e^{-\int\limits_0^t [R_s-\pi_s]ds} = 0. \tag{10}
$$

Equations (6) – (10) fully describe the optimal decision making of the representative household for whom the time path $[(\tau_t, T_t, R_t, \pi_t)]_{t=0}^{+\infty}$ is exogenously given.

2.2 The Government

The government consists of a fiscal authority and a monetary authority. The consolidated government prints money, issues nominal bonds, collects taxes to the amount of $\tau_t y_t$ where y_t is output, and rebates to the households a real lump-sum transfer T_t. The government's instantaneous budget constraint, denominated in dollars, is therefore given by $R_t B_t + P_t T_t = \dot{M}_t + \dot{B}_t + P_t \tau_t y_t$, where P_t is the nominal price of a consumption bundle, $\dot{M}_t$ and $\dot{B}_t$ are net changes in the money and bond supply, respectively, and R_t is the nominal interest paid over outstanding debt. Let $a_t^G \equiv \frac{M_t}{P_t} + \frac{B_t}{P_t}$ denote the real value of all government's liabilities. Dividing both sides of the nominal budget constraint by P_t and rearranging yields that (the real) government liabilities evolve according to

$$
\dot{a}_t^G = \underbrace{(R_t - \pi_t)\, a_t^G}_{\text{interest payments on the debt}} - \underbrace{R_t m_t}_{\text{seigniorage}} + \underbrace{T_t - \tau_t y_t}_{\text{primary deficit}} \tag{11}
$$

where $\pi_t \equiv \frac{\dot{P}_t}{P_t}$ is the rate of inflation. Equation (11) shows that deficits are financed via increments to government debt. All in all, government liabilities increase with the primary deficit and with the real interest paid over outstanding debt, and decrease with seigniorage. Note that in eq. (11) T_t, τ_t, R_t denote the instruments of fiscal and monetary policies. To allow scrutiny of first-order consequences of policy changes, we take what is by now the conventional approach in the literature and consider simple rules. In particular, we assume that monetary policy follows the interest rate feedback rule,

$$R\left(\pi_t\right) = \rho + \pi^* + \alpha(\pi_t - \pi^*) \qquad \text{where} \qquad \alpha > 0, \tag{12}$$

where a monetary rule that exhibits $\alpha > 1$ is called an active monetary policy, while $\alpha < 1$ corresponds to a passive monetary policy.[1] We also assume that the fiscal authority sets its policy instruments according to rules which embed two features. First, there may be some automatic stabilizer component to movements in fiscal variables. This is modeled as a contemporaneous response to deviations of output from the steady state. Second, the income-tax rate is permitted to respond to the state of government debt. Altogether, the fiscal authority sets the income-tax rate according to

$$\tau(y_t, a_t^G) = \tau^* + \beta\frac{y_t - y^*}{y^*} + \gamma\frac{a_t^G - a^*}{a^*} \qquad \text{where} \qquad \beta, \gamma \geq 0 \tag{13}$$

and y^*, a^* are long-run output and a debt target, respectively.[2] Finally, and to keep the model simple, we assume an exogenous path for lump-sum transfers $T_t = T^*$.

2.3 General Equilibrium

In equilibrium, a) the goods market clears

[1] See Leeper (1991).

[2] This rule is consistent with much of the empirical literature. Papers who emphasize fiscal rules include Bi (2012), Bi and Traum (2012), Bi et. al. (2013), Leeper and Yang (2008) and Leeper et. al. (2010). See also Battaglini and Coate (2008) and Barseghyan et. al. (2013) who arrive at similar conclusions from a political economic perspective.

$$f(k_t) = y_t = c_t + I_t, \tag{14}$$

b) the money market clears

$$m_t = \frac{1}{\nu}\left(c_t + I_t\right), \tag{15}$$

and c) the assets market clears $a_t \equiv a_t^G = a_t^H$.

Using the monetary policy rule and the fiscal rules, imposing market clearing conditions, and assuming that the elasticity of intertemporal substitution in consumption is constant, we obtain that in equilibrium the aggregate dynamics satisfy the following ODE system:

$$\frac{\dot{c}_t}{c_t} = \sigma\left\{\left[\frac{1-\tau(f(k_t), a_t)}{1+\frac{1}{\nu}R\left(\pi_t\right)}f'(k_t) - \delta\right] - \rho\right\} \tag{16}$$

$$\dot{\pi}_t = \frac{\nu + R\left(\pi_t\right)}{\alpha}\left\{\left[R\left(\pi_t\right) - \pi_t\right] - \left[\frac{1-\tau(f(k_t), a_t)}{1+\frac{1}{\nu}R\left(\pi_t\right)}f'(k_t) - \delta\right]\right\} \tag{17}$$

$$\dot{k}_t = f(k_t) - c_t - \delta k_t \tag{18}$$

$$\dot{a}_t = \left[R\left(\pi_t\right) - \pi_t\right]a_t + T_t - \left[\tau(f(k_t), a_t) + \frac{1}{\nu}R\left(\pi_t\right)\right]f(k_t). \tag{19}$$

Equation (16) is an Euler equation, where $\sigma > 0$ denotes the elasticity of intertemporal substitution in private consumption. In our economy the marginal product of capital is distorted by the income-tax and liquidity constraints. Notice that with no distortions equation (16) becomes the familiar Ramsey-type Euler equation. Equation (17) was obtained by taking a time derivative from the first-order condition ($5a$) and substituting in equation (6). It corresponds to a Fisher equation in which the nominal rate of interest varies with expected inflation and the real rate of interest. Since capital and bonds are perfect substitutes, at the private level, the net return on capital investments should equal the real interest received from holding the financial asset. Of course, this condition is satisfied in the steady state but not necessarily at every instant. Along an equilibrium path, if these returns are not equal, the difference must arrive from an expected change in inflation. Finally, equations (18)-(19) were obtained by substituting market clearing conditions (14)-(15) into equations (8)-(9). So, we can characterize an equilibrium, in our model, as a set of time paths $\left\{[(c_t, \pi_t, k_t, a_t, \tau_t, T_t, R_t)]_{t=0}^{+\infty}\right\}$ that satisfy (16)-(19) given $k_0, a_0 > 0$.

Steady State Equilibrium: In a steady state,

$$f'(k^*) = (\rho + \delta) \frac{1 + \frac{1}{\nu} R^*}{1 - \tau^*}, \tag{20}$$

where τ^* denotes a long-run income-tax rate and R^* is a steady-state rate of interest. Note the distorting effect of income taxes and interest rates on long-run output as the marginal product of capital increases with both distortions. From equations (17) and (20), R^* must satisfy

$$R^* = \rho + \pi^* \tag{21}$$

where π^* is the long-run rate of inflation. Equation (18) implies that the steady-state consumption is

$$c^* = f(k^*) - \delta k^*. \tag{22}$$

Finally, equation (19) shows that in a steady-state equilibrium, government liabilities must satisfy $a^* = \frac{1}{\rho}\left[f(k^*)(\tau^* + \frac{1}{\nu}R^*) - T^*\right]$. Let $\widetilde{a}^* \equiv \frac{a^*}{f(k^*)}$, $\widetilde{T}^* \equiv \frac{T^*}{f(k^*)}$, denoting debt/GDP and transfers/GDP in the steady state, respectively. So, a sustainable debt must satisfy

$$\widetilde{a}^* = \frac{1}{\rho}\left[\tau^* + \frac{1}{\nu}R^* - \widetilde{T}^*\right]. \tag{23}$$

An important implication of this model is that the selection of long-run tax and inflation rates determine steady state output and consumption. However, the long-run level of public debt has no direct role in determining long run output. Eq. (23) shows that in a steady state that is determined by τ^* and π^*, adjustments to the long run debt/GDP can be made as long as transfers/GDP are adjusted accordingly. As a result, the government should have three policy targets. We assume throughout that the targets are $(\tau^*, \pi^*, \widetilde{a}^*)$, and that this is common knowledge. Note that proclaiming a debt/GDP target immediately implies, via eq. (23), the size of primary deficit that is sustainable in the steady state.

3 The various regimes in the policy parameter space

In our model economy, the slope of the government revenues schedule with respect to the tax-rate is related to the degree to which a tax cut is self-financing. We define the rate of self financing of a tax cut as the ratio of additional tax revenues due to general equilibrium effects and the lost tax revenues due to the tax cut. The degree to which a tax cut is self-financing, denoted by $\mathcal{RSF}$, is calculated as

$\mathcal{RSF} = 1 - \frac{1}{f(k^*)} \frac{d\left[f(k^*)(\tau^* + \frac{1}{\nu}R^*)\right]}{d\tau^*}$

where $f(k^*)(\tau^* + \frac{1}{\nu}R^*)$ are total tax revenues in the steady state. For example, if there were no endogenous changes in allocations following a tax change, the loss in tax revenue due to a one-percentage-point reduction in the tax rate would be one percent of $f(k^*)$, and the self-financing rate would calculate to 0. Similarly in a non-monetary economy, at the peak of the income-tax Laffer curve, tax revenue would not change at all in the wake of a one-percentage-point reduction in the tax rate, and the self-financing rate would be 1. This self-financing rate would become larger than 1 beyond the peak of the Laffer curve. Note, however, that in our economy seigniorage is a source of revenue. Thus, tax cuts may affect seigniorage revenues via general equilibrium effects. All in all, we find that the rate of self-financing near the steady state depends on the elasticity of tax revenues, the tax-rate target, and the inflation target, and reads

$$\mathcal{RSF}^* = 1 - \frac{\varphi^* - 1}{\tau^*}\left[\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}\right]. \tag{24}$$

where $\varphi_{(R_t)} \equiv \frac{\partial \ln(\tau_t y_t)}{\partial \ln(\tau_t)} = 1 + \frac{\partial \ln(y_t)}{\partial \ln(\tau_t)}$ is the marginal revenue generated from an increase in taxes holding the nominal interest rate constant.[3] We can thus conclude that the long-run rate of self-financing is increasing with the long-run rates of nominal-interest and income-tax. In an accompanying paper[4] we obtain a condition that is crucial to understanding the extent

[3] The second term is negative as higher taxes decrease output, so the elasticity of tax revenue with respect to tax rates is less than one. In this economy $y_t = f(k_t)$, accordingly $\varphi_{(\tau_t, y_t)} = 1 + \frac{\tau_t}{f(k_t)}\frac{\partial f(k_t)}{\partial \tau_t} = 1 + \frac{\tau_t}{f(k_t)} f'(k_t) \frac{dk_t}{d\tau_t}$. Implementing the implicit function theorem on eq. (20) yields that $\frac{dk^*}{d\tau^*} = \frac{1}{1-\tau^*}\frac{f'(k^*)}{f''(k^*)}$, and therefore near the steady state $\varphi_{(\tau^*)} = 1 + \frac{\tau^*}{1-\tau^*}\frac{\left[f'(k^*)\right]^2}{f(k^*)f''(k^*)}$. Assuming that production technology is of the form $f(k_t) = k_t^{\epsilon}, \epsilon < 1$, we get $\varphi_{(\tau^*)} = 1 + \frac{\tau^*}{1-\tau^*}\frac{\epsilon}{\epsilon - 1}$.

[4] See Gliksberg (2016).

to which the government is able to stabilize its debt. Consider the expression $\rho\widetilde{a}^{*}\frac{d\ln a_t}{d\ln y_t}$ that shows the rate of growth of debt/GDP when the economy is not in the steady state. Also consider the expression $(1-\mathcal{RSF}^{*})\frac{\tau^{*}}{1-\varphi^{*}}$ which is the slope of the government's revenue schedule, taking into account the general equilibrium effect of tax increases on all sources of revenue [i.e., tax collections and seigniorage]. Whenever the economy resides in an environment such that the debt/GDP grows faster than the government's ability to raise revenues via tax increases, monetary policy must come into play so as to restore fiscal solvency. Where the government has access only to distortionary taxation, tax revenues become a feature of equilibrium. In this case, output, inflation, and the tax rate are determined simultaneously in equilibrium, and the government cannot fully control its revenues. In such an environment, distorting taxes bring about a natural limit to revenue growth. As a result, the government is unable to finance its commitments entirely through direct tax collections. The bottom line is that three regimes exist in our model economy. The main characteristics of these regimes are provided in Table 1:

Table 1 - Regimes under distortionary financing

Regime	Feasibility	Monetary policy	Fiscal policy
Taylor-rules	$\rho\widetilde{a}^{*}\frac{d\ln a_t}{d\ln y_t}<(1-\mathcal{RSF}^{*})\frac{\tau^{*}}{1-\varphi^{*}}$	$\alpha>1$	$\gamma>\rho\widetilde{a}^{*}$
QE	$\rho\widetilde{a}^{*}\frac{d\ln a_t}{d\ln y_t}>(1-\mathcal{RSF}^{*})\frac{\tau^{*}}{1-\varphi^{*}}$	$\alpha<1$	$\gamma>\rho\widetilde{a}^{*}$
Debt devaluation	$\rho\widetilde{a}^{*}\frac{d\ln a_t}{d\ln y_t}>(1-\mathcal{RSF}^{*})\frac{\tau^{*}}{1-\varphi^{*}}$	$\alpha<1$	$\gamma<\rho\widetilde{a}^{*}$

In regimes that feature Taylor rules, deficit-financed tax cuts or spending increases do not affect aggregate demand because the private sector expects the resulting increase in government debt to be exactly matched by future tax increases or spending reductions. This statement is backed by the feasibility condition which shows that there is enough space for fiscal policy to stabilize its debt via tax hikes. This fiscal behavior relieves monetary policy of fiscal financing concerns, freeing the central bank to target inflation. As a result, under a Taylor rule, the central bank responds aggressively to deviations of inflation from its long run level by increasing the nominal interest rate more that the increase in inflation - thus bringing about an increase in the real rate of interest.

In the QE regime the government is committed to stabilize public debt (specifically γ is greater than $\rho\widetilde{a}^*$). As a result, debt shocks are expected to cause future changes in either fiscal or monetary policies. Fiscal policy is designed to allow the fiscal authority to fund debt increases via tax collections. However, if taxes are already high, or if the debt level is too high, the government's ability to finance its commitments through taxes is limited. In the QE regime, since households expect that either the government will cut back on some of its promised transfers or that inflation will be above its target, Ricardian equivalence continues to hold even after a debt shock. In such events, in order to reduce the level of public debt back to its level prior to the shock the central bank absorbs government bonds via open market operations. This instrument has at least three merits: first, it reduces the amount of government debt held by the private sector hence preventing it from taking an explosive path; second, exchanging bonds for money increases money supply, thus increasing government revenues via seigniorage; finally, the monetary stance allows the fiscal authority to service its debt at lower costs.

Finally, in the regime that brings about debt devaluation, the fiscal authority is committed to smoothing the income tax rate. As a result, the income-tax rate responds only weakly to changes in government debt ($\gamma < \rho\widetilde{a}^*$), and fiscal expansions are financed via debt increases. The fiscal expansion is supported by an appropriate monetary policy which increases current demand for goods and drives up the price level. However, letting government liabilities take an explosive path is inconsistent with equilibrium. The commitment to stabilize the tax rate excludes the possibility of restoring equilibrium via tax increases. As a result debt revaluation comes to play. An important characteristic of such regimes is that they arrive from a policy choice, and that they are feasible even when the income tax is low! In such regimes, a commitment to stabilize the tax rate may cause a breakdown of Ricardian equivalence. As a result, government debt must devalue to restore equilibrium.

To gain intuition with respect to the bounds of the regimes in advanced economies, we calibrate the model at an annual frequency to the parameters of the US and EU economies. We set the annual (subjective) rate of time preference to 0.02, and the elasticity of intertemporal substitution to 2. Following the results of Trabandt and Uhlig (2011), we calibrate the elasticity of production technology, ϵ, so as to induce maximal capital tax rates of 0.63 and

0.48 for the US and EU-14 economies, respectively, and the elasticities of tax revenues of 0.5 and 0.2 for the US and EU economies, respectively. We set money velocities in the steady state so as to correspond to the US M2 and the EU M2 money velocities in October 2013.[5] An overview of the calibration is provided in Table 2:

Table 2 - Structural parameters and Calibrations

Parameter	Description	US	EU
ρ	Subjective rate of time preference (%, annual)	2	2
σ	Elasticity of intertemporal substitution	0.5	0.5
δ	Rate of capital depreciation (%, annual)	7	7
β	Tax response to output	1	1
$1-\epsilon$	Maximal tax rate (%)	63	48
φ^*	Elasticity of tax revenues	0.5	0.2
ν	M2 money velocity	1.5	0.97

There are two main differences in the structural parameters of the two economies. First, the production technology in the EU is more capital intensive than in the US. This implies that the maximal income tax rate in the EU is lower than in the US.[6] Second, an implication of the smaller maximal tax rate in Europe is that the European income-tax Laffer curve is flatter than the US curve. This, together with higher income tax rates in Europe implies that the elasticity of tax revenues in Europe is lower than in the US.

[5] Sources: (I) Federal Reserve Bank of St. Louis - Velocity of M2 Money Stock, Ratio, Quarterly, Seasonally Adjusted. (II) Eurostat and ECB calculations.

[6] Specifically, it can be shown that in a real economy with CRS production technology with inelastic labor supply, the maximal tax rate is one minus the capital share in procduction. See a similar argumant in Trabandt and Uhlig (2011) with endogenous labor supply.

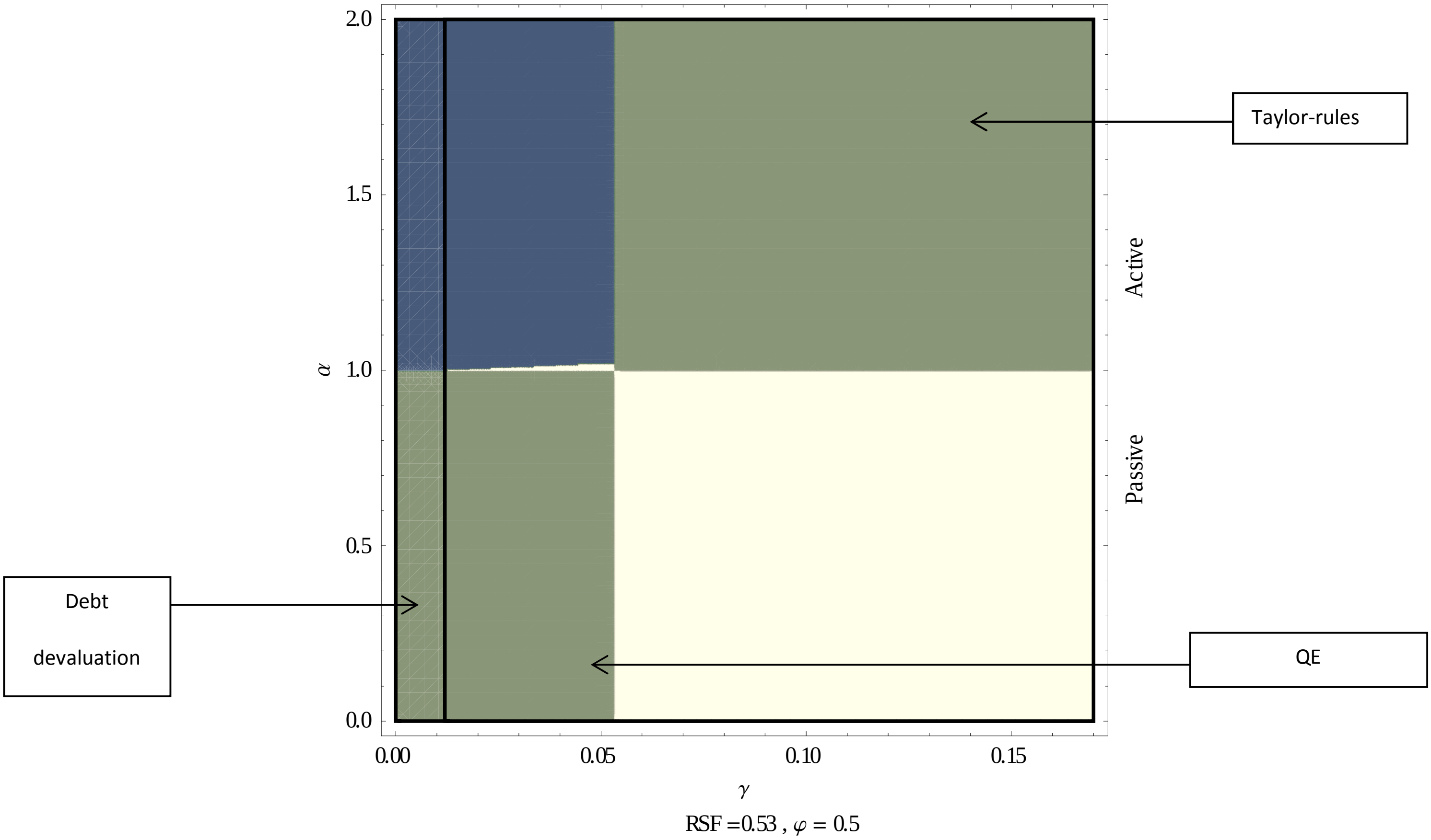

Figure 1: Phase portraits in the policy parameter space - calibration for the US economy.

(Light grey area – Unique equilibrium; Dark Blue area - no equilibrium; White area – multiple equilibria)

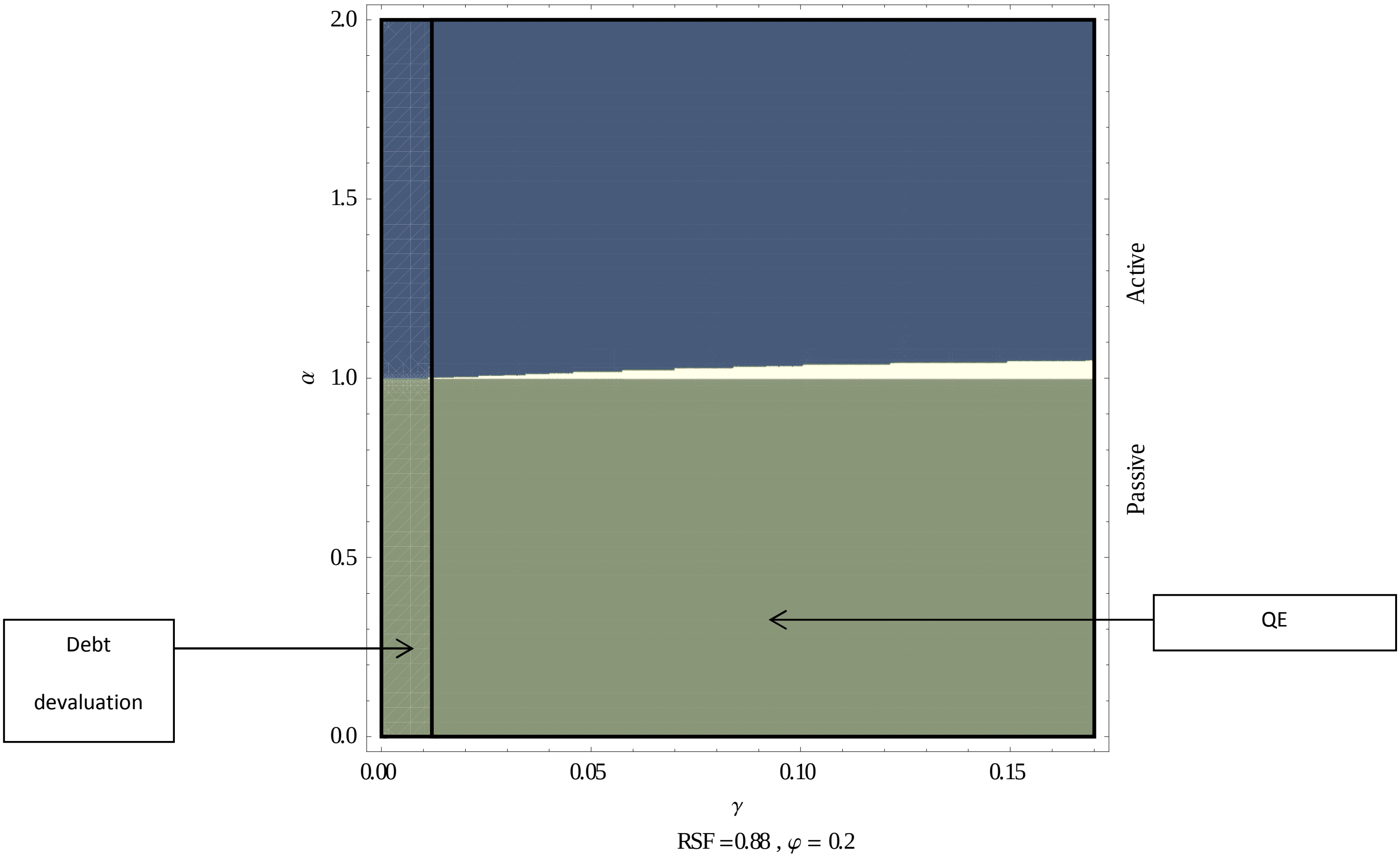

Figure 2: Phase portrait in the policy parameter space - calibration for the EU.

(Light grey area – Unique equilibrium; Dark Blue area - no equilibrium; White area – multiple equilibria)

Figures 1 and 2 show the phase portraits of the US and EU economies, respectively. The horizontal axis measures the magnitude of income tax responses to percentage deviations of public debt from its long run level. The vertical axis measures the magnitude of nominal interest responses to deviations of inflation from its long run level.

In the context of implementable regimes, we find striking differences between the two economies. Notice figure 1. In the US economy, the conditions for a unique equilibrium, based on eigenvalues, split the determinacy regions in the (γ, α) space into two distinct spaces, shown by figure 1 as the grey areas. In addition, the demarcation line that indicates a violation of the government's budget constraint – the vertical line at $\gamma = \rho\widetilde{a}^*$ – further splits the fiscal conditions when monetary policy is passive into two different regimes, debt devaluation and QE.

Figure 1 shows that in the US there are three distinct implementable regimes, though not necessarily efficient. Each regime takes a non-negligible size in the parameter space, and corresponds to one of the regimes provided in Table 1. The interpretation is that given a fiscal shock, the government can implement either Taylor rules, or debt devaluation regimes, or QE regimes, in the sense that they all induce a unique rational expectations equilibrium which is consistent with a bounded public debt. In particular, all the three regimes are expected to stabilize public debt in the sense that they induce a convergence of public-debt levels back to their proclaimed target.

We obtain different results for the EU economy. Notice figure 2. In the EU, the conditions for a unique equilibrium, based on eigenvalues, bring about one space shown by figure 2 as the grey area. In addition, the demarcation line that indicates a violation of the government's budget constraint further splits the fiscal conditions when monetary policy is passive into debt devaluation and QE. The absence of a Taylor rule regime from the phase portrait of the EU is striking. The interpretation is that the structural parameters of the EU economy and the high income taxes bring about a flat revenue schedule. When the slope of the revenue schedule is low, a regime based on an active monetary stance may become inconsistent with a rational expectations equilibrium with a bounded public debt. In such cases, government responses to a debt shock, via active monetary stances will only exacerbate the fiscal conditions and lead public debt into an explosive path. We conclude this section with the observation that given the current structural parameters in the EU only regimes that emphasize a passive monetary stance are implementable. Thus, when experimenting with policies designed to control public-debt, we should consider only policies that reside either in the debt devaluation or the QE regimes.

4 Quantitative Exercises

We now study the dynamic effects of reducing public debt under various scenarios. We compare equilibrium paths and consequent welfare gains under three regimes. Consider an economy that resides at a steady state x^* corresponding to the calibration provided in Table 2. Let $U^* = \frac{1}{\rho}u(c^*)$ denote a measure of the representative household's welfare in this steady state. We measure welfare gains as $\widehat{U} \equiv \frac{U_0 - U^*}{U^*}$, where U_0 measures welfare according to eq. (1) along the (unique) trajectory that goes back to x^* following a debt shock. It is straightforward to obtain a first order approximation $\widehat{U} \simeq \frac{u'(c^*)c^*}{u(c^*)}\rho \int_0^\infty e^{-\rho t}\frac{c_t - c^*}{c^*}dt$, where $[c_t]_{t=0}^{+\infty}$ is the time path of consumption along the equilibrium trajectory, and $\frac{u'(c^*)c^*}{u(c^*)} \leq 1$, is a constant. Hence, welfare gains are proportional to the present value of percentage deviations of consumption from its level long run level. Accordingly, we rank different policy experiments by their respective measures of $\widehat{U}$.

We simulate impulse responses, in the US and EU under implementable regimes, to an overnight increase of public-debt to GDP levels from 60% to 100%. An overview of the main results is provided in Table 3:

Table 3 - Scenarios of reducing public debt

	QE		Taylor rule		Debt Devaluation	
	US	EU	US	EU	US	EU
Welfare gains (%)	0.85	0.36	-0.07	NA	-0.27	-0.098
$\tilde{a}_0$	1	1	1		1	1
$\tilde{a}^*$	0.6	0.6	0.6		0.6	0.6
φ^*	0.5	0.2	0.5		0.5	0.2
$\pi^*(\%)$	2	2	2		2	2
$\tau^* - \widetilde{T}^*$(%, implied)	-1.47	-2.92	-1.47		-1.47	-2.92

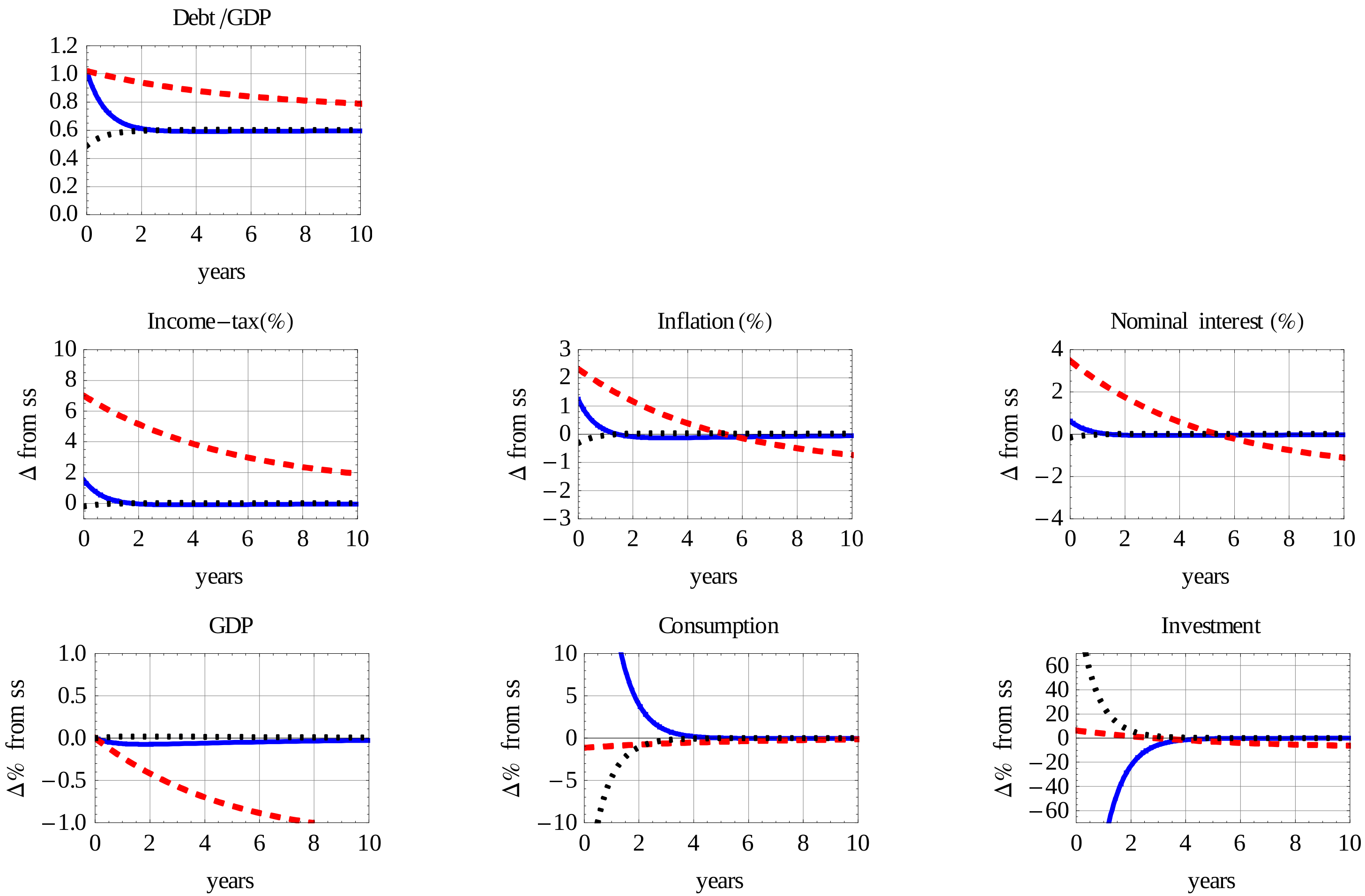

Figure 3: Responses to a public-debt shock in the **US** economy.

The solid blue lines are responses under QE, dashed red lines are responses under a Taylor-rule, and dotted black lines are responses with debt devaluation.

In the US, there are three implementable regimes. The Taylor rule regime is characterized by an aggressive monetary response to inflation ($\alpha > 1$) that interacts with a strong response of the tax rate to public debt ($\gamma > \rho\widetilde{a}^{*}$). The consequent dynamics of this policy is a slow adjustment of public debt to its long run target. On impact, the fiscal authority raises the tax rate in response to the debt shock. However, current and future tax hikes are not enough to bring about surpluses that will eventually reduce public debt to its target. This is where inflation must kick in in order to produce enough seigniorage revenues. But this causes the central bank only to further increase the nominal interest rate in order to fight inflation. Since the main instrument of the central bank in our model is the rate of nominal interest, adhering to the Taylor rule, the central bank must issue new debt so as to bring about higher nominal interest. All in all, under the regime that emphasizes Taylor rules, the central bank slows the process of debt reduction because a) the service of the debt is now more expensive, and b) it continuously issues new debt in order to keep real interest at high levels. The upshot of this scenario is a combination of tight monetary and fiscal policies. This combination sends the economy to a recession, while private consumption drops as a result of a negative intertemporal substitution effect. We know that welfare gains are proportional to the present value of percentage deviations of consumption from its level long run level. Hence the attempt to reduce public debt via Taylor rules induces welfare losses.

The dynamics of public debt under a regime that induces debt devaluation is quite different. Under this regime, tax rates respond very weakly to the state variables. Since the long-run targets (debt, inflation, tax rate) before and after the shock are the same, with weak policy responses equilibrium is restored via debt devaluation. In our model, since we assumed nominal debt, the debt devaluation is achieved by an instantaneous price jump. The rapid adjustment to the debt level is consistent with a mild response of the policy instruments, and this is an equilibrium outcome. All in all the most noticeable result is the effect on consumption. Debt devaluation has an instantaneous negative effect on households wealth and as a result consumption and welfare drop.

The QE regime emphasizes tax increases combined with tolerance towards inflation. This is the only regime that delivers considerable welfare gains from reductions to public debt. In this regime the central bank wants to bring about only weak responses to inflation ($\alpha < 1$). To prevent the nominal interest from increasing as a result of rational expectations, the central bank must repurchase part of the debt in the open market. This reduces the debt to GDP ratio. All in all, via QE the central bank speeds the process of debt reduction because a) the cost of debt service decreases, and b) it continuously purchases debt from the households in order to keep real interest at low levels. Consumption now increases as a result of a positive intertemporal substitution effect and all in all this scenario delivers welfare gains.

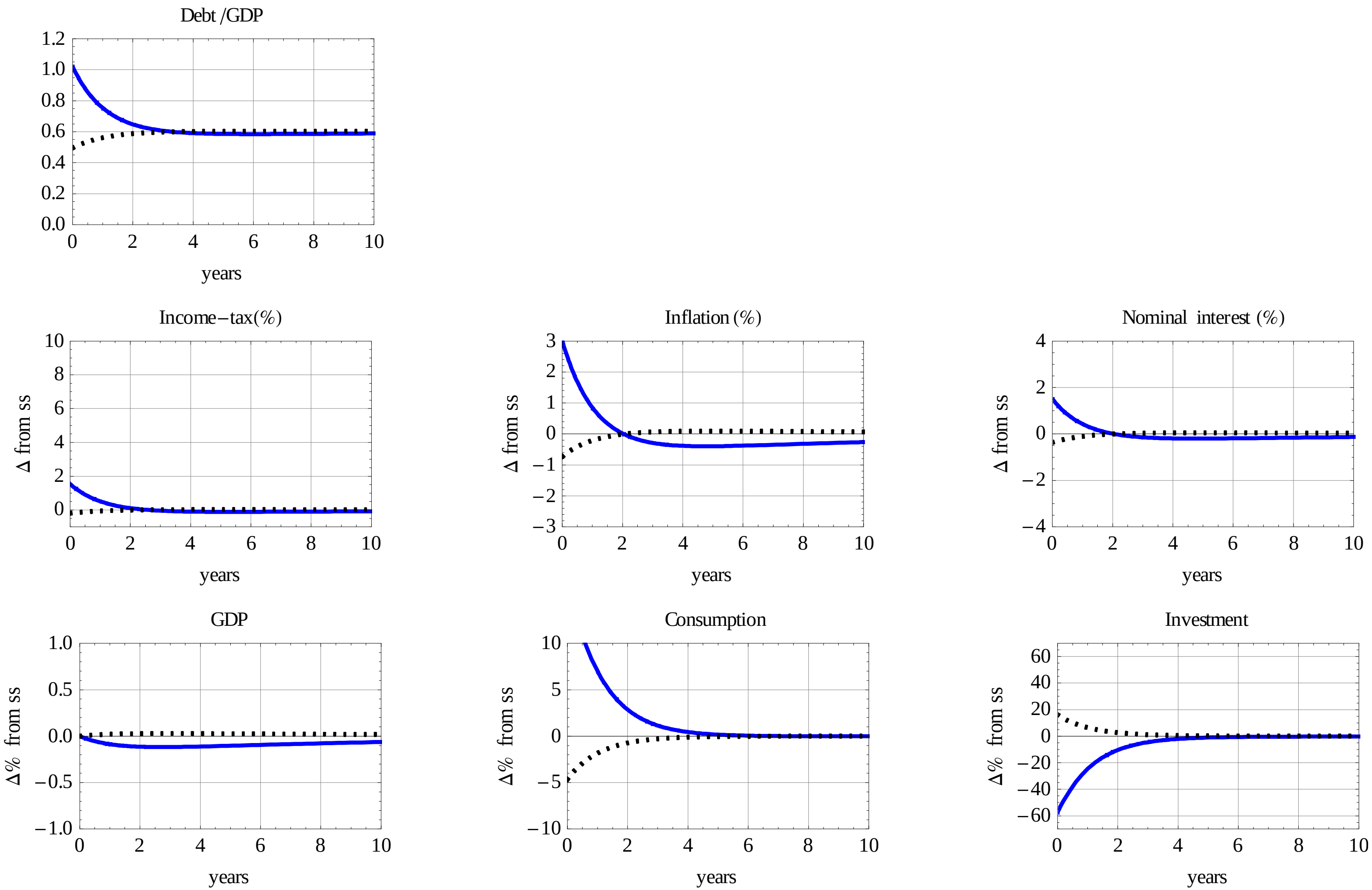

Figure 4: Responses to a public-debt shock in the **EU**.

The solid blue lines are responses under QE, dotted black lines are responses with debt devaluation.

In the EU, there are two implementable regimes, QE and debt devaluation . Under a regime that emphasize debt devaluation tax rates respond very weakly to the debt shock. So, equilibrium is restored via a price jump that brings about a debt devaluation. With this negative effect on households wealth, consumption and welfare drop.

By contrast, the QE regime emphasizes mild and transitory tax increases combined with tolerance towards inflation. In this regime there is a temporary tax hike and, more importantly, the ECB responds only weakly to inflation. To prevent the nominal interest from increasing, the ECB must repurchase part of the debt in the open market. This reduces the debt to GDP ratio and temporarily increases inflation. Via QE the central bank achieves two ends: it speeds the process of debt reduction, and it induces a positive intertemporal substitution effect. All in all, this scenario for the EU increases consumption and delivers welfare gains, while letting public debt converge to its level prior to the shock.

5 Concluding Remarks

This paper derives the dynamics of major macroeconomic variables when the debt to GDP ratio converges from 100% to 60%. The response to the debt shock depends on the mix of fiscal and monetary policies that attempt to cut the excessive debt. There are three possibilities: a price shock that devalues the nominal debt, a tight fiscal-monetary mix as in a Taylor rule, and expansionary monetary policy in the form of QE. Calibration of the model to structural parameter values that characterize the EU economy finds out that a Taylor rule is not a feasible response. The basic rationale of this result is a flat tax revenue curve generated by high income tax rates. For the US economy, on the other hand, a Taylor rule is possible. However, both in the US and the EU, a QE policy delivers the best policy as measured by consumer surplus.

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