THE CHASE OF A MULTI-ARMED ECONOMIST FOR THE ELUSIVE SOCIAL DISCOUNT RATE

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Abstract. The social discount rate debate has been plagued by the use of inadequate one-agent models and vague analysis. The Weitzman-Gollier puzzle (resolution) is picked here to illustrate.
J.E.L. classification numbers: D61, H30. *Keywords:* policy evaluation, social discount rate, social welfare.

1. Introduction

Policy-makers have to regularly decide whether to finance public projects that are to last several years. To evaluate such a project the classical cost-benefit analysis suggests to calculate its net present value (NPV), i.e., to attach a value to the stream of its benefits and costs. Our job as economists is to give a clear rationale for this calculation and, if appropriate, suggest the social discount rate to be used. There is, probably, no better area of economic research where the old joke about the two-(or multi-)handed economist applies: suggestions are plentiful, often contradictory, and one can endlessly amuse himself by writing yet another "back-of-the-envelope" calculation "deriving" the next social discount rate. The main message here is to remind us that we have a decent analytic hand to deal with such policy questions, so we can leave the rest for fantasizing over coffee.

2. The representative agent fallacy

Owing, most likely, to the ingenious contribution of Ramsey (1928), our intuition heavily relies on the representative agent model. However, the problem of social discounting inevitably involves several generations, especially if projects are long-term, and therefore calls for a welfare analysis in a general equilibrium framework. Almost 90 years after Ramsey we are able to analyse evolution of equilibrium prices, even with uncertainty luring; we have thought long and hard about aggregating infinite streams of individual utilities (see [10], [1], [5] among

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others); besides, understanding the connection between the Kaldor-Hicks criterion (the basis for cost-benefit analysis) and welfare criteria became a part of an undergraduate curriculum, [4].

Yet, in the social discount rate debate we still hope that the (assumed) unique solution to the representative agent optimization problem will have the same allocation as all possible equilibria of the relevant economy. Moreover, in some popular models the time preferences of the phantom individual "pin down" the interest rate, which becomes the basis for our social discount rate calculation. Others go even further in ascribing human fallacies to the fictional agent, hyperbolic discounting, for example (making the whole optimization exercise fruitless, as the agent is not even consistent). What is next? Shall we design policies to help the poor phantom to learn faster how to play the prisoner's dilemma, stop over-generalizing based on small samples and be aware of tiny-probability events?

Justifying application of the NPV calculation requires spelling out the underlying objective, as a first step. Ducking away from the answer often leads to implicit ad-hoc assumptions that force the objective to "look like" the desired NPV, usually alluding to the utility of the phantom representative agent, if at all. If the objective is not formulated, any social discount rate is as good as any other. There is no intelligent way to proceed with the debate then. If the objective is the utility of the phantom, the approach has to be justified.

Further, if we are to do the job properly and aggregate individual responses to policies, we shall rely on equilibrium analysis. It is, as many would agree, harder than solving the optimization problem of a representative agent. But that does not mean that we have to search for all the answers under the same street light: having an ample body of knowledge enabling comparative statics in a variety of equilibrium models (see [3], [9], [2], [7] among others), we should be able to tackle the analysis.

And when we do search for the answers, we have to at least take ourselves seriously.

3. The vague smoothness and other technicalities

Not to overburden the reader with details of too many papers in the area, let me pick one stream of discussions that is in the "must-refer-to" list, namely, the Weitzman-Gollier puzzle and its resolution, [6]. The authors admit that both suggested somewhat heuristic arguments supporting the two contradicting suggestions (use the lowest/highest interest rate as a social discount one), so the resolution is to provide a formal argument clarifying the issue. Here is one possible interpretation of the model.

Time is discrete, the world starts a bit before time zero when the policy maker is to decide whether to undertake an investment that costs ε at time zero and pays δ at some future time t. The policy-maker knows that at time 0 uncertainty is to be realized and one of n states will prevail, determining marginal productivity of capital, which will remain constant forever after. There is one representative agent in the economy and a single consumption good which is produced using a linear technology from capital. Capital fully depreciates and there is no storage, all next year's capital has to be produced out of investment (= capital not consumed by the agent). One should guess that the initial stock of capital, K_0, is strictly positive and equal in all states.

According to the description, the agent must be an expected utility maximizer and can choose a contingent plan in each of n possible states. In each state his life-time utility V is defined on an infinite stream of consumption $\mathbf{C} = (C_0, C_1, \dots, C_t, \dots)$, $C_t \in \mathbb{R} \quad \forall t \in \mathbb{Z}$. There are no explicit assumptions imposed on consumption streams, no mentioning of the space they belong to, one can only guess that they probably have to satisfy some form of summability (when multiplied by an exponent?) since otherwise the implied resource constraint of the agent (defined only recursively in the paper as: $K_{t+1} = e^{r_i}[K_t - C_t]$ for a finite collection of $r_i > 0$ corresponding to each state) will become dubious and more explanation will be required to cover the cases of divergent series. We also learn that V "...is smoothly differentiable, is strictly concave, has positive first derivatives, and satisfies some kind of generalized Inada conditions that will guarantee unique interior solutions". The meaning of differentiability here is non-existent, since no definition of closeness for two arguments of the function (consumption streams) is given, i.e., no metric is provided for the function domain. Luckily, there is an example for the value of consumption, $V(\mathbf{C}) = \sum_{t=0}^{\infty} e^{-\rho t} U(C_t)$, with $\rho > 0$, and our only hope at this stage is that at least some ideas will become clear through it. U is assumed to be differentiable with $\lim_{x\to 0} U(x) = \infty$ and $\lim_{x\to\infty} U(x) = 0$. Before embarking on the search for the social discount rate with the societal objective being the agents' life-time utility (which is implicit in the analysis), the authors offer condition (11) that should describe the optimal consumption plan of the representative agent. Unfortunately, since the derivatives on neither side of the equation are well defined, it is rather uninformative. Luckily, it might remind one of the condition known (in the macro literature) as an Euler equation. Using the example, this condition should read (for strictly positive C_t and C_0)

$$U'(C_0) = U'(C_t)e^{(r_i-\rho)t}, \quad \rho > 0$$

for any state i. Does it describe a solution to the optimization problem?

If the productivity of capital, r_i, happens to equal ρ, the solution to the optimization problem of the agent is the classical consumption smoothing, $C_0 = C_t = C^*$ for all t. For a given $C^* > 0$ the capital path has to satisfy $K_{t+1} = e^{r_i}[K_t - C^*]$. It is easy to show that the optimal capital path has to be constant, $K_t = K_0$.[1] Thus, $C^* = K_0(1 - e^{-r_i})$.

However, if $r_i \neq \rho$, to satisfy the optimality condition, the derivative $U'(C_t)$ times $e^{(r_i-\rho)t}$ has to be a constant, hence the consumption path has to be chosen in such a way that $U'(C_t)$ depends exponentially on time and so has the form $Ae^{(\rho-r_i)t}$. However, this might require a consumption path which is not sustainable. Let us take a textbook example, $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma > 0$, which then implies $U'(C_t) = C_t^{-\sigma}$. Putting the two requirements together we get $C_t = Ae^{\frac{(r_i-\rho)t}{\sigma}}$, so, $A = C_0$. Re-write the capital path (feasibility) equation using $k_t = \frac{K_t}{C_t/C_0}$:

$$k_{t+1} = B[k_t - C_0], \text{ where } B = e^{r_i - \frac{r_i-\rho}{\sigma}} > 0 \tag{1}$$

If $B \leq 1$, or equivalently, $r_i\sigma \leq r_i - \rho$, then k_t has to be strictly decreasing, dropping each period by at least BC_0 as long as $k_t > 0$, and so it has to fall below $C_0 > 0$ at some finite time t for any initial stock $k_0 = K_0$, thus yielding strictly negative k_{t+1}. In other words, in this case the "interior solution" requires consumption to grow too fast and so there is no "interior solution". Thus, even to start working with this model, clear assumptions have to be put forward (as opposed to "some kind of generalized Inada conditions"). Besides, these are assumptions on the preferences of a fictitious agent in a model that, over the past decades, has, probably, generated more empirical "puzzles" than it solved.

But even if we put aside all the "technicalities" and conceptual problems mentioned above, further analysis does not become easier to digest. Recall that the policy to be evaluated here is a decrease in consumption today that generates the same benefit at time t, independent of a state of the world (productivity of capital). Moreover, our fictitious agent can not trade against the future increase in income or smooth

[1] Assume the contrary that, e.g., $K_{t+1} > K_t$ along the candidate optimal path for some t. Then it would have been possible to increase consumption in $t+1$ by the difference $K_{t+1} - K_t$ and then follow the candidate optimal path one period later. Similar improvement is possible for $K_{t+1} < K_t$, thus showing that the proposed consumption path was not optimal.

out the time-zero hit (although calculating an impulse response in such models is a standard exercise). The agent is not even allowed to (optimally) decide to save some of the benefit for future use, once the increase is observed. As a result, one is left to believe that the change in the utility of the agent due to this suboptimal change in the consumption plan has to somehow reflect the way the society evaluates a public project...

And this "...rigorous analysis..." is supposed to give us some insights into social discount rate and resolve the related controversy?!

Unless we, as economists, take our work seriously, we will remain the endless source of jokes, and so will our suggestions.

References

[1] d'Aspremont, C. (2007). Formal welfarism and intergenerational equity. In J. E. Roemer and K. Suzumura (Eds.), *Intergenerational Equity and Sustainability*, Number 143 in IEA Conference Volumes, pp. 113–130. Palgrave Macmillan.

[2] Chichilnisky, G. and Y. Zhou (1998). Smooth infinite economies. *Journal of Mathematical Economics 29*, 27–42.

[3] Debreu, G. (1976, May). Regular differentiable economies. *The American Economic Review 66*(2), 280–287. Papers and Proceedings of the Eighty-eighth Annual Meeting of the American Economic Association.

[4] Feldman, A. M. and R. Serrano (2006). *Welfare Economics and Social Choice Theory*. Springer.

[5] Fleurbaey, M. and P. Michel (2003). Intertemporal equity and the extension of the Ramsey criterion. *Journal of Mathematical Economics 39*, 777–802.

[6] Gollier, C. and M. L. Weitzman (2010). How should the distant future be discounted when discount rates are uncertain? *Economics Letters 107*, 350–353.

[7] Mertens, J.-F. and A. Rubinchik (2014). Essential properties of $L_{p,q}$ spaces (the amalgams) and the implicit function theorem for equilibrium analysis in continuous time. *Journal of Mathematical Economics 50*, 187–196.

[8] Ramsey, F. P. (1928, December). A mathematical theory of saving. *The Economic Journal 38*(152), 543–559.

[9] Shannon, C. and W. R. Zame (2002). Quadratic concavity and determinacy of equilibrium. *Econometrica 70*(2), 631–662.

[10] Zuber, S. and G. B. Asheim (2012). Justifying social discounting: The rank-discounted utilitarian approach. *Journal of Economic Theory 147*, 1572–1601.