DISCOUNTING AND WELFARE EVALUATION OF POLICIES

JEAN-FRANÇOIS MERTENS† AND ANNA RUBINCHIK‡

Abstract. If policy discounting is to have any welfare relevance, it must be a derivative of a social welfare function. If that derivative is to have a net present value (NPV) form, the baseline allocation must be stationary. Given a stationary baseline in an overlapping generations growth economy the inter-generationally fair discount rate under the relative utilitarian welfare function equals the growth rate of per-capita consumption, roughly, 2% for the U.S. This differs from the interest rate, even in the golden rule equilibrium unless population growth is null.

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1. Introduction

"Benefit-cost analysis is a primary tool used for regulatory analysis" is stated in Circular A-4 of the U.S. Office of Management and Budget [15, p. 2], which was developed to review and coordinate regulatory programs of the U. S. Federal agencies.[1] It is suggested there that a proposed program can be rationalized in several different ways:

> [...] you should try to explain whether the action is intended to address a significant market failure or to meet some other compelling public need such as improving governmental processes or *promoting intangible values such as distributional fairness* or privacy. (*Emphasis* is ours.)

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†CORE, Université Catholique de Louvain; 34, Voie du Roman-Pays; B-1348 Louvain-la-Neuve; Belgique.
‡Dept. of Economics; University of Haifa; Mount Carmel, Haifa, 31905; Israel. E-mail: arubinchik@econ.haifa.ac.il.
[1] In accordance with sect. 2(b) of Executive Order 12866, "Regulatory Planning and Review."

As is well-known,[2] the classical "efficiency" rationale used in CBA is the Kaldor-Hicks criterion, which is based on the total monetized net benefit, and thus is seemingly independent of the distribution of benefits. The same applies to projects that span several years, in which case the regulators are to use a discounted value of future net benefits, and hence to evaluate the net present value (NPV) of the monetized benefits using the interest rate for discounting.[3]

The Circular also encourages the regulators to analyze distributional effects of the proposed action, in addition to the standard CBA: "[b]enefits and costs of a regulation may also be distributed unevenly over time, perhaps spanning several generations."

Since it is impossible to make a consistent choice using any two distinct objectives (unless they are perfectly aligned), we offer yet another way to reconcile the efficiency and equity considerations with a single objective function that is consistent with the NPV calculation and that explicitly incorporates the distributional concerns, or at least, make it clear what distributional concerns *are* taken into account when the classical CBA criterion is applied (and no compensating transfers are made).[4]

First point is that any policy evaluation based on the Kaldor-Hicks criterion embeds *some* distributional judgment. And if we take utilitarian view, it pins down welfare weights, which are equilibrium-dependent. Such weights are not always easy to justify. Consider an example, which is somewhat contrived, but makes the point clear.

Example (A toy OG model). There are three different private goods and one public good. Preferences of all agents are represented by differentiable utility functions. The utility u_A of A, who belongs to the first generation increases in his apples consumption (c_1) and the size of the public park (g) and B's utility, u_B, from the last generation, increases in his consumption of pheasants (c_2) and decreases in g. There is also C, whose lifespan overlaps with A and B, he does not care for the park but enjoys both apples and pheasants, with marginal rate of substitution between the two being constant and equal to some $\theta > 0$. Utility of each individual is linear in the quantity of the third private good, for C a pheasant is worth a unit of the third good. Markets for the private goods are perfectly competitive, A owns all the apples and B has all the pheasants and C owns the third good, so the markets should clear at prices θ for apples and 1 for pheasants and the third good. A regulator considers a small expansion of the current park (g). Assume for now the direct costs are zero and the regulator knows that A is ready to give up no more than 5 apples for this project, whereas

[2] Cf. Mishan [13].

[3] U.S. Office of Management and Budget [15, p. 31-36].

[4] Merits of using an explicit welfare function for policy analysis has been advocated by Drèze and Stern [3].

B will have to be compensated by at least 2 pheasants if the park is expanded (as his hunting grounds would shrink). If the regulator is to simply monetize the net benefit using market prices (or perform what we call later a market evaluation), he should accept the project iff its value, $5\theta - 2$, is positive. In other words, if "marginal analysis" is applicable, the value of the project is $\frac{\partial u_A(\cdot)/\partial g}{\partial u_A(\cdot)/\partial c_1}\theta + \frac{\partial u_B(\cdot)/\partial g}{\partial u_B(\cdot)/\partial c_2}$, as 5 is the marginal rate of substitution ($\frac{\partial u_A(\cdot)/\partial g}{\partial u_A(\cdot)/\partial c_1}$) between park and apples for A and, $2 = \frac{\partial u_B(\cdot)/\partial g}{\partial u_B(\cdot)/\partial c_2}$. In a (baseline) competitive equilibrium consumers optimize, hence marginal utilities with respect to private goods consumed in strictly positive quantities are proportional to prices, so $\partial u_A(\cdot)/\partial c_1 = \theta\mu_A$ and $\partial u_B(\cdot)/\partial c_2 = \mu_B$, where μ_i is the marginal utility of income for agent i. Then the value of the project becomes

$$\frac{\partial u_A(\cdot)/\partial g}{\mu_A} + \frac{\partial u_B(\cdot)/\partial g}{\mu_B} \tag{1}$$

This value is equal to the derivative of utilitarian welfare if the welfare function is of the form

$$W = \frac{1}{\mu_A}u_A + \frac{1}{\mu_B}u_B \tag{2}$$

So, the welfare weights are the reciprocals of the marginal utility of income.[5] The absurd feature of this extreme example is that the weights are determined, in part, by the preferences of individual C (his parameter θ), who does not even care about the park: $\mu_A = \frac{\partial u_A(c_1,g)/\partial c_1}{\theta}$.

The above aspect is further explored in section 2.2. Next, in section 2.3, it is shown that imposing discounting (requiring welfare differential to have a discounting form) presupposes stationarity of the baseline allocation. To simplify the exposition, we consider just a transfer policy (or a private consumption variation which is equivalent to some regulation or a public good).

Finally, in section 3, we give our answer to the question, what should be a discount factor that incorporates both efficiency and equity in a competitive market economy? In doing so, we use our previous findings [10] to identify the discount factor that is consistent with welfare policy evaluation that incorporates general equilibrium effects, i.e., allowing agents to "re-trade" after the policy is enacted. We show that imposing interest rate as a social discount rate can generate equilibrium-dependent distributional implications.

Analyzing equilibrium effects of a policy is a daunting task on its own, as it entails solving a complex fixed point problem in the infinite-dimensional space of equilibrium paths (for infinite economies), and is therefore often avoided by considering instead a "representative agent"

[5] This on its own is not surprising, it is consistent with Kotlikoff [8, eq. (12)], a measure of aggregate incidence of a policy, which, in this example, is well-defined since the economy is finite.

model, thus reducing it to an optimization problem,[6] and losing thereby the potentially important equilibrium effects. Most importantly, in the latter case, time-preference parameters of the fictitious agent are often confused with welfare weights associated with different generations.

If one is to address long-term projects, hence inevitably involving some form of intergenerational transfers, the model should contain (overlapping) generations, thus, in particular, different agents. Using [10], one can overcome the difficulty of equilibrium evaluation and derive the discount rate on policies from the primitives of an OG economy, even with heterogeneous agents, in the neighborhood of a BGE — allowing thus to meaningfully address the discount rate debate. Here we demonstrate such an evaluation for a particular class of policies.

Proofs that are not in the text are in the appendices.

2. Basic results

2.1. **Discounting and monetization: definitions.** By and large, policy discounting, and in particular NPV calculation, is a weighted average of policy changes (over time). As policies here we take either (1) a set of endowment changes $\delta\omega_{x,s,\tau,i}$, where i is the index for physical goods, τ and s denote type and age on an individual, x denotes his age; or (2) a set of consumption changes, $\delta c_{x,s,\tau,i}$. Endowment changes represent transfer policies (actual changes in individual endowments). Changes of consumer bundles stand for either actual changes that occur after the policy is enacted, or, alternatively, consumers' willingness to pay (WTP) for a policy, i.e., potential changes. We measure the WTP in commodities, in order to avoid its dependence on arbitrary (say, period-by-period) price normalization. Otherwise, the definition is fully in the spirit of the Kaldor-Hicks criterion used in CBA.

In order to relate the standard policy evaluation to welfare analysis, we define a welfare function W on a set of policies described above.

First step in the analysis is to assume that the derivative of W with respect to policy exists,[7] so one can construct a "first order approximation" to the welfare change as a result of a change in policy. If so, the derivative has to be a weighted sum of the changes, ($\delta\omega_{t,s,\tau,i}$ in the first case), where the weights $(q_{t,s,\tau,i})$, in general, are unrestricted:

$$\delta W = \sum_{t,s,\tau,i} q_{t,s,\tau,i} \delta\omega_{t-s,s,\tau,i} \tag{3}$$

In the second case, the derivative, similarly, is the weighted sum of the consumption changes.

[6] See, e.g., Arrow and Kurz [1].

[7] Verifying this assumptions might be a non-trivial task, but it is beyond the scope of the current paper, cf. Mertens and Rubinchik [11].

On the other hand, evaluating projects using discounting, including NPV form, allows only particular weights, and we study implications of these restrictions. To be more precise,

Definition 1. If the weights in equation 3 are anonymous, so that $q_{t,s,\tau,i} = q_{t,i}$, then we call the evaluation *distribution-neutral*, i.e., the value of the project depends only on the aggregates. If, further, those weights are the corresponding prevailing prices of the goods, $q_{t,i} = p_{t,i}$, it is a *market evaluation.*

Clearly, for long-term projects the goods are dated and so market evaluation is equivalent to an NPV with the interest rate as discount rate.

We allow for a *general interpretation of discounting*: it does not necessarily imply that the evaluation is distribution-neutral, neither does it require a constant discount factor. More precisely,

Definition 2. If the weights $q_{t,s,\tau,i}$ in equation 3 can be represented as a product of a time-independent factor $\kappa_{s,\tau}$ with a type-independent factor $H_{t,i}$, we say that the welfare differential, δW, has a *discounting form.*

2.2. **Distribution-neutral discounting imposes welfare weights.** Let us establish formally that calculating the market value of a policy is not "welfare free", rather, it corresponds to a very particular utilitarian welfare function.

First, it will be demonstrated for the simple case in which W is defined on the set of individual consumption changes, in this form one could reasonably call it a classical result (a "folk wisdom").

Proposition 1. *Consider an economy with differentiable utilities u_τ and utilitarian social welfare function (SWF) $W = \sum_\tau w_\tau u_\tau$, differentiable on a set of feasible consumption changes. Then the market value of these changes is the first order approximation to welfare W if and only if the welfare weights w_τ are the reciprocals of the marginal utilities of income.*

Proof. The baseline is a competitive equilibrium (consumption, production and prices), (c, y, p), of the economy. Every consumer τ chooses optimal bundle, so there exists $\mu_\tau > 0$ such that $D_c u_\tau(c, \cdot) \leq \mu_\tau p$, where equality holds for all goods consumed in strictly positive quantity, $i \in I_\tau$. Each consumer evaluates a public project in terms of equivalent consumptions changes $\delta c_{\tau,i}$ in terms of the goods he consumes, $i \in I_\tau$, so $\delta c_{\tau,i} = 0$ if $i \notin I_\tau$. Then the differential of welfare is

$$(4) \quad \delta W = \sum\nolimits_\tau w_\tau \delta u_\tau = \sum\nolimits_\tau w_\tau \sum\nolimits_i \frac{\partial u_\tau}{\partial c_{i,\tau}} \delta c_{i,\tau} = \sum_\tau w_\tau \mu_\tau \sum_i p_i \delta c_{\tau,i}$$

It equals the market value of consumption changes, $\sum_i p_i \sum_\tau \delta c_{\tau,i}$, if and only if w_τ is equal (or proportional) to the reciprocal of μ_τ. ■

Clearly, this argument does not require the economy to be finite: both the set of consumers and that of goods can be infinite, provided, of course, the derivative of welfare and the market value of the consumption changes are well-defined.

For practical purposes, however, it is much easier to calculate a market of value of a policy itself, take transfer policies for simplicity. When will such evaluation be a good approximation to welfare change?

Allowing the agents to re-trade the benefits complicates the problem and at this stage we formulate it for a finite economy satisfying a mild (Lipschitz) condition.

Proposition 2. *Consider a finite economy with differentiable utilities u_τ and utilitarian social welfare function (SWF) $W = \sum_\tau w_\tau u_\tau$, differentiable on a set of feasible endowment changes $(\delta\omega)$. Then the market value of $\delta\omega$ equals the derivative of welfare W at a locally Lipschitz equilibrium*[8] *if and only if the welfare weights w_τ are the reciprocals of the marginal utilities of income.*

In both claims the evaluation is *distribution-neutral*: the change in welfare depends only on the aggregates. It is also obvious that with any other welfare weights the derivative will not have the desired form, which implies that NPV is not even the first order approximation to the welfare change.

As mentioned above, expressing W as a function of willingness to pay in terms of *potential* transfers, as in proposition 1, easily rationalizes the evaluation of projects by the market value in an infinite economy, including the OG one. However, extending proposition 2 for the OG economy, is not immediate.[9] And this final step is crucial for the applicability of (at least transfer) policy analysis: calculating the value of the endowment change is clearly much easier than predicting the resulting consumption variation.

It follows that even to rationalize market evaluation a new approach is desirable.

Even more so, it is needed when the purpose is to investigate what discount rate corresponds to the concept of intergenerational equity.

[8] An equilibrium (allocation and prices) (c, y, p) of an economy with endowment ω is locally Lipschitz if $\exists K \exists \varepsilon > 0$: for any equilibrium (c', y', p') of the economy with endowment ω', $\|(c', y', p') - (c, y, p)\| \leq \varepsilon \Rightarrow \|(y', p') - (y, p)\| \leq K\|\omega - \omega'\|$, where $\|\cdot\|$ is a norm on $\mathbb{R}^n$. This is a weak form of regularity, requiring no smoothness.

[9] If one is to follow the steps of the argument above, reproducing the orthogonality condition (2) would need a detailed proof. Expressions like $\langle p, \delta c\rangle$ are not necessarily well-defined, the sums being potentially infinite, and the Lipschitz condition is nontrivial, as the production set for capital-investment is not smooth, being a linear subspace with non-negativity constraints.

2.3. **Discounting implies stationarity.** While market evaluation is so restrictive, the following simple model shows that any plausible form of discounting — short of *defining* it as the derivative of welfare — can be rationalized only at a stationary baseline allocation.

The model here is very simple, and intentionally so — we want to flash out the main difficulty of making welfare differential to be of a discounting form.

Time is discrete, $t \in \mathbb{Z}$. At any period t there are 2 individuals, each of a different type $\tau \in \{1, 2\}$, each lives a single period. There are 2 goods, $i \in \{1, 2\}$, and utilities U_τ are strictly concave, monotone and differentiable. Aggregate endowment $\Omega \in \mathbb{R}^2$ is constant over time. Policy is measured in terms of (bounded) changes in consumption, $\delta c_t \in \mathbb{R}^4$. Baseline consumption is $\bar{c}_t \in \mathbb{R}^4$, with $p > 0$ as supporting price vector.

Consider the classical utilitarian welfare function[10]

$$W(\delta c) = \sum\nolimits_{t=-\infty}^{\infty} e^{-\beta t} \sum_{\tau} w_\tau (U_\tau(\bar{c}_{t,\tau} + \delta c_{t,\tau}) - U_\tau(\bar{c}_{t,\tau})) \tag{1}$$

Proposition 3. *Assume the welfare function is differentiable at the baseline. If the differential has a discounting form, the baseline allocation is constant over time. If it is a distribution-neutral discounting, then, in addition, the baseline allocation is the welfare maximizer.*

Proof. By construction, the differential of W w.r.t. a change in policy is

$$\delta W(\delta c) = \sum\nolimits_{t,\tau,i} e^{-\beta t} w_\tau D_i U_\tau(\bar{c}_{t,\tau}) \delta c_{t,\tau,i} \tag{2}$$

where $D_i U_\tau(\bar{c}_{t,\tau})$ is the derivative of U_τ at the baseline allocation $\bar{c}_{t,\tau}$ with respect to consumption of good i. On the other hand, if welfare is differentiable,

$$\delta W = \sum_{t,\tau,i} q_{t,\tau,i} \delta c_{t,\tau,i} \tag{3}$$

so the weight associated with $\delta c_{t,\tau,i}$ is $q_{t,\tau,i} = e^{-\beta t} w_\tau D_i U_\tau(\bar{c}_{t,\tau})$. By definition 2, the differential δW being of a discounting form implies there are functions (of type) κ_τ and (of time and commodity) $H_{t,i}$ such that $q_{t,\tau,i} = \kappa_\tau H_{t,i}$. Hence $H_{t,i} = e^{-\beta t} w_\tau \kappa_\tau^{-1} D_i U_\tau(\bar{c}_{t,\tau})$ is independent of τ, which implies, in particular, that $w_\tau \kappa_\tau^{-1} D_i U_\tau(\bar{c}_{t,\tau})$ is also independent of τ. Thus, the baseline consumption allocation, $\bar{c}$, is a distribution of the total endowment that equates weighted individual marginal utilities, hence, by concavity of U, it solves the problem

$$\max_{c \in \mathbb{R}^4_+ :\ \sum_\tau c_\tau = \Omega} w_\tau \kappa_\tau^{-1} U_\tau(c_\tau) \tag{4}$$

for every t. Therefore the consumption allocation is constant in time, by strict concavity.

[10]Subtracting the baseline utility to make sure W is well-defined.

The second assertion follows (using definition 2) by setting κ_τ to unity. ■

Even if one wants to extend definition 2, allowing the weights that are multiplicatively separable just in commodity and the rest of the parameters (hence allowing them to be functions of time and agent: type and birth-date), one would deduce similarly that $p_{t,i} = \pi_i H_t$: all $\bar{c}_t$ have the same supporting price π. This is compatible with a non-constant baseline only in degenerate cases; e.g., for CES utility functions $U_\tau(c_{t,\tau}) = (\theta_\tau c_{t,\tau,1}^{\sigma_\tau} + (1-\theta_\tau)c_{t,\tau,2}^{\sigma_\tau})^{1/\sigma_\tau}$, a non-constant $\bar{c}_t$ requires the ratio $\frac{\theta_\tau}{1-\theta_\tau}\left(\frac{\Omega_1}{\Omega_2}\right)^{\sigma_\tau - 1}$ to be independent of τ; call it ρ. Indeed, if this holds the contract curve is the diagonal of the Edgeworth box and all its points are supported by the relative price ρ; and if not, any two different Pareto optima have different supporting prices.

It is not hard to see now that if one was to allow an arbitrary endowment, then at a competitive equilibrium as a baseline, and with any transfer policy the same argument would have gone through, with the additional adjustment of the re-equilibrating effect $\partial c/\partial\omega$, a 4 by 4 matrix of derivatives that depend on t.

In the next section we show that discounting is still consistent with balanced growth, even in a full-fledged model.

3. Discounting in an OG model with exogenous growth

Consider the general form of the classical exogenous-growth model in an overlapping generations setting, as in Mertens and Rubinchik [10].

3.1. **The Economy.** Time is the real line, $\mathbb{R}$. Individuals differ by type $\tau \in \Theta$ (Θ is finite) and by birth-date, $x \in \mathbb{R}$. They have life-length T_τ, and population grows exponentially at a constant rate ν, such that the distribution of age-groups and types is stationary. $N_x^\tau dx \stackrel{\text{def}}{=} N_0^\tau e^{\nu x} dx$ ($N_0^\tau > 0$) individuals of type τ get born in $[x, x+dx]$, $\forall x \in \mathbb{R}$.

Instantaneous consumption of any individual is a non-negative bundle of n consumption goods and h fractions of total time allocated to h different types of labour. Individual preferences over lifetime streams of time allocation and consumption bundles are described by a utility function U_τ, homogeneous[11] of degree $1-\rho_\tau$ in consumption.

Instantaneous production transforms m capital goods and l types of effective labour inputs into n consumption goods and m types of investment goods, with constant returns to scale.[11] The fraction of time, $z_{\tau,j}(s,t)$, devoted at date t to activity j by an agent of type τ and age s multiplied by a non-negative and integrable efficiency factor $\zeta_{\tau,j}(s)$,[12]

[11]Homogeneity and constant returns to scale are necessary for balanced growth.

[12]For example, setting $\zeta_{\tau,j}(s) = 1$ in a first part of life and 0 thereafter (or vice-versa) corresponds to the classical 2-period models.

and then multiplied by $e^{\gamma t}$ (representing labour-saving technological progress) is the productive labour, $e^{\gamma t}\zeta_{\tau,j}(s)z_{\tau,j}(s,t)$, for activity j contributed by type τ.

There are m capital goods (K_j), and a corresponding investment good (I_j) for each, linked by the usual capital accumulation equation, $K_j'(t) = I_j(t) - \boldsymbol{\delta}_j K_j(t)$ with the corresponding depreciation factor $\boldsymbol{\delta}_j$.

3.2. **Balanced Growth Paths.** We focus on balanced growth equilibria (BGE).[13] On a balanced growth path, individual labour is independent of the birth-date, individual consumption grows at rate γ, and all aggregate inputs and outputs at rate $\gamma + \nu$, as in the standard (1 type, 1 good) case, e.g., Arrow and Kurz [1], King et al. [6].

3.3. **Transfer Policies.** Assume now that policies consist of lump-sum real taxes and subsidies that might vary over time. As before, ω denotes distribution ("density") of endowments that depend on date of birth (x), age (s) and type (τ) of an individual, with values in $\mathbb{R}^n$, corresponding to the n consumption goods. The transfers can be both positive and negative and are not required to sum up to zero.

Fix an initial policy $\bar{\omega}$ that satisfies $\bar{\omega}(x,s,\tau) = e^{\gamma h}\bar{\omega}(x-h,s,\tau)$ for any $h \in \mathbb{R}$, i.e., is consistent with a balanced growth equilibrium. Observe that the initial policy of no-transfers, i.e., $\bar{\omega}(x,s,\tau) = 0$ satisfies this requirement.

In order to derive the inter-generationally fair discount rate for this case and compare it to the interest rate, we will apply [10]. The set of policies used there is rather abstract, here we want to work with a set for which verifying the assumptions needed to apply the results from [10] is much easier.

The set of transfer policies here will be denoted by F^λ. The policies in the set are required to converge to the initial policy no slower than at exponential rate λ with time (thus, implying $\omega(x,\cdot,\cdot)$ converges exponentially to $\bar{\omega}(x,\cdot,\cdot)$ as $x \to \infty$). Further, we require the aggregate transfers to grow no faster than the overall resources, and we bound the life-time transfers to assure individual consumption is non-negative. Formally,

Definition 3. F^λ is the space of $\mathbb{R}^n$-valued functions f defined for any $(x,s,\tau) \in \mathbb{R} \times \cup_\tau([0,T_\tau] \times \{\tau\})$ satisfying

(i) Exponential convergence:

$$\sum_\tau N_0^\tau \int e^{\lambda x} f(x,s,\tau)\,dx\,ds < \infty$$

[13] Cf. sect. 2.1 for the justification.

(ii) Bound on aggregate transfers:

$$\sup_t e^{-\gamma t} \sum_\tau N_0^\tau \int |f(t-s,s,\tau)| ds < \infty$$

(iii) Bound on life-time transfers:

$$\sup_{x,\tau} e^{-\gamma x} \int |f(x,s,\tau)| ds < \infty$$

The first restriction is necessary to assure existence of the derivative of welfare. Indeed, as mentioned before, the derivative is, roughly speaking, a weighted sum of transfers, with the weights reflecting the welfare weights of individual utilities and the amount of affected individuals of each type at any future point in time (if the weights are only type-specific, cf. eq. (1) below). Since population grows exponentially in this model, requiring just the sum of per capita transfers to be summable over time, is, clearly, not sufficient. Hence, we require exponential convergence, which is also an assumption required for the main results of Mertens and Rubinchik [10] to hold.[14] The required rate of convergence λ will be specified later (with each statement below).

The other two restrictions come from feasibility: material balance and budget constraints, and are shown to be sufficient for differentiability of equilibrium with respect to transfer policy at a BGE of a generic economy which is a particular case of the one considered here, [11, thm. 2]. Recall that we measure transfers in per capita terms and since per-capita consumption grows at the rate of exogenous growth of productivity, γ, at a BGE, the transfers are re-scaled accordingly to assure existence of a uniform bound over time.

Based on the argument in appendix B, the transfer policy space that we defined above can be mapped into the space of policies $\delta\pi(t)$ that satisfies the corresponding definition in [10], hence we can apply the results from that paper.

To present the summary of the results, let us define two welfare functions. Both functions are of the form

$$W = \sum_\tau N_0^\tau \int w_{\tau,x} e^{\nu x} \delta U_{\tau,x} dx \tag{1}$$

Here $\delta U_{\tau,x}$ is the change in individual utility of an individual of type τ born at x, and since in this model at any time the set of possible types is the same, $\delta U_{\tau,x} = U_\tau(c_{x,\tau}, l_{x,\tau}) - U_\tau(\bar{c}_{x,\tau}, \bar{l}_{x,\tau})$, where $(\bar{c}_{x,\tau}, \bar{l}_{x,\tau})$ is the BGE bundle of such individual under the initial policy $\bar{\omega}$ and $(c_{x,\tau}, l_{x,\tau})$ is the new equilibrium bundle that results after the transfer policy is implemented.

[14] Cf. the definition of the set Z_F there.

The first welfare function we consider is the *traditional utilitarian* with some welfare weights $w_{\tau,x}$: recall individuals differ by type and date of birth in this model. In particular, one could take $w_{\tau,x} = e^{-\beta t}\hat{w}_\tau$ for some time (generational) discount rate $\beta > 0$ and type-specific weights $\bar{w}_\tau$.

The second welfare function is *relative utilitarian*: individual utilities are normalised between 0 and 1 on an exogenously given set of "all feasible and just" alternatives (policies), and then summed.[15] Assume that this set of acceptable policies is shift invariant, cf. assumption 3 in [10], so that any feasible and just policy, when shifted in time, remains feasible and just; and that each individual utility is bounded on this set. It is also shown in that paper that the normalization in this case amounts to choosing welfare weights $w_{\tau,x} = e^{-(1-\rho^\tau)\gamma x}$.

We will use an extension of RU by allowing the utilities to be discounted in addition,[16] so that $w_{\tau,x} = e^{-[\beta+(1-\rho^\tau)\gamma]x}$, $\beta > 0$.

Now we can formulate the implications of the main theorem in [10]. In each case we state the necessary assumption specifying the minimal rate of convergence of the feasible policies (for the welfare differential to be well-defined) in definition 3 of the set F^λ.

Implications. (i) Assume $\lambda = -\min_\tau\{\beta + \rho_\tau\gamma\} + \nu$.

There is no way to rationalize discounting as a derivative of welfare with heterogenous individuals, if welfare is the traditional utilitarian one even with the separable utility weights: $w_{\tau,x} = e^{-\beta t}\hat{w}_\tau$.

Indeed, time- and type- weights of $\delta\omega = \omega - \bar{\omega}$ in the welfare differential are not multiplicatively separable as required by definition 2: the weight of $\delta\omega_{t-s,s,\tau}$ is proportional to $e^{-\rho_\tau\gamma t}$, cf. [10, cor. 3].

(ii) Assume $\lambda = \nu - \gamma$.

With relative utilitarian welfare, discounting can be rationalized. If $\beta = 0$, so all generations have equal weight (as required by RU), the implied discount rate is γ, cf. [10, cor. 7].

(iii) Assume $\lambda = \nu - r$.

With extended RU (allowing $\beta \neq 0$), (real) interest rate r can be rationalized as a discount rate only if $\beta = r - \gamma$, i.e., if utility weights are taken to be equal the difference between the interest rate and the rate of growth of per-capita consumption, cf. [10, cor. 7].

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[15] Introduced and axiomatized in Dhillon and Mertens [2] for a finite set of agents.

[16] This violates the anonymity axiom, used to derive RU. We conjecture it should be replaced, for this, by anonymity within generations plus a form of stationarity.

4. Conclusions

Our objective here was to rationalize the common CBA approach and, in particular, check when market evaluation of a public project, including discounting at the market interest rate, can be welfare-justified. We analyze this question in perfectly competitive environments, thus setting aside the related dispute in the presence of distortions (see Murty [14] for the most recent theoretical generalization of the related results and the overview, and Harberger and Jenkins [5] for the recent estimates for practical use in the presence of distortions).

First lesson, based on a finite economy, is that classical "efficiency-based" CBA is not distribution-free. Whenever it can be made consistent with classical utilitarian welfare, market evaluation (monetization) imposes price-dependent welfare weights associated with utilities of different individuals.

Second, stationarity (including balanced growth) is necessary even in simple models to rationalize discounting. The implications above are, most likely, not valid beyond a balanced growth path as a baseline: Mertens and Rubinchik [11] show that while the derivative of welfare (in a neighbourhood of a BGE) typically still exists, even in a simple model it will no longer have a net present value (NPV) form, while it is of this form at a BGE. To stress, beyond the BGE, NPV is not even a first order approximation to the derivative of welfare.

Extending the first lesson to the infinite economy with overlapping generations, we found, again, that imposing evaluation at market prices requires the welfare weights (for different generations) to depend on the price system (interest rate) and hence the inter-generationally fair discount rate (based on relative utilitarian welfare), which equals the per-capita growth rate γ, is typically different from the prevailing interest rate, even in an efficient equilibrium. To wit, in a subclass of economies considered here, the golden rule equilibrium interest rate is $\nu + \gamma$, (cf. [12]), and so is not equal to the discount rate derived above as long as the population growth rate, ν, is non-null. Let us stress that the reason for this discrepancy is purely distributional, and has no relation to the gap between prices and the "true social value" of resources that might be present in an economy with distortions.

Finally, stationarity of the baseline allocation is not sufficient for discounting: it can not be calculated from the derivative of traditional utilitarian welfare even with balanced growth equilibrium as a baseline for the economy with heterogeneous agents. Relative utilitarian function, in contrast, provides a rationale for discounting, with the caveats mentioned above.

Appendix A. Proposition 2: its proof and related examples

A.1. **The proof.**

Proof of proposition 2. Start with a baseline competitive equilibrium (c, y, p) and let the equilibrium after perturbation $(\delta\omega)$ be (c', p', y'). Using previous proposition (1), $\delta W = \sum_i p_i \sum_\tau \delta c_{i,\tau}$ Since by market clearing $\delta c = \delta\omega + \delta y$, the claim, $\delta W = \sum_i p_i \sum_\tau \delta\omega_{i,\tau}$, will follow if the value of the variation in production δy (induced by the endowment perturbation) is negligible, or simply, the *orthogonality condition*,[17]

$$\lim_{\|\delta\omega\|\to 0} \frac{|\sum_i p_i \delta y_i|}{\|\delta\omega\|} = 0 \tag{2}$$

This follows from the Lipschitz condition. Indeed, by the profit maximization, $\sum_i p_i \delta y_i \leq 0$, and similarly, $\sum_i p'_i \delta y_i \geq 0$, so $0 \geq \sum_i p_i \delta y_i \geq \sum_i (p_i - p'_i)\delta y_i$. So, by the local Lipschitz condition (fn. 8), $|\sum_i p_i \delta y_i| \leq \varepsilon \|\delta\omega\|$ for $\|\delta\omega\|$ sufficiently small. ■

A.2. **The "locally Lipschitz" condition.**

This example[18] shows that even in smooth economies[19] the orthogonality condition (2) of proposition 2 can be violated (at a critical point).

A.2.1. *Economy.* There are two goods and two individuals with utilities $u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{2}x_{21}^{-2}$ and $u_2(x_{12}, x_{22}) = x_{22} - \frac{1}{2}x_{12}^{-2}$ and the corresponding endowments $(1, r)$ and $(r, 1)$, with $r > 0$. Both individuals share profits equally. The aggregate production set is $Y = \{(y_1, y_2) \in \mathbb{R}^2 | y_1 + y_2 \leq \min\{0, \alpha y_1 y_2\}\}$, where $\alpha > 0$.

A.2.2. *Equilibria.* Let $p = \frac{p_1}{p_2}$.

A.2.3. *Profit maximization.* $\max_{y_1} py_1 + \frac{-y_1}{1-\alpha y_1}$. So, the optimal production plan satisfying $y_1 + y_2 \leq 0$ is $y_1(p) = \frac{1}{\alpha}\left(1 - \frac{1}{\sqrt{p}}\right)$, $y_2(p) = \frac{1}{\alpha}\left(1 - \sqrt{p}\right)$. Hence, the profit is $\frac{(1-\sqrt{p})^2}{\alpha} p_2$.

A.2.4. *Consumer demand.* Given the budget constraints of the first and the second consumer, respectively,

$$\begin{aligned} px_{11} + x_{21} &= p + r + \frac{(1-\sqrt{p})^2}{2\alpha} \\ px_{12} + x_{22} &= 1 + rp + \frac{(1-\sqrt{p})^2}{2\alpha} \end{aligned}$$

[17] The condition does not hold everywhere, even for smooth economies, cf. app. A.

[18] Based on Mas-Colell et al. [9].

[19] Indifference curves hit the axes, but the equilibria considered are away from that.

their demands are

$$(1 + rp^{-1} + \frac{(\sqrt{p}-1)^2}{2p\alpha} - p^{-2/3}, p^{1/3})$$
$$(p^{-1/3}, 1 + rp + \frac{(\sqrt{p}-1)^2}{2\alpha} - p^{2/3})$$

We will only look at equilibria in the neighborhood of $p = 1$, where all those quantities are strictly positive.

A.2.5. *Market clearing.* Take the market for the second good:
$p^{1/3} + 1 + rp + \frac{(\sqrt{p}-1)^2}{2\alpha} - p^{2/3} = 1 + r + \frac{1}{\alpha} - \frac{\sqrt{p}}{\alpha}$
Let $z = p^{1/6}$, so the market clearing condition becomes

$$F(z, r) = (r + \frac{1}{2\alpha})(z^6 - 1) - z^2(z^2 - 1) = 0$$

Note that $z = \pm 1$ solves the equation, for any r and α.
Hence the other equilibria are the positive (z) roots of
$G(z, r) = F(z, r)/(z^2 - 1) = (r + \frac{1}{2\alpha})(z^4 + z^2 + 1) - z^2 = 0.$

A.2.6. *Perturbation of endowment (r) around a critical point.* The equilibrium graph can thus be described as the graph of the function $z^2 + z^{-2} + 1$ ($z > 0$), together with the vertical through $z = 1$, thinking of the vertical axis as the parameter $(r + \frac{1}{2\alpha})^{-1}$ of the economy and of the horizontal axis as the parameter z of the equilibrium. Obviously this has 1 critical point: $z = 1$, $r + \frac{1}{2\alpha} = \frac{1}{3}$.

So, consider e.g. $\alpha = 3$, $r = 1/6$, and perturbations of the form $r = \frac{1}{6} - \rho$, $\rho > 0$.

Since for $z = p = 1$ the optimal production is zero,

$$\begin{aligned} \lim_{\rho\to 0} \frac{1}{\rho}\langle p, \delta y\rangle &= \lim_{\rho\to 0} \frac{y_1(1+\delta p) + y_2(1+\delta p)}{\rho} \\ &= \lim_{\rho\to 0} \frac{-\left(\sqrt{1+\delta p} - 1\right)^2}{3\rho\sqrt{1+\delta p}} \end{aligned}$$

Letting $z = 1 + \zeta$, express now $\sqrt{1+\delta p}$ as $(1+\zeta)^3$ and use the previous equation to express also ρ in terms of $1 + \zeta$: $\rho = \frac{1}{3} - \frac{z^2}{z^4+z^2+1} = \frac{1}{3}\frac{(z^2-1)^2}{z^4+z^2+1}$, so $3\rho \sim \frac{4}{3}\zeta^2$ and thus:

$$\lim_{\rho\to 0} \frac{1}{\rho}\langle p, \delta y\rangle = -\lim_{\zeta\to 0} \frac{3(3\zeta + 3\zeta^2 + \zeta^3)^2}{4\zeta^2(1+\zeta)^3} = -\frac{27}{4} \neq 0$$

Appendix B. Space of policy changes satisfies the assumptions in [10]

Our objective here is to show how to define the set of policies satisfying definition 2 in Mertens and Rubinchik [10] based on distribution of endowments ω defined in in section 3.3.

First, for any set of Borel subsets S_τ of $\mathbb{R}$ (groups of agents of different ages, indexed by type), define endowment measure $\psi\colon (t, S_\tau) \mapsto \sum_\tau N_0^\tau \int_{S_\tau} e^{\nu(t-s)}\omega(t-s, s, \tau)ds$. Also, using the initial policy from section 3.3, let $\bar{\psi}(t, S_\tau) = \sum_\tau N_0^\tau \int_{S_\tau} e^{\nu(t-s)}\bar{\omega}(t-s, s, \tau)ds$.

Next, define the normalized measure $\pi\colon t \mapsto \pi(t) = e^{-(\gamma+\nu)t}\psi(t, \cdot)$. Also let $\bar{\pi}(t) = e^{-(\gamma+\nu)t}\bar{\psi}(t, \cdot)$. Then, for any t, $\pi(t)$ is integrable with respect to age and type with values in $\mathbb{R}^n$, hence belongs to the Banach space $L_1^{\mathbb{R}^n}(\cup_\tau([0, T_\tau] \times \{\tau\}))$.

By construction, for any $t \in \mathbb{R}$ the value $e^{-(\gamma+\nu)t}\bar{\psi}(t, \cdot)$ is independent of t, as $e^{-\gamma x}\bar{\omega}(x, s, \tau) = \bar{\omega}(0, s, \tau)$ is independent of x, and so for any $\bar{\omega}$ (and hence for any $\bar{\psi}$) one can define $b_0\colon S_\tau \mapsto e^{-(\gamma+\nu)t}\bar{\psi}(t, S_\tau)$, then $\bar{\pi}(t) = b_0$, hence satisfying the requirement of baseline policy, Mertens and Rubinchik [10, def. 2(i)].

It follows then by definition 3 that if $\omega - \bar{\omega}$ is in F^λ, then $\pi - \bar{\pi}$ is in $F^{\lambda'}$, where $\lambda' = \lambda + \gamma$.

Finally, it is left to verify that space F^λ satisfies definition 1(iii) of Mertens and Rubinchik [10] with F^λ standing for F and $L_1^{\mathbb{R}^n}(\cup_\tau([0, T_\tau] \times \{\tau\}))$ (or L_1 for short) standing for E.

For that, observe first that F^λ is shift invariant by construction, hence part (1) of definition 1(iii) (*ibid*). Further, define an auxiliary space (K_E) for part (3) of definition 1(iii) (*ibid*):

Definition 4. K_{L_1} is the space of infinitely differentiable functions $\varphi\colon \mathbb{R} \to L_1$ with compact support.

$\varphi_n \in K_{L_1}$ converges to 0 if it and its successive derivatives converge uniformly to 0 and $\exists h \in \mathbb{R}\colon |x| \geq h \Rightarrow \varphi_n(x) = 0$ for all n. $K_{L_1}^\star$ is the space of linear functionals ψ on K_{L_1} s.t. $\psi(\varphi_n) \to 0$ when $\varphi_n \to 0$ in K_{L_1}.

For the remaining part (2) it is sufficient to prove the following

Lemma 1. *K_{L_1} is dense in F^λ for any $\lambda \in \mathbb{R}$.*

Proof. The two steps imply the statement by transitivity.

Step 1. *K_{L_1} is dense in $H = \{t \mapsto \sum_i^N b_i \mathbb{1}_{S_i}(t)\}$, the space of simple functions, where $S_i \subset \mathbb{R}$ are Borel sets of finite measure, $b_i \in L_1$ are constants, and $\mathbb{1}_{S_i}$ are indicator functions.*

Define $\phi(x) = e^{-\frac{1}{a(1-x)x}}$ for $0 < x < 1$ and 0 otherwise. ϕ is infinitely differentiable (cf. Kolmogorov and Fomin [7, p. 243]) clearly has finite support, and so belongs to K. ϕ converges to the indicator $\mathbb{1}_{[0,1]}$ in L_1 by the (Lebesgue) dominated convergence theorem. This implies K is dense in the set of indicator functions of $[0, 1]$. The statement then follows by linearity and given that the set of indicator functions of finite disjoint intervals is dense (in L_1 norm) in the set of indicators of Borel sets of finite measure.

Step 2. *H is dense in F^λ.*

That the set of simple functions is a dense subset of integrable functions is a basic property of Bochner integral (sometimes taken as its definition, cf. e.g., Dunford and Schwartz [4, def. III.2.17]). Hence the statement is proved for $\lambda = 0$.

For an arbitrary $\lambda \in \mathbb{R}$, and any $\phi \in K_{L_1}$, define function $g(t) \stackrel{\text{def}}{=} e^{\lambda t}\phi(t)$, then $g \in K_{L_1}$. Given the commutative property, Mertens and Rubinchik [11, lemma 6] the statement holds for any $\lambda \in \mathbb{R}$. ■

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