Rational Choice with Category Bias*

Amnon Maltz†

October 8, 2015

Abstract

This paper develops, using the revealed preference approach, a model of choice with an initial endowment and in the presence of alternatives that are grouped into categories. Our model generalizes the classical individual choice model which is rationalized by utility maximization, and reduces to that model in the absence of an initial endowment. Given an exogenous endowment, our decision maker follows a 3-step procedure: First, she identifies the best alternative in the choice set which belongs to the same category as her endowment. This alternative serves as her endogenous reference point which in turn, at the second step, induces a "psychological constraint". Finally, she chooses the best feasible alternative in her constraint set according to her reference-free utility. The model gives rise to a "category bias" which generalizes the status quo bias by attracting the decision maker towards the endowment's category but not necessarily towards the endowment itself. It also accommodates recent experimental findings on the absence of status quo bias among goods which belong to the same category. We apply the model to a financial choice problem and show that category bias may lead to a risk premium even with risk neutral agents.

Keywords: Status Quo Bias, Categories, Reference Dependence, Risk Premium, Revealed Preference.
JEL Codes: D03, D11.

*I would like to thank Efe Ok, Andrew Schotter, Pietro Ortoleva, Oded Nov and Giorgia Romagnoli for invaluable advice. For helpful comments I would also like to thank Ariel Rubinstein, Andrew Caplin, Guillaume Frechette, Hiroki Nishimura, Michael Richter, Kfir Eliaz, Alexander Coutts, and participants at the Micro Student Seminar at NYU.

†Department of Economics, University of Haifa. Address: Department of Economics, University of Haifa, Mount Carmel, Haifa, 3498838, Israel. Email: amaltz@econ.haifa.ac.il

1 Introduction

Many empirical studies, both within economics and psychology, have established that individuals' decisions are dependent on references. One such reference effect, the Status Quo Bias (SQB) was originally introduced by Samuelson and Zeckhauser (1988) and describes decision makers' tendency to not give up their initial endowment. In a set of field and lab experiments they showed that individuals choose to keep their endowment more frequently than would be predicted by the standard model of choice. Following their original findings, evidence for this bias has emerged from a wide range of markets and for different types of goods.[1]

Despite the abundance of evidence in support of the bias, some experimental studies have shown that its emergence is context dependent. One example comes from the endowment effect literature in the world of uncertainty. In this literature the bias takes the form of a gap between the minimum compensation demanded by an agent for a good that she owns and the maximum price she is willing to pay for the same good (Thaler 1980). Many studies report this gap for risky lotteries, i.e., gambles which pay monetary outcomes with known probabilities, as well as for ambiguous bets, i.e., gambles which pay monetary payoffs contingent on realizations of states which have no specified probabilities.[2] However, as one would expect, such a gap does not arise in evaluations of fixed monetary payoffs as shown by Kahneman et al. (1991).

More recently, Maltz and Romagnoli (2015) test directly for SQB in the world of uncertainty and find that when the endowment and the alternative are risky lotteries, the bias is absent.[3] They also report its absence when both alternatives are ambiguous bets. However, they find that when the endowment is risky and the alternative ambiguous, or vice versa, the bias emerges, a finding supported by Roca et al. (2006). Table I summarizes these recent experimental findings alongside those from the endowment effect literature. It reports whether SQB is present or absent for all combinations of the type of endowment and alternative in the world of uncertainty. The table reveals a pattern which can be described using the notion of categories: when the endowment and alternative belong to the same category, i.e., monetary payoffs, risky lotteries or ambiguous bets, SQB is absent. When they belong to different categories, the bias emerges.

Our work is motivated by this observation. The main goal is to develop a choice theoretical model which formally incorporates categories into the space of alternatives and is able to accommodate these findings. We assume exogenous categories and do not attempt to derive them from choice. Thus, we restrict attention to applications in which

[1] See among many others: Knetsch (1989) as well as Knetsch and Wong (2009) who study the phenomenon using everyday ordinary goods, Madrian and Shea (2001) and Choi et al. (2004) for a close examination of 401(K) retirement plans, Johnson and Goldstein (2003) for a study on organ donations and Kempf and Ruenzi (2006) for evidence from mutual fund markets.

[2] See for example Knetsch and Sinden (1984), Eisenberger and Weber (1995) and Isoni et al. (2011).

[3] Dean (2008) and Ren (2014) report similar findings for small choice sets consisting of risky lotteries.

		Alternative		
		Monetary Payoff	**Risky Lottery**	**Ambiguous Bet**
	Monetary Payoff	*NO SQB*	*SQB*	*SQB*
SQ	**Risky Lottery**	*SQB*	*NO SQB*	*SQB*
	Ambiguous Bet	*SQB*	*SQB*	*NO SQB*

Experimental evidence from the endowment effect literature and SQB experiments. The row and column represent the type of status quo option (SQ) and alternative respectively. Each entry specifies whether SQB was found for that SQ-Alternative pair.

Table I: SQB Findings under Uncertianty

products are grouped into categories that an outside observer can infer based on objective information, such as brand name, country of origin or whether probabilities are specified or non-specified. Moreover, our categories consist a partition of the grand space.[4] As an interpretation, we follow Schwartz (2000) and think of alternatives which belong to different categories as being difficult to compare due to their different nature. Within a category, we imagine comparisons being straightforward since its alternatives share common features.[5]

Our representation maintains a strong rationality structure and reduces to standard utility maximization in the absence of a status quo alternative. When facing a choice given an initial exogenous endowment, our agent follows a 3-step procedure: **First**, she identifies the best feasible alternative which belongs to her endowment's category, according to her reference-free utility (if there is more than one such alternative, she picks among them randomly). This alternative serves as her "psychological endogenous reference point" which in turn, at the **second** step, generates a "psychological constraint set". The alternatives in this set are the ones which are deemed "choosable" from the endogenous reference point's perspective. **Finally**, the decision maker evaluates all feasible alternatives in her constraint set and picks the one that maximizes her reference-free utility function. Figure I illustrates this choice procedure (notice that higher utility corresponds to a higher physical position of the alternative in the figure).

The first step of the above procedure is the novel aspect of this model as it links

[4]The classical view of categorization in psychology and philosophy claims that categories are discrete entities, characterized by necessary and sufficient conditions for membership. For a review of theories of categorization in psychology, see for example Smith and Medin (1981) and Komatsu (1992).

[5]Schwartz (2000): "Within each category, it may be relatively easy to express preferences. Between categories, however, expressing preferences is more problematic."

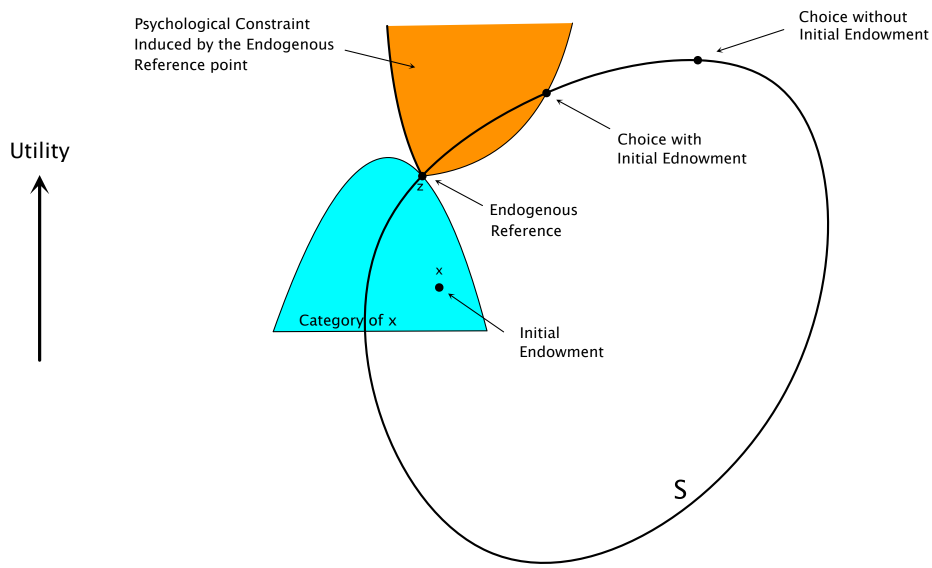

Figure I: 3-Step Choice Procedure

the agent's exogenous endowment to an endogenous reference point. It is as if the decision maker starts off by conducting an "easy evaluation" - one in which she considers only goods in her endowment's category. Among those she identifies the best one (in the reference-free utility sense) and treats it, rather than her exogenous endowment, as her reference point. From this point onwards the newly identified good will potentially be compared to other alternatives before a final decision is made, while the exogenous endowment becomes irrelevant for choice since it is clearly inferior to the new reference.

The steps that follow are identical to those developed in Masatlioglu and Ok (2014) which they dub "constrained utility maximization". In the second step a "psychological constraint" is induced by the endogenous reference point. This set contains all alternatives which the decision maker considers from that reference point's perspective and may very well include alternatives which are outside the endowment's category. Finally, the decision maker chooses the best feasible alternative in the constraint set according to her reference-free utility.

This choice procedure allows for reference effects that do not reflect status quo biased behavior but are closely related to it. For example, consider an agent who, when unemployed, chooses a non-academic job from a set S of possible work places. Once employed at a college however, she feels more drawn to academia and chooses a position in a large academic university from the same set S. Such behavior cannot be labeled status quo biased since the agent does not stay at her current position. However, she chooses an alternative "close" to her endowment, i.e., in the endowment's category of academic positions, exhibiting what we dub "category bias". In the context of a specific product market, when categories are viewed as the different competing brands, this bias may lead to brand loyalty, i.e., individuals' tendency to repurchase from the same brand (although not necessarily the same good).

Status quo biased behavior is also consistent with the described procedure and may show up whenever the exogenous endowment is the best feasible alternative within its category. However, the model predicts the absence of SQB when the choice set comprises alternatives from a single category. The reason is that the agent finds no difficulty realizing what the best available option in her endowment's category is and, in fact, tracks it down at the very first step of the choice procedure. Therefore, in the presence of only such alternatives, she will simply pick the best one, as she would absent an endowment. Thus, the model accommodates the pattern of the motivating findings reported in Table I.

The main axiom of our model, which we name "Categorical Referential Equivalence" (CRE) states that alternatives from the same category have the same effect on choice. More formally, for any set S and any two alternatives x and y that belong to the same category and to S, we have $c(S, x) = c(S, y)$. We also impose the Weak Axiom of Revealed Preference (WARP) across all problems with the same initial endowment which translates into the strong rationality structure exhibited by our decision maker. Two more axioms formally state the extent to which the agent's choices reflect status quo biased behavior. Our main result shows that these four axioms are satisfied if and only if the agent follows the 3-step procedure described above.

After stating our representation theorem we illustrate by means of an example the differences between our model and the model of Masatlioglu and Ok (2014) in the presence of categories.[6] Specifically, we show that the emergence of the endogenous reference point allows the agent to be drawn towards her endowment's category without being drawn towards the endowment itself. Finally, we turn to a financial application in which we define two categories: risky investments and riskless investments. In this set up we show that category bias may generate risk premium even when the agent is risk neutral.

The remainder of the paper is organized as follows: In section 2 we briefly review the related literature. Section 3 outlines the model and states the main result followed by examples and a comparison to the SQB model. Section 4 presents our application while section 5 concludes. Proofs are relegated to the Appendix.

2 Related Literature

Our work ties together two strands of behavioral choice models. The first is the class of reference dependent models which are able to accommodate SQB. By far the most well

[6] In the paper we will sometimes refer to the model of Masatlioglu and Ok (2014) as *the SQB model*.

known approach is the loss aversion model by Kahneman and Tversky (1979, 1991). More recently, another approach which describes choice as a constrained optimization procedure has been suggested by Masatlioglu and Ok (2005, 2014) and Ortoleva (2010). The second class can be described as models of choice with categories, such as Barbos (2010), Manzini and Mariotti (2012) and Furtado et al. (2015). These incorporate the notion of categories to study behavioral phenomena related to menu dependence (such as the attraction effect) and do not incorporate an endowment into the choice problem. Following our motivating findings presented in Table I, we propose a model which designates an endowment as part of the choice problem and makes use of categories to shed light on SQB.

The theoretical development we propose is most closely related to the SQB model of Masatlioglu and Ok (2014). We use a similar set up to theirs and add an exogenous partition on the grand space of alternatives. In the finest partition case, i.e., when every category is a singleton, our model reduces to theirs. Moreover, in that case our axioms also reduce to theirs. The departure from their model, emerges exactly in those cases in which the partition is not the finest and the space contains alternatives which are distinct from each other yet belong to the same category. It is in these cases that our reference equivalence axiom has bite and leads, together with the other postulates, to the formation of the endogenous reference point.

We also relate to models of endogenous references, such as Ok et al. (2015). In their model, when facing a choice set, the agent highlights an endogenous reference point from which a constraint set is induced. The models share the emergence of the endogenous reference point but differ in the set up as well as the axiomatic approach. Most notably, their model, which captures behavioral biases such as the attraction effect, examines choice problems with no endowment and does not deal with categories.

3 Model

3.1 The basic framework

We adopt the framework of the SQB model by Masatlioglu and Ok (2014). We designate a finite set X to act as the universal set of all mutually exclusive alternatives. The set X is thus viewed as the grand alternative space and is kept fixed throughout the exposition. The members of X are denoted as x, y, z, etc.. We assume there exists an exogenous partition on the set X, denoted by $X = X_1 \sqcup X_2, ..., \sqcup X_n$ for some $n \in \mathbb{N}$. Each cell in the partition should be thought of as a category of goods, i.e., a group of alternatives linked by some characteristic. For every $z \in X$ we denote by X_z the cell in the partition which z belongs to.

We designate the symbol $\Diamond$ to denote an object which does not belong to the set X. We shall use the symbol σ to denote a generic member of $X \cup \{\Diamond\}$. We let Ω_X denote the set of all nonempty subsets of X. By a choice problem we mean a list (S, σ) where $S \in \Omega_X$

and $\sigma \in S \cup \{\Diamond\}$.[7] The set of all choice problems is denoted by $\mathcal{C}(X)$. The interpretation of a choice problem (S, x) with $x \in S$ is that the decision maker is confronted with choosing an alternative from S while currently endowed with $x \in S$. Alternatively, a choice problem $(S, \Diamond)$ is interpreted as a choice from S absent an endowment.[8]

By a choice correspondence in this set up we mean a function $c : \mathcal{C}(X) \to \Omega_X$, such that

$$c(S, \sigma) \subseteq S \quad \text{for every} \quad (S, \sigma) \in \mathcal{C}(X).$$

(Notice that a choice correspondence on $\mathcal{C}(X)$ must be non-empty-valued by definition.)

3.2 Axioms

We introduce four axioms. The first two are taken directly from Masatlioglu and Ok (2014) while the others tie together the choice behavior to the underlying category structure.

We begin our axiomatic development by introducing a rationality property familiar from the classical theory of revealed preference. As in that theory, this property warrants that some type of utility maximization does take place in the decision-making procedure.

Weak Axiom of Revealed Preference (WARP). *For any* (S, σ) *and* (T, σ) *in* $\mathcal{C}(X)$,

$$c(S, \sigma) \cap T = c(T, \sigma)$$

provided that $T \subseteq S$ *and* $c(S, \sigma) \cap T \neq \emptyset$.

This property conditions the behavior of a decision maker across two choice problems whose endowment structures are identical. In this sense, it is merely a reflection of the classical weak axiom of revealed preference to the framework of individual choice in the (potential) presence of an exogenously given reference alternative. When $\sigma = \Diamond$, our formulation of WARP reduces to the classical formulation of this property.

Next, we describe behaviorally our decision maker's (weak) tendency to choose her endowment.

Weak Status Quo Bias (WSQB). *For any* $x, y \in X$,

$$y \in c(\{x, y\}, x) \quad \textit{implies} \quad y \in c(\{x, y\}, \Diamond)$$

and

$$y \in c(\{x, y\}, \Diamond) \quad \textit{implies} \quad y \in c(\{x, y\}, y.)$$

[7] Note that by this formulation, the endowment is always available for choice.

[8] While the use of the symbol $\Diamond$ is clearly redundant here, it is convenient as it allows us to describe a choice problem with or without an endowment in a unified manner.

This property was originally introduced by Masatlioglu and Ok (2014). It is a very natural postulate to impose on a decision maker who may be vulnerable to the status quo bias phenomenon (at least in the weak sense). Put simply, it states that serving as an endowment can only enhance an alternative's possibility of being chosen over another. More specifically, if y is chosen over x when x is the status quo option then it must also be chosen over x in the absence of a status quo option. The second part is similar - if y is revealed preferred to x without a status quo option then it must also be revealed preferred to it when acting as the status quo.[9] It is important to note that the axiom is stated in a "weak" fashion which allows the decision maker to exhibit, or not to exhibit status quo bias, depending on the alternatives at hand. Specifically, a rational agent who completely ignores her endowment will satisfy this axiom.

The first two axioms are independent of the underlying category structure imposed on the space of alternatives. Our next behavioral postulate, which we consider the main axiom of our model, links choice behavior across different endowments from the same category. Putting succinctly, it states that two alternatives which belong to the same category have the same referential effects.

Categorical Referential Equivalence (CRE). *For any given* $S \in \Omega_X$,

$$c(S,x) = c(S,y)$$

provided that $x, y \in S \cap X_x$.

CRE states that replacing an alternative which serves as an endowment with another from the same category should have no impact on choice. If WSQB is viewed as weakly highlighting the status quo option in the decision maker's mind then, combined with CRE, it implies that this option weakly highlights its category rather than itself. This axiom is at the heart of the model and the representation to follow and is novel as far as we know.

The last axiom describes situations in which the presence of a status quo option would not have any effect on choice. Roughly, it states that if the status quo option belongs to a "bad" enough category the decision maker will disregard it.

Categorical Status Quo Irrelevance (CSQI). *For any given* (S, x), *suppose that* $c(T, x) \nsubseteq X_x$ *for every subset* T *of* S *such that* $[S \cap X_x] \subset T$. *Then* $c(S, x) = c(S, \Diamond)$.

Suppose the decision maker faces a choice problem (S, x). Moreover, suppose that for any subset T of S, that contains all alternatives in S which belong to x's category and at least one alternative which does not, the decision maker chooses an alternative outside the category of x. According to CSQI, the choice from (S, x) will be the same as the choice from $(S, \Diamond)$. In other words, if the agent chooses "outside" her endowment's category whenever

[9] For a more elaborate discussion of this axiom see Masatlioglu and Ok (2014).

possible (for all subsets of her feasible set which allow her to do so), her endowment would be irrelevant.

3.3 Partial Characterization

We start by stating a lemma en route to our main representation theorem.

Lemma 1 *Let X be a non-empty finite set such that $X = X_1 \sqcup X_2, ..., \sqcup X_n$ for some $n \in \mathbb{N}$. Let c be a choice correspondence on $\mathcal{C}(X)$. If c satisfies WARP, WSQB, CRE and CSQI, then there exists a (utility) function $U : X \to \mathbb{R}$ and a self-correspondence Q on X such that*

$$c(S, \diamondsuit) = \arg\max U(S) \tag{1}$$

and for every $(S, x) \in \mathcal{C}(X)$,

$$c(S, x) = \arg\max U(S \cap Q(x')) \tag{2}$$

where $x' \in \arg\max U(S \cap X_x)$. Moreover, for any $x_1, x_2 \in X$ such that $x_1 \in X_{x_2}$

$$U(x_1) \geq U(x_2) \;\; \Leftrightarrow \;\; Q(x_1) \subseteq Q(x_2). \tag{3}$$

Lemma 1 provides sufficient conditions for the representation outlined in the introduction. To understand its nature let c be a choice correspondence on $\mathcal{C}(X)$, U a real function on X, Q a self-correspondence on X and suppose that the representation holds for any choice problem $(S, \sigma) \in \mathcal{C}(X)$. When dealing with a choice problem without an initial endowment, an agent whose choice behavior is modeled through c makes her decisions by maximizing the (ordinal) utility function U. That is, in this case, her final choice is realized by solving the problem:

$$\text{Maximize } U(\omega) \text{ subject to } \omega \in S.$$

In turn, when facing a choice problem with a given status quo option, say (S, x) the agent proceeds by following a 3-step procedure. In the first step she identifies the best alternative in S that belongs to the same category as x which we denote by x'. We can think of the decision maker as starting off by making an "easy" evaluation, i.e., only considering alternatives which are in the same category as x (by our category interpretation this evaluation is straightforward). Formally, x' is the solution to the following maximization problem:

$$\text{Maximize } U(\omega) \text{ subject to } \omega \in S \cap X_x.$$

In the second step, x' serves as the agent's endogenous reference point and induces a "psychological constraint set" $Q(x')$. All alternatives which do not belong to this set are excluded from any further consideration. Put differently, the agent forms the set $S \cap Q(x')$ from which the final choice will be made. (This set is non-empty since $x' \in S$ and by (2) and the fact that $\{x\} = c(\{x\}, x)$ we have $x \in Q(x)$ for every $x \in X$.) Property (3) imposes structure on the relationship between the constraint set Q and the utility function U. It states that within a category, the constraint sets are monotonically ordered. Specifically, they shrink as one considers reference points with higher utility. In other words, within a category a lower valued alternative leads the agent to consider (weakly) more alternatives.[10]

In the third and final step, the agent evaluates all alternatives in the set $S \cap Q(x')$ and chooses the best one according to her reference-free utility function U. Formally:

$$\text{Maximize } U(\omega) \text{ subject to } \omega \in S \cap Q(x').$$

If only x' belongs to S and $Q(x')$ then the decision maker will pick x', thus exhibiting "category bias". If in addition $x' = x$, i.e., x is the best alternative in $S \cap X_x$, the agent will exhibit SQB. On the other hand, if there are other alternatives in $S \cap Q(x')$ besides x', she may or may not stay within her endowment's category.

The role of the exogenous endowment x is simply highlighting the category to which it belongs and "directing" the decision maker to the best feasible alternative in that category to be treated as an endogenous reference point. The role of the endogenous reference point x' is to generate the constraint set $Q(x')$. The reference effect kicks in whenever the constraint set eliminates alternatives which are utility superior to x'. Notice that the fact that x' is highlighted in the first step, before the constraint set is constructed, implies the agent will not exhibit SQB when her choice set comprises only alternatives which belong to a single category.

In summary, the choice model given by Lemma 1 describes a reference dependent phenomenon which we dub "category bias". An agent whose choice behavior abides by this model is indistinguishable from a standard utility maximizer in the context of a choice problem without a status quo option. Given an initial entitlement, the agent utilizes it to highlight an endogenous reference point which is the best alternative in her endowment's category. This reference point induces a constraint set from which the agent finally chooses the best alternative according to her reference free utility function.

3.4 Characterization of the Main Model

Lemma 1 provides sufficient conditions for the existence of U, Q and the representation given by equations (1),(2) and property (3) to hold. However, one can easily provide an example to show that they are not necessary for this type of representation.[11] In this section we add structure on the relationship between U and Q which will ensure the axioms are also necessary for the representation given in Lemma 1. In order to do so we need the following definition. Suppose that, as in Lemma 1, U is a real function on X and Q is a self-correspondence on X.

[10] This property may be formally stated as within category negative comonotonicity of U and Q.

[11] The two axioms which fail are WARP and CSQI. With a very slight modification of the representation we may also ensure CSQI to hold. However, a more substantial addition to the representation is required in order to ensure WARP is satisfied as well.

Definition 1 *We say that Q is* U***-monotonic with respect to dissimilar alternatives*** *if for every $x \in X$ and $y_1, y_2 \notin X_x$ such that:*

1. $U(y_1) \geq U(y_2)$
2. $y_2 \in Q(x)$
3. $y_1 \in Q(x')$ *for some* $x' \in X_x$

we have $y_1 \in Q(x)$.

Imposing this structure on U and Q is intuitively close to stating that if an alternative y_2, outside the category of x, is considered from the perspective of x, then so will any other alternative y_1 which doesn't belong to the category of x and has greater utility than y_2. This statement is accurate if one only considers the first two requirements. Our formal statement given in the definition is slightly weaker by adding the third requirement - y_1 needs to be considered from the point of view of some other reference point x' in the category of x.

Let's illustrate this definition with a toy example. Suppose the agent considers a small size jeep from the perspective of her family car where our categories are $\{$*jeeps*$\}$ and $\{$*family cars*$\}$. Would she also consider a large jeep from this perspective? If the large jeep is better than the small one in the reference-free sense *and* would be considered from the point of view of *some* family car, then the answer is yes. Thus the definition requires that the large jeep not only carries higher utility then the small one for it to be considered but also that it is deemed choosable from the "family cars category's perspective."

We are now ready to state our main result.

Theorem 1 *Let X be a non-empty finite set such that $X = X_1 \sqcup X_2, ..., \sqcup X_n$ for some $n \in \mathbb{N}$. Let c be a choice correspondence on $\mathcal{C}(X)$. Then c satisfies WARP, WSQB, CRE and CSQI if, and only if, there exists a (utility) function $U : X \rightarrow \mathbb{R}$ and a self-correspondence Q on X such that*

$$c(S, \Diamond) = \arg\max U(S) \tag{4}$$

and for every $(S, x) \in \mathcal{C}(X)$,

$$c(S, x) = \arg\max U(S \cap Q(x')) \tag{5}$$

where $x' \in \arg\max U(S \cap X_x)$ and

1. *For any $x_1, x_2 \in X$ such that $x_1 \in X_{x_2}$*
$$U(x_1) \geq U(x_2) \ \Leftrightarrow \ Q(x_1) \subseteq Q(x_2).$$
2. *Q is U-monotonic with respect to dissimilar alternatives.*

The representation given in Theorem 1 describes the exact same choice procedure as in Lemma 1. The only difference is that it imposes more structure on the relationship between U and Q. First, as in Lemma 1 they satisfy negative comonotonicity within categories, i.e., the worse the alternative the greater the constraint set it induces. Second, it requires the type of monotonicity of Q with respect to U given by Definition 1.

Remark. The uniqueness structure of the pair (U, Q) found in Theorem 1 is as follows: Let U and V be two real functions on X, and let P and Q be two self-correspondences on X such that (4) and (5) hold for every $(S, x) \in \mathcal{C}(X)$ and for each (U, Q) and (V, P). Then, and only then, there is a strictly increasing map $f : U(X) \to \mathbb{R}$ such that $V = f \circ U$ and $Q(x) \cap x^{\uparrow} = P(x) \cap x^{\uparrow}$, where $x^{\uparrow}$ is the set of all $y \in X$ with $y \in c(\{x, y\}, \Diamond)$, for every $x \in X$.

3.5 Examples

This section presents a few examples that illustrate the type of choice behavior allowed by the above model.

Example 1. *(Rational Choice)* A decision maker who is not vulnerable to any reference effects and simply maximizes utility can be described by our model. In fact, we can describe such an agent by imposing two different restrictions - one on the exogenous partition, the other on the endogenous constraint set.

- **Example 1.1** *(Coarsest Partition Rational Choice)* Consider the coarsest exogenous partition on the grand set X where all alternatives belong to one category. Suppose the agent faces choice problem (S, x). In the first step of her choice procedure, she identifies the best feasible alternative in x's category, say alternative y, which in this example is the best alternative overall in S. Since $y \in Q(y)$, our agent will consider y in her final maximization stage (perhaps alongside other alternatives) and choose it as she would absent an endowment.

- **Example 1.2** *(Largest Constraint Set Rational Choice)* Our agent may also act as a utility maximizer when the partition on X contains more than one category. This may be the case if her constraint set is so large that she considers the whole space X from the viewpoint of every alternative. Formally, we describe such behavior by setting $Q(x) := X$, for every $x \in X$. Here, the endogenous reference has no bite in terms of choice since the decision maker deems all alternatives choosable prior to arriving at her final choice.

Example 2. *(No Extreme SQB)* An interesting feature of our model is that aside for the finest partition case, our decision maker cannot exhibit extreme SQB, i.e. $c(S, x) = \{x\}$, for every choice problem (S,x). To this end, consider two alternatives in the same category x_1 and x_2 and assume $U(x_1) > U(x_2)$.[12] If the agent faces the problem (S,x_2) where $x_1 \in S$, she will never keep x_2 as her final choice. Rather, she will choose from $S \cap Q(z)$, where $z \in X_{x_1}$ and $U(z) \geq U(x_1)$. Since $z \in Q(z)$, x_2 will not be chosen at her final maximization stage.[13]

Example 3. *(Brand Loyalty)* A natural interpretation of exogenous categories in our framework is that of brand names within a specific product market (goods of the same brand are easier to compare due to their similar features while across brands the comparison may be harder). Given this interpretation, our model gives rise to brand loyalty, i.e., individuals' tendency to repurchase products of the same brand over a long period of time. Due to the category bias exhibited by our decision maker, she may well switch among different goods (thus not exhibiting status quo bias) but she is more likely to stick to the same brand.

Example 4. *(Extreme Category Bias)* Consider a decision maker whose choice behavior is vulnerable to the category bias at the highest level. Such an agent is captured by our model when setting $Q(x) \subseteq X_x$ for every $x \in X$. Such an agent would never leave the category which her exogenous endowment belongs to. In the context of our previous example, such a consumer would be 100 percent loyal to her brand.

Example 5. *(No Cycles)* Our choice model does not allow behavior that exhibits cycles. For instance, for any distinct alternatives x, y and z, the following situation is incompatible: $\{y\} = c(\{x,y\}, x), z \in c(\{y,z\}, y)$ and $x \in c(\{x,z\}, z)$. For, by the representation derived in Theorem 1, these statements would entail $U(y) > U(x)$, $U(z) \geq U(y)$ and $U(x) \geq U(z)$, yielding a contradiction. Note that we reach this contradiction regardless of the category structure on the space. Thus, our model embodies a considerable amount of rationality.[14]

Example 6. *(Attraction Effect)* In the procedure followed by our decision maker the initial endowment plays the role of highlighting the best feasible alternative which belongs to its category. Thus, in a very natural way, our model may lead to the attraction effect relative to the endowment. That is, it allows an agent to choose x over y in the absence of a status quo option and choose y over x and z, when z is the status quo option. This may hold for example, if $U(x) > U(y) > U(z)$, y and z belong to the same category which is different

[12] This is without loss of generality. If $U(x_1) = U(x_2)$ in this example, the agent will still not end up choosing her endowment alone.

[13] The SQB model of Masatlioglu and Ok (2014) allows as a special case for extreme status quo type of behavior, hence this example is one illustration of the difference between the two models in the presence of a non-trivial partition.

[14] Notice that our model allows $c(\{x,y\}, x) = \{x\}, c(\{y,z\}, y) = \{y\}$, and $c(\{x,z\}, z) = \{z\}$. This is not a cycle because the decision maker always stays with the endowment. Cycles occur when the decision maker moves away from the status quo option in each choice problem and comes back to where she started.

than X_x and $x \notin Q(y)$.

3.6 A Comparison to the SQB Model

As discussed earlier, this model uses the set-up of the SQB Model of Masatlioglu and Ok (2014). In this section we briefly describe their main representation and outline the similarities as well as a major distinction between the two models.

The decision maker in the SQB Model behaves as if she has a (utility) function U and a self-correspondence Q on X. Unlike in our model the constraint set is always induced by the exogenous endowment. Formally, the decision maker's choices are described by:

$$c(S, \Diamond) = \arg\max U(S)) \tag{6}$$

and,

$$c(S, x) = \arg\max U(S \cap Q(x)) \tag{7}$$

Both models share the constrained maximization step. The difference between the two lies in which alternative generates the constraint set. In the SQB Model it is the exogenous endowment while in our model it may or may not be that alternative. When facing choice sets in which the exogenous endowment is the best feasible alternative (in the reference-free sense) in its category, it will also be the generator of the constraint set. Specifically this occurs whenever it is the only alternative available in that category. Thus, in the case of the finest partition on X the models coincide. However, when a utility improving alternative in its category is available, our decision maker will abandon her exogenous endowment and the constraint set will be induced by a different alternative in the endowment's category.

Despite the common features of the models, they predict different choices in the presence of a non-trivial partition on X. To illustrate, consider a decision maker facing three job opportunities: A high pay academic position (A), a low pay academic position (a) and a consultant position at a private firm (P) which pays more than both academic jobs. The categories in this simplified example will naturally be *academic jobs* and *non-academic jobs*. Assume that when the agent considers the positions while unemployed she ranks the private firm on top, then the high pay academic job followed by the low pay academic job.

Suppose further that when the agent is working at the low pay academic job, she feels drawn to academia and reveals it by choosing A from these three options. Given the above, what would the agent choose when currently at the low pay academic job and facing an offer only from the private firm?

According to both models the first ranking reveals that $U(P) > U(A) > U(a)$. According to the SQB Model, that together with the second choice is only compatible with P not being considered from the point of view of a, i.e., $P \notin Q(a)$. This implies the low pay academic job would be chosen over the private firm. Formally,

$$c(\{a, P\}, a) = \{a\}.$$

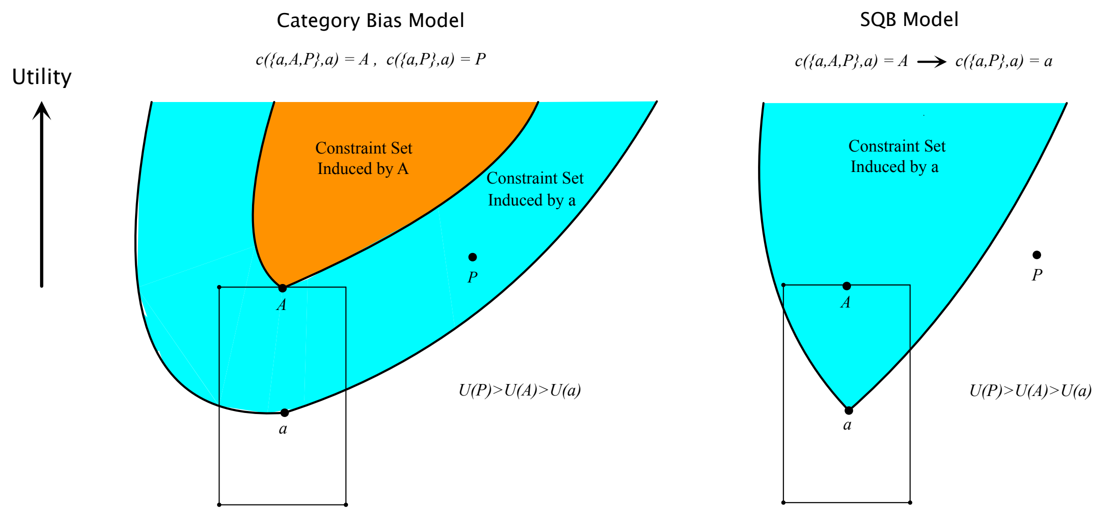

Figure II: SQB Model and Category Bias Model

This observation highlights a certain rigidity of the SQB Model. In order for the low pay academic job to highlight academia it needs to enhance itself to the level that it too, is preferred to the option outside the academic world. This seems quite unreasonable in situations in which the low ranked endowment is a "really bad option" in the reference-free sense which nevertheless generates a liking to alternatives of similar nature. However, according to the category bias model, the private job is considered from the point of view of the high pay academic job since it belongs to the same category (academia) as the current position. Thus, after observing the second choice we may only conclude that $P \notin Q(A)$. Hence, it is completely plausible that $P \in Q(a)$. In this case, our model predicts

$$c(\{a, P\}, a) = \{P\}.$$

Thus, the category bias model allows to relax the rigidity of the SQB Model through the first step of the choice procedure, namely the link between the exogenous endowment and the endogenous reference point. It is only from the latter's perspective that some alternatives are being excluded from the final maximization stage. Figure II illustrates the different constraint sets implied by the above choices according to the two models.

4 An Application: A Novel Source of Risk Premium

We now turn to apply our representation to the analysis of a simple financial market.[15] In this example we will describe an agent who not only follows the procedure described in Theorem 1 but also has more structure on her constraint sets. This structure has been developed by Ortoleva (2010) in his model accounting for SQB under uncertainty. His model describes a constrained maximizer very much like the SQB model of Masatlioglu and Ok (2014). However, his set up uses a more structured space which translates into added structure on the constraint set. In his paper, he uses the example we are about to borrow to show how a status quo biased agent may generate risk premium even if she is risk neutral as long as she is "pessimistic enough" in a manner to be described below. Using our model with categories, we will show that such premium may be generated for a risk neutral agent even if we relax the degree of pessimism.

Consider an economy in which there is one risk neutral representative agent (with $U(x) = x$), a government bond and a stock. There are two possible states of the world: s_g, the good state, and s_b, the bad state. The bond is traded for the price p_b and yields, with certainty, $\$B$. The stock, priced at p_{st}, yields payoffs $\$M$ and $\$m$, respectively, in the two states of the world, where $m < B < M$. The representative agent currently holds her money in the checking account which yields no interest and can choose whether to buy the stock, the bond, or not to invest (keep the money in the checking account). To keep the analysis simple, we assume that only one of these three actions can be taken. We take the categories in this set-up to be *uncertain investments* (stock) and *riskless investments* (Bond and Checking).

More formally, define $S := \{s_g, s_b\}$; $X := \mathbb{R}$ and $\mathcal{F}$ the set of all acts, that is the set of all functions from S into X. We focus on the preferences over three acts: buy the stock, st; buy the bond, b; keep the money in the checking account, c. For any given M, B, m, p_{st}, p_b, define these acts as: $c(s) := 0$ for all $s \in S$; $b(s) := B - p_b$ for all $s \in S$; $st(s_g) := M - p_{st}$ and $st(s_b) := m - p_{st}$. Suppose there is a market prior ρ (that the agent uses when choosing without an endowment) such that $\rho(s_g)M + \rho(s_b)m = B$.

The agent makes choices according to the procedure described in Theorem 1 with additional structure on her correspondence Q. Formally, the agent has a set Π of possible priors over the states which are interpreted as a set of possible models of the world in her mind. Since we assume the agent uses ρ absent an endowment, according to Ortoleva (2010) we have that $\rho \in \Pi$.[16] For any act f the agent's constraint set is defined by:

$$Q(f) := \{g \in \mathcal{F} | E_\pi(g) > E_\pi(f) \text{ for all } \pi \in \Pi\}.$$

Such an agent may be thought of as pessimistic and cautious. She knows the market prior ρ and trusts it absent an endowment. However, given an endowment, she considers

[15] The example set up is taken from an earlier version of Ortoleva (2010) available at http://gtcenter.org/Archive/Conf07/Downloads/Conf/Ortoleva498.pdf.

[16] Moreover, his representation ensures that ρ is in the relative interior of Π.

different models of the world according to some of which the good state occurs with a lower probability than $\rho(s_g)$. Before making her final choice, she considers only those investments that generate a higher expected payoff than her endowment according to every model of the world she has in mind. If no feasible investment satisfies this condition, she remains with her status quo option. Define as $\underline{\pi}$ the prior in Π which assigns the lowest probability to s_g.

We will now analyze the agent's choices in this environment given that

$$p_b < \underline{\pi}(s_g)M + \underline{\pi}(s_b)m < B \tag{8}$$

First notice that since $B > p_b$ the bond is a better investment than leaving the money in the checking account. Second, recall that the agent's exogenous endowment is the checking account but her endogenous reference is the bond since it is the best feasible alternative in the endowment's category (of riskless investments). In order for the stock to be traded it must be considered from the bond's perspective. Thus, if $p_b = p_{st}$ we have from (8)

$$\underline{\pi}(s_g)M + \underline{\pi}(s_b)m - p_{st} < B - p_b$$

In other words $s \notin Q(b)$ and the stock would not be traded.

Therefore if the stock is traded in the market we must have $s \in Q(b)$ which in our context translates into

$$\underline{\pi}(s_g)M + \underline{\pi}(s_b)m - p_{st} > B - p_b$$

which implies $p_{st} < p_b$. The implication is that the stock must be priced below the price of the bond for it to be sold although the bond and the stock have the same expected payoff according to the market analysts and according to the agent when she has no status quo.

Let us now compute the risk premium in this economy. For the bond, the rate of return is simply $r_b := \frac{B-p_b}{p_b}$. The stock's rate of return according to the market is

$$r_{st} := \frac{\rho(s_g)M + \rho(s_b)m - p_{st}}{p_{st}} = \frac{B - p_{st}}{p_{st}}.$$

$p_{st} < p_b$ therefore implies $r_{st} > r_b$. Thus we find a **positive risk premium** in this economy, **even though the agent is risk neutral.** As noted earlier a risk premium in this setting can also be generated by the SQB model of Ortoleva (2010). The difference is that in his model, the agent needs to exhibit a higher degree of pessimism than that ascribed to her by (8). Specifically, in order to deliver a premium, one needs to change (8) into

$$\underline{\pi}(s_g)M + \underline{\pi}(s_b)m < p_b < B \tag{9}$$

This implies that in the case of $p_b = p_{st}$, the agent would not choose the stock over the checking account even if those were the only available options. Thus, her $\underline{\pi}$ needs to assign a lower probability to s_g than that required by (8). In other words, Ortoleva (2010) shows how the channel of SQB may lead to risk premium given a sufficiently high degree of pessimism as reflected through the agent's endogenous "worst case scenario" probability.

Our model allows generating a risk premium through the category channel while relaxing the agent's pessimism. Specifically, in the first step of the choice procedure the bond becomes the endogenous reference point for the agent. In turn, the bond sets a "higher bar" for the stock if it is to be chosen compared to the "bar" that would have been set if the checking account was taken as the reference point. This requires the stock to be more attractive in order to be chosen - driving its price down and generating a risk premium.

Notice that if an external observer studied this market, but disregarded the role of category bias by using the standard expected utility model, she would erroneously deduce that the agent is risk averse. In fact, if the agent is really risk averse, then the risk premium would be even higher, owing both to risk aversion and category bias. Consequently, if the observer disregarded the role of the latter, then she would attribute to the agent a much higher, possibly implausible, level of risk aversion. This situation is dubbed in the macro-finance literature the equity premium puzzle, where extremely high levels of risk aversion are required to justify the risk premium observed in financial markets. Ortoleva (2010) provides the SQB channel as a possible missing ingredient to the puzzle requiring a certain degree of pessimism in the process. Our model adds the category bias channel which supports the emergence of a risk premium while allowing for both more reasonable levels of risk aversion and pessimism.

5 Conclusion

Our work is motivated by a pattern highlighted in experiments that study the endowment effect and SQB in the realm of uncertainty. This pattern implies that the presence of categories may play an important role in the emergence of SQB. We use the revealed preference approach to develop a model of choice with an initial endowment and in the presence of goods that are grouped into categories. With no endowment our decision maker is a standard utility maximizer. Given an endowment, she first focuses on goods that belong to her endowment's category. Among those she identifies the one with greatest (reference-free) utility and treats it as her endogenous reference point. This reference generates a constraint set from which she makes her final choice by picking the utility maximizing alternative.

The model is a generalization of the SQB model by Masatlioglu and Ok (2014) and reduces to their model when the partition into categories is finest. Our decision maker's behavior could be summarized as "rational choice with category bias". Rather than an inclination to choose her endowment she is biased towards picking an alternative which is in her endowment's category. The model allows for status quo biased behavior but predicts rational choice in the presence of goods which belong to the same category, in line with the motivating experimental findings.

6 Appendix

A Proof of Theorem 1

We first prove the "if" part of the theorem. Let $U : X \to R$ be a function and Q a self-correspondence on X, that satisfy properties (1) and (2) and take any choice correspondence c on C(X) that satisfies (4) and (5) for any $(S, \sigma) \in \mathcal{C}(X)$. We first make the following observations:

Claim 1.1. For every $x, x' \in X_x$, $U(x) = U(x') \Rightarrow Q(x) = Q(x')$.

Proof of Claim 1.1. Follows immediately from property (1).

Claim 1.2. $x \in Q(x)$ for every $x \in X$.

Proof of Claim 1.2. Since c is a choice correspondence on $\mathcal{C}(X)$, we must have $c(\{x\}, x) = \{x\}$ for any $x \in X$. Our claim thus follows from (5).

WARP. For the case in which $\sigma = \Diamond$ it is trivial. So take $x \in X$ and choice problems $(S, x), (T, x)$ such that $T \subseteq S$ and $c(S, x) \cap T \neq \emptyset$.
Case 1: $\arg\max U(S \cap X_x) \cap \arg\max U(T \cap X_x) \neq \emptyset$. In view of claim 1.1 and (5) this is obvious.
Case 2: $\arg\max U(S \cap X_x) \cap \arg\max U(T \cap X_x) = \emptyset$. Let $\bar{z} \in \arg\max U(S \cap X_x)$ and $z \in \arg\max U(T \cap X_x)$. By case 2, we have $U(\bar{z}) > U(z)$. Let $y \in c(S, x) \cap T$. By (5), $y \in \arg\max U(S \cap Q(\bar{z}))$. Thus, $y \in T \cap Q(\bar{z})$. By property (1), $Q(\bar{z}) \subseteq Q(z)$. Therefore, $y \in T \cap Q(z)$. Let $t \in \arg\max U(T \cap Q(z))$. We are to show that $U(y) \geq U(t)$.

2.1. $y \in X_x$. In this case, $y \in \arg\max U(S \cap X_x)$. Since $T \subseteq S$ this implies $y \in \arg\max U(T \cap X_x)$, Contradicting case 2.

2.2 $y \notin X_x$. By the choice of t, $U(t) \geq U(y)$. If $t \in X_x$, then $U(\bar{z}) > U(z) \geq U(t)$. We obtain $U(y) \geq U(\bar{z}) > U(t)$, a contradiction. If $t \notin X_x$ then by property (2) we obtain $t \in Q(\bar{z})$. Hence, $t \in S \cap Q(\bar{z})$ and so $U(y) \geq U(t)$.

To show the second inclusion, let $y \in c(T, x)$. By (5) $y \in \arg\max U(T \cap Q(z))$. Let $q \in \arg\max U(S \cap Q(\bar{z})) \cap T$. Since $Q(\bar{z}) \subseteq Q(z)$ it follows that $q \in T \cap Q(z)$. By choice of y, we have $U(y) \geq U(q)$. We are thus left to show that $y \in S \cap Q(\bar{z})$. Since $y \in T \subseteq S$, this reduces to $y \in Q(\bar{z})$. We consider the following three cases:

2.3. $q \in X_x$. We must have $q \in \arg\max U(S \cap X_x)$ which in turn implies that $q \in \arg\max U(T \cap X_x)$ contradicting case 2.

2.4. $q \notin X_x, y \notin X_x$. Since $q \in Q(\bar{z})$, $y \in Q(z)$, and $U(y) \geq U(q)$ we can use property (2) to obtain $y \in Q(\bar{z})$.

2.5. $q \notin X_x, y \in X_x$. Since $y \in T$ we have $y \in T \cap X_x$. By case 2, $U(\bar{z}) > U(y)$. By claim 1.2 $\bar{z} \in Q(\bar{z})$ and hence $\bar{z} \in S \cap Q(\bar{z})$. We obtain $U(q) \geq U(\bar{z}) > U(y)$, a contradiction.

WSQB. Take any $x, y \in X$ and suppose that $x \in c(\{x, y\}, y)$. By (5), $x \in \arg\max U(\{x, y\} \cap Q(z))$ where $z \in \arg\max U(\{x, y\} \cap X_y)$. If $y \in \arg\max U(\{x, y\} \cap X_y)$, we have $x \in Q(y)$. By claim 1.2 we obtain $U(x) \geq U(y)$, which by (4) is equivalent to $x \in c(\{x, y\}, \Diamond)$. If $y \notin \arg\max U(\{x, y\} \cap X_y)$ then $x \in X_y$ and $U(x) > U(y)$ which by (4) yields $\{x\} = c(\{x, y\}, \Diamond)$. On the other hand, if $x \in c(\{x, y\}, \Diamond)$, that is, $U(x) \geq U(y)$, then $x \in \arg\max U(\{x, y\} \cap X_x$. and in view of claim 1.2, $x \in \arg\max U(\{x, y\} \cap Q(x)$. By (2) we obtain $x \in c(\{x, y\}, x)$ as we sought.

CSQI. Take any $(S, x) \in \mathcal{C}(X)$ and let $z \in \arg\max U(S \cap X_x)$. Suppose that $c(T, x) \not\subseteq X_x$ for every subset T of S such that $[S \cap X_x] \subset T$. If S is itself a singleton we have $c(S, x) = \{x\} = c(S, \Diamond)$ by virtue of c being a choice correspondence. If S is not a singleton then, by hypothesis, $y \in c(\{y\} \cup (S \cap X_x), x)$ for every $y \in S \setminus X_x$. That is, $y \in Q(z)$ for every $y \in S \setminus X_x$. Moreover, by claims 1.1 and 1.2 we obtain $\arg\max U(S \cap X_x) \subseteq Q(z)$. Note that

$$S = [S \setminus X_x] \cup [\arg\max U(S \cap X_x)] \cup [(S \cap X_x) \setminus (\arg\max U(S \cap X_x))].$$

Denote the sets in the decomposition by A, B and C respectively. We have already obtained $A \subseteq Q(z)$ as well as $B \subseteq Q(z)$. Note that

$$\arg\max U(S) = \arg\max U(S \setminus C).$$

Hence,

$$\begin{aligned}
&c(S, x) = \arg\max U(S \cap Q(z)) = \arg\max U([A \cup B \cup C] \cap Q(z)) = \\
&\arg\max U([A \cap Q(z)] \cup [B \cap Q(z)] \cup [C \cap Q(z)]) = \\
&\arg\max U([A \cap Q(z)] \cup [B \cap Q(z)]) = \arg\max U(A \cup B) = \\
&\arg\max U(A \cup B \cup C) = \arg\max U(S) = c(S, \Diamond)
\end{aligned}$$

where the fourth equality follows from $z \in B \cap Q(z)$ and $U(z) > U(c)$ for every $c \in C$.

CRE. Let $S \in \Omega_X$ such that $x, y \in S \cap X_x$. Take any $z \in \arg\max U(S \cap X_x)$. By (5) $c(S, x) = \arg\max U(S \cap Q(z)) = c(S, y)$, as we sought.

We now move to prove the "only if" part of Theorem 1.[17] Let c be a choice correspondence on $\mathcal{C}(X)$ that satisfies WARP, WSQB, CSQI and CRE. Define the binary relation $\succsim$ on X by

$$y \succsim x \text{ if and only if } y \in c(\{x, y\})$$

A standard argument based on WARP shows that $\succsim$ is a complete preorder on X and that

$$c(S, \Diamond) = \{\omega \in S : \omega \succsim x \text{ for all } x \in S\} \text{ for every } S \in \Omega_X.$$

Furthermore since X is finite there exists a real function U on X such that $y \succsim x$ if and only $U(y) \geq U(x)$ for any $x, y \in X$. Therefore:

$$c(S, \Diamond) = \arg\max \{U(\omega) : \omega \in S\} \text{ for every } S \in \Omega_X.$$

Claim 1.3. Let $X_i \subset X$ be a cell in the partition. For any $x, y \in X_i$, we have: $c(\{x, y\}, x) = c(\{x, y\}, y) = c(\{x, y\}, \Diamond)$.

Proof of Claim 1.3. Suppose $c(\{x, y\}, x) = \{x\}$. By CRE $c(\{x, y\}, y) = \{x\}$. By WSQB $c(\{x, y\}, \Diamond) = \{x\}$. The other cases are handled similarly.

Claim 1.4. Let $X_i \subset X$ be a cell in the partition. Take any $S \in \Omega_X$ such that $S \subseteq X_i$. Then, $c(S, s) = c(S, \Diamond)$ for every $s \in S$.

Proof of Claim 1.4. Follows from claim 1.3 and WARP.

Claim 1.5. Let $X_i \subset X$ be a cell in the partition. For any $x, y \in X_i$ and $z \in X$ we have:

$$x \in c(\{x, y\}, \Diamond) \text{ and } z \in c(\{x, z\}, x) \Rightarrow z \in c(\{y, z\}, y).$$

Proof of Claim 1.5. Let $x, y \in X_i$ and $z \in X$. If $z \in X_i$ then by claim 1.3 and WARP the proof is complete. So suppose $z \notin X_i$. Consider the following three cases:

Case 1: $z \in c(\{x, y, z\}, y)$. WARP implies $z \in c(\{y, z\}, y)$ as we sought.

Case 2: $x \in c(\{x, y, z\}, y)$. By CRE $x \in c(\{x, y, z\}, x)$. WARP implies:

$$c(\{x, y, z\}, x) \cap \{x, z\} = c(\{x, z\}, x).$$

By assumption $z \in c(\{x, z\}, x)$ so we conclude $z \in c(\{x, y, z\}, x)$. By CRE once again we have that $z \in c(\{x, y, z\}, y)$ and we are back in Case 1 and so $z \in c(\{y, z\}, y)$.

Case 3: $y \in c(\{x, y, z\}, y)$. CRE implies $y \in c(\{x, y, z\}, x)$. By WARP $y \in c(\{x, y\}, x)$.

[17] This part also provides the proof for Lemma 1.

By assumption $x \in c(\{x, y\}, \lozenge)$. Using WSQB we obtain $x \in c(\{x, y\}, x)$. Combining, we obtain $\{x, y\} = c(\{x, y\}, x)$. Using WARP once again we have

$$c(\{x, y, z\}, x) \cap \{x, y\} = c(\{x, y\}, x) = \{x, y\}.$$

We may conclude $x \in c(\{x, y, z\}, x)$ and using CRE that $x \in c(\{x, y, z\}, y)$ which brings us back to Case 2. Conclusion: $z \in c(\{y, z\}, y)$ which completes the proof of Claim 1.5.

Now define

$$Q(x) := \{y \in X : y \in c(\{x, y\}, x)\}.$$

Claim 1.6. U and Q are categorically-negatively-comonotonic.

Proof of Claim 1.6. Take $x_1, x_2 \in X$ such that $x_1 \in X_{x_2}$. Suppose $U(x_1) \geq U(x_2)$. By the first part of the proof, $x_1 \in c(\{x_1, x_2\}, \lozenge)$. If $y \in Q(x_1)$ then $y \in c(\{y, x_1\}, x)$. By Claim 1.5 $y \in c(\{y, x_1\}, x_2)$ thus by defintion $y \in Q(x_2)$. Now suppose that $U(x_1) > U(x_2)$. Following the same steps we have that $Q(x_1) \subseteq Q(x_2)$. Note that, by definition $x_2 \in Q(x_2)$. If $x_2 \in Q(x_1)$ then $x_2 \in c(\{x_1, x_2\}, x_1)$. By claim 1.3 we obtain $x_2 \in c(\{x_1, x_2\}, \lozenge)$. But by first part of the proof this is true iff $U(x_2) \geq U(x_1)$, a contradiction. Thus, we conclude $Q(x_1) \subset Q(x_2)$.

Claim 1.7. Q is U-monotonic with respect to dissimilar alternatives.

Proof of Claim 1.7. Let $z' \in X$. Take any $y, x \notin X_{z'}$ such that:

- $U(y) \geq U(x)$
- $x \in Q(z')$
- $y \in Q(z'')$ for some $z'' \in X_{z'}$.

We are to show that $y \in Q(z')$. By definition and the eariler part of the proof, our assumptions can be rewritten as $y \in c(\{y, z''\}, z''), x \in c(\{x, z'\}, z'), y \in c(\{x, y\}, \lozenge)$. If $\{z''\} \in c(\{z', z''\}, \lozenge)$ then $U(z'') \geq U(z')$. By Claim 1.6 we have $Q(z'') \subseteq Q(z')$ and thus $y \in Q(z')$ as we sought. So suppose $\{z'\} = c(\{z', z''\}, \lozenge)$. Define $A = \{x, y, z', z''\}$. We examine the following four cases:

Case 1: $y \in c(A, z')$. WARP implies $y \in c(\{y, z'\}, z')$ so $y \in Q(z')$ as we sought.

Case 2: $z'' \in c(A, z')$. By CRE $z'' \in c(A, z'')$. WARP implies

$$c(A, z'') \cap \{y, z''\} = c(\{y, z''\}, z'').$$

By assumption $y \in c(\{y, z''\}, z'')$ and thus $y \in c(A, z'')$ Using CRE once again we have $y \in c(A, z')$ and we are back to Case 1 which completes the proof.

Case 3: $x \in c(A, z')$. CRE implies $x \in c(A, z'')$ and using WARP:

$$c(A, z'') \cap \{x, z'', y\} = c(\{x, z'', y\}, z''). \tag{10}$$

Thus $x \in c(\{x, z'', y\}, z'')$. We also have by WARP that $x \in c(\{x, z''\}, z'')$ and by assumption $y \in c(\{y, z''\}, z'')$. We can thus use CSQI with respect to the choice problem $(\{x, z'', y\}, z'')$ to obtain

$$c(\{x, z'', y\}, z'') = c(\{x, z'', y\}, \Diamond) \tag{11}$$

Now if $y \in c(\{x, y, z''\}, \Diamond)$, (10) and (11) imply that $y \in c(A, z'')$ and using CRE we are back to Case 1. If $x \in c(\{x, z'', y\}, \Diamond)$ then by WARP and the assumption $y \in c(\{x, y\}, \Diamond)$ we may conclude that $y \in c(\{x, z'', y\}, \Diamond)$ once again. Finally, suppose $z'' \in c(\{x, z'', y\}, \Diamond)$. By WARP

$$c(\{x, z'', y\}, \Diamond) \cap \{z'', y\} = c(\{z'', y\}, \Diamond). \tag{12}$$

By assumption $y \in c(\{z'', y\}, z'')$ and hence by WSQB $y \in c(\{z'', y\}, \Diamond)$. By (12) we have yet again $y \in c(\{x, z'', y\}, \Diamond)$.

Case 4: $z' \in c(A, z')$. WARP alongside our assumption that $x \in c(\{x, z'\}, z')$ implies $x \in c(A, z')$ which brings us back to Case 3 and completes the proof of Claim 7.

We are left to show (5), that is

$$c(S, x) = \arg\max U(S \cap Q(z)), \text{ where } z \in \arg\max U(S \cap X_x).$$

Take any $(S, x) \in \mathcal{C}(X)$ and $z \in \arg\max U(S \cap X_x)$. By CRE $c(S, x) = c(S, z)$. We now prove two final claims.

Claim 1.8. $c(S, z) = c(S \cap Q(z), z)$.

Proof of Claim 1.8. Let $T := S \cap Q(z)$. and pick any $y \in c(S, z)$. By WARP $y \in c(\{y, z\}, z)$ and hence $y \in Q(z)$. which implies $y \in T$. Conclusion: $c(S, z) \subseteq T$. Therefore $c(S, z) \cap T = c(S, z)$, which ensures that this is a non-empty set. We may thus apply WARP to conclude that $c(S, z) = c(S, z) \cap T = c(T, z)$ and we are done.

Claim 1.9. $c(S \cap Q(z), z) = c(S \cap Q(z), \Diamond)$.

Proof of Claim 1.9. If $S \cap Q(z) \subseteq X_x$ then by claim 1.4 we are done. So Suppose $S \cap Q(z) \not\subseteq X_x$. Take any $T \subseteq S \cap Q(z)$ such that $[S \cap X_x] \subset T$. We wish to show that

$c(T,z) \nsubseteq X_x$. There exists $\omega \in T$ such that $\omega \notin X_x$ and such that $\omega \in Q(z)$. By definition of $Q(z)$, we have

$$\omega \in c(\{\omega, z\}, z). \tag{13}$$

Suppose $c(T,z) \subseteq X_x$. Take any $y \in c(T,z)$. $c(T,z) \cap \{y,z,\omega\} \neq \emptyset$. We may apply WARP to obtain $c(T,z) \cap \{y,z,\omega\} = c(\{y,z,\omega\}, z)$. Thus, given our assumption, we have $c(\{y,z,\omega\}, z) \subseteq X_x$. This means that

$$\omega \notin c(\{y,z,\omega\}, z). \tag{14}$$

Moreover, $y \in c(\{y,z,\omega\}, z)$. We may apply WARP once more to get

$$c(\{y,z,\omega\}, z) \cap \{y,z\} = c(\{y,z\}, z). \tag{15}$$

$z \in X_x$ and by assumption also $y \in X_x$ and thus in view of Claim 1.3, we obtain $c(\{y,z\}, z) = c(\{y,z\}, \Diamond)$. Combined with (15), we may conclude

$$c(\{y,z,\omega\}, z) \cap \{y,z\} = c(\{y,z\}, z) = c(\{y,z\}, \Diamond).$$

By the fact that $z \in \arg\max U(S \cap X_x)$ and $y \in X_x$ we have that $z \in c(\{y,z\}, \Diamond)$. Therefore, $z \in c(\{y,z,\omega\}, z)$. Apply WARP once again to obtain $c(\{y,z,\omega\}, z) \cap \{z,\omega\} = c(\{z,\omega\}, z)$. By (14) this implies that $\omega \notin c(\{z,\omega\}, z)$. Hence, $z = c(\{z,\omega\}, z)$ which contradicts (13). Conclusion: $c(T,z) \nsubseteq X_x$. Thus we may apply CSQI to conclude that Claim 1.9 holds. Together with Claim 1.8 and CRE we obtain:

$$c(S,x) = c(S,z) = c(S \cap Q(z), z) = c(S \cap Q(z), \Diamond)$$

which in view of (4) completes the proof of Theorem 1.

References

Barbos, A. (2010): "Context effects: A representation of choices from categories," *Journal of Economic Theory*, 145(3), 1224–1243.

Choi, J. J., D. Laibson, B. C. Madrian, and A. Metrick (2004): "For better or for worse: Default effects and 401(k) savings behavior," in *Perspectives on the Economics of Aging*, ed. by D. Wise, University of Chicago Press.

Dean, M. (2008): "Status Quo Bias in Large and Small Choice Sets," *Working Paper*.

Eisenberger, R. and M. Weber (1995): "Willingness-to-pay and willingness-to-accept for risky and ambiguous lotteries," *Journal of Risk and Uncertainty*, 10(3), 223–233.

Furtado, B. A., L. Nascimento, and G. Riella (2015): "Rational Choice with Categories," *Working Paper*.

Isoni, A., G. Loomes, and R. Sugden (2011): "The Willingness to Pay-Willingness to Accept Gap, the "Endowment Effect," Subject Misconceptions, and Experimental Procedures for Eliciting Valuations: Comment," *American Economic Review*, 101(2).

Johnson, E. J. and D. G. Goldstein (2003): "Do Defaults Save Lives?" *Science*, 302, 1338–1339.

Kahneman, D., J. L. Knetsch, and R. H. Thaler (1991): "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias," *Journal of Economic Perspectives*, 5(1), 193–206.

Kahneman, D. and A. Tversky (1979): "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, 47(2), 263–292.

——— (1991): "Loss Aversion in Riskless Choice: A Reference Dependent Model," *Quarterly Journal of Economics*, 106(4), 1039–1061.

Kempf, A. and S. Ruenzi (2006): "Status Quo Bias and the Number of Alternatives: An Empirical Illustration from the Mutual Fund Industry," *Journal of Behavioral Finance*, 7(4), 204–213.

Knetsch, J. L. (1989): "The Endowment Effect and Evidence of Nonreversible Indifference Curves," *American Economic Review*, 79(5), 1277–1284.

Knetsch, J. L. and J. Sinden (1984): "Willingness to Pay and Compensation Demanded: Experimental Evidence of an Unexpected Disparity in Measures of Value," *Quarterly Journal of Economics*, 99(3), 507–521.

Knetsch, J. L. and W. K. Wong (2009): "The endowment effect and the reference state: Evidence and manipulations," *Journal of Economic Behavior and Organization*, 71(2), 407–413.

Komatsu, L. K. (1992): "Recent Views of Conceptual Structure," *Psychological Bulletin*, 112(3), 500–526.

Madrian, B. C. and D. F. Shea (2001): "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior," *Quarterly Journal of Economics*, 116(4), 1149–1187.

Maltz, A. and G. Romagnoli (2015): "The Effect of Ambiguity on Status Quo Bias: An Experimental Study," *Working Paper*.

Manzini, P. and M. Mariotti (2012): "Categorize Then Choose: Boundedly Rational Choice and Welfare," *Journal of the European Economic Association*, 10(5), 1141–1165.

Masatlioglu, Y. and E. A. Ok (2005): “Rational Choice with Status Quo Bias,” *Journal of Economic Thoery*, 121(1), 1–29.

——— (2014): “A Canonical Model of Choice with Initial Endowments,” *The Review of Economic Studies*, 81(2), 851–883.

Ok, E. A., P. Ortoleva, and G. Riella (2015): “Revealed (P)Reference Theory,” *American Economic Review*, 105(1), 299–321.

Ortoleva, P. (2010): “Status quo bias, multiple priors and uncertainty aversion,” *Games and Economic Behavior*, 69(2), 411–424.

Ren, Y. (2014): “Status quo bias and Choice Overload: An Experimental Approach,” *Working Paper*.

Roca, M., R. M. Hogarth, and A. J. Maule (2006): “Ambiguity seeking as a result of the status quo bias,” *Journal of Risk and Uncertainty*, 32(3), 175–194.

Samuelson, W. and R. Zeckhauser (1988): “Status Quo Bias in Decision Making,” *Journal of Risk and Uncertainty*, 1(1), 7–59.

Schwartz, B. (2000): “Self-determination: The tyranny of freedom,” *The American psychologist*, 55(1), 79–88.

Smith, E. E. and D. L. Medin (1981): *Categories and Concepts*, Harvard University Press.

Thaler, R. (1980): “Toward a positive theory of consumer choice,” *Journal of Economic Behavior and Organization*, 1(1), 39–60.