Monetary Policy and Fiscal Limits with No-Default

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Abstract

This paper discusses monetary and fiscal policy interactions that stabilize government debt. Two distortions prevail in the economy: income taxes and liquidity constraints. Possible obstructions to fiscal policy include a ceiling on the equilibrium debt-to-GDP ratio, zero or negative elasticity of tax revenues, and a political intolerance of raising tax rates. In states of fiscal stress two mechanisms restore solvency: fiscal inflation, which reduces the real value of nominal debt, and open market operations, which diminish the size of government debt held by the private sector. Three regimes achieve this goal. In all regimes monetary policy is passive. In all regimes a muted tax response to government debt is consistent with equilibrium. The propensity of a fiscal authority to smooth output is found to determine what is an acceptable response (in the form of tax rate changes) to the level of government debt, while monetary policy determines the timing and magnitude of fiscal inflation. Impulse responses show that the inflation and tax hikes needed to offset a permanent shock to transfers are lowest under nominal interest rate pegs, whereas most of the reduction in government debt arrives from open market purchases.

JEL Codes: E42; E62; E63; H60.
Keywords: Distorting Taxes; Finance Constraint; Fiscal Limits; Fiscal Rules; Fiscal Theory of Prices; Monetary Fiscal Regimes; Fiscal Consolidation; Open Market Operations; Debt Devaluation;

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1 Introduction

This paper discusses stabilizing fiscal-monetary regimes at the 'fiscal limit' where the government is able to increase tax revenues only by small amounts. Fiscal limits imply that current deficits are financed mainly by increasing government debt. Assuming no outright default, this regime is sustainable only if market equilibrium brings about fiscal solvency. In this situation, the valuation method of nominal government debt is the celebrated fiscal theory of the price level whose canonical foundations are set in Leeper (1991), Sims (1994), Woodford (1995), and Bassetto (2002). The contribution of this paper is to extend the conventional treatment of monetary-fiscal policy interactions, in which there is lump-sum taxation only, to an economy with distortionary taxation, capital accumulation, and liquidity constraints. As a result, the issue of nominal and real determinacy implies a new role for monetary policy in a fiscal theoretic equilibrium that is not the classical passive stance. Specifically, this paper recognizes the crucial role of open market operations in the stabilization of government debt, and specifies policy prescriptions that restore fiscal solvency. Moreover, it demonstrates that some interactions can substantially moderate the extent of fiscal inflation and fiscal consolidation required to stabilize government debt.

Under the conditions I will discuss in this paper fiscal limits arise endogenously from the underlying distortions and the requirement to run a balanced budget in the long run. Capital accumulation and liquidity constraints induce limitations on fiscal policy in a convincing way. In addition, the interaction between monetary and fiscal policy is encumbered by actual practice of fiscal policy in at least two dimensions:

I. In practice, fiscal authorities have limited or no access to lump-sum taxation and therefore mostly implement their policies by distorting taxes. This practice can become very limiting as the economy approaches the peak of its Laffer curves.[1] Accordingly, my analysis does not rule out government borrowing subject to zero elasticity tax revenues.

II. Political intolerance to raising tax rates can markedly limit fiscal policy. Accordingly, my analysis does not rule out fiscal rules according to which tax rates exhibit weak or mute responses to the state variables.

Prominent work on fiscal stress includes Cochrane (2011), Davig, Leeper and Walker (2010, 2011), Davig and Leeper (2010), Sims (2011), and Bi, Leeper and Leith (2012). This literature typically assumes that a blunt default is

[1] Trabandt and Uhlig (2010) estimate that tax rates in fourteen EU countries are slightly below the peak of their respective Laffer curves, and Denmark and Sweden are on the wrong side of the Laffer curve for capital income taxation.

inconceivable to both lender and borrower and examines the impact of alternative fiscal and monetary policy adjustments that ensure government solvency. Fiscal solvency is mostly restored through fiscal inflation and an expected future fiscal consolidation. Although in these models rapid bursts of inflation are a feature of the equilibrium, they are considered very low probability events that affect inflation expectations only through the small probability that households attach to those bursts. However, as households attach more probability to policy makers' attempts to stabilize debt with passive monetary policy, upward drift in inflation expectations and inflation itself become more pronounced. In particular, Davig et. al. (2011) show that without significant and meaningful fiscal policy adjustment, the task of meeting inflation targets will become increasingly difficult.

My contribution builds on this literature. Capital accumulation and distortionary taxation add a complication to aggregate dynamics since capital and bonds are perfect substitutes in the households' portfolio. However, these stores of value are in fact different from one another since the marginal product of capital is distorted by a liquidity constraint and an income tax. As a result, any policy response - either via the nominal rate of interest or via the income tax rate - potentially drives households to transfer wealth between bonds and capital, hence creating an arbitrage effect. In this story, if the government encounters a fiscal stress it has access to two mechanisms to devalue its debt: fiscal inflation that reduces the real value of nominal debt, and open market purchases that reduce the amount of government debt held by the private sector. However, these mechanisms are not straightforward. Fiscal inflation can successfully reduce nominal debt only if the fiscal policy brings about wealth effects. Similarly, open market operations can reduce government debt if the private sector has an incentive to transfer wealth from bonds to capital. Only then, purchases of government debt in the open market substantially reduce fiscal inflation. The results are threefold. First, the government must explicitly proclaim exactly three policy targets [For example: an inflation target, lump-sum transfers, and a debt-to-GDP ratio]. Remaining long run levels are implied by a condition that the consolidated budget is balanced in the steady state. Second, once debt deviates from its sustainable level, under all types of policy interactions the government must let its debt grow at a faster rate than the growth in tax revenues. This result is consistent with the fiscal theory of prices. Finally, the usual characterization of regime-F no longer obtains.[2]

[2] According to the conventional paradigm - "regime M" - debt financed tax cuts do not affect aggregate demand because the private sector expects the resulting increase in

Three regimes arise in which monetary policy is passive and fiscal policy adjusts income tax rates in response to debt and output. In these regimes, a sensible fiscal rule both motivates households to transfer wealth from bonds to productive capital at the private level and, at the same time, impedes self-fulfilling herd runs from bonds to capital at the social level. Open market purchases can then reduce the amount of government debt held by the households resulting in lower future inflation. Importantly, this mechanism is effective even when the elasticities of tax revenues are negative or when tax rates respond very weakly to the state variables. An important feature of this mechanism is that the interaction between monetary and fiscal policies must induce a return on capital investment that, after taking into account all the distortions, is higher than the real rate of interest. Liquidity constraints on investment that operate as capital adjustment costs enable this discrepancy to exist without implying an arbitrage.

The rest of the paper is organized as follows: Section 2 describes the

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government debt to be matched by future tax increases or spending reductions. Thus, expansions in government debt do not raise wealth. According to the fiscal theory - "regime F" – a deficit today is financed by an increment to nominal debt and is not expected to imply future tax hikes. As a result, households initially perceive the increase in nominal debt to be an increase in their wealth and try to convert higher wealth into consumption. Where output is predetermined, rising demand causes rising prices until real wealth falls back to its pre-deficit level. In this regime an active fiscal policy must interact with a passive monetary policy. By preventing nominal interest rates from rising sharply with inflation, monetary policy prevents debt service from growing too rapidly.

economic environment, the optimal decision-making of the representative household, and the evolution of government debt. Section 3 contains a detailed general equilibrium analysis. (All proofs are deferred to the appendix.) Section 4 describes stabilizing monetary-fiscal regimes. Section 5 discusses the computation of equilibrium, shows how the economy responds to changes in the proclaimed inflation and tax rate targets under the scenarios of lump-sum and distortionary taxation [some figures are deferred to appendix B] and provides impulse responses to a permanent increase in transfers. Section 6 contains the conclusion.

2 The Economic Environment

Time is continuous. The economy is closed and populated by a continuum of identical infinitely long-lived households, with measure one. The representative household enjoys consumption, is endowed with perfect foresight and one unit of time per "period" which it supplies inelastically. Accordingly, the representative household's lifetime utility is given by

$$U_t = \int_t^\infty e^{-\rho s} u(c_s) ds \tag{1}$$

where $\rho > 0$ denotes the rate of time preference, c_s denotes consumption per capita, $u(\cdot)$ is twice differentiable, strictly increasing, strictly concave, and satisfies the usual limit conditions. Production takes place in a competitive sector via a constant returns to scale production technology $f(k_t)$ where k_t denotes per capita capital which depreciates at a rate δ. Finally, $f(k_t)$ is concave and twice differentiable. Money enters the economy via a liquidity constraint on all transactions. Let m_t denote the stock of money denominated in the consumption good, then a formal representation of the liquidity constraint is:

$$c_t + I_t \leq \nu m_t \tag{2}$$

where I_t denotes per-capita investment and ν is money velocity.[3] Assuming the existence of government bonds, the representative household's budget constraint is

[3]This version of cash-in-advance is similar to Rebelo and Xie (1999). Let $\frac{1}{\nu}$ denote the inverse of money velocity, then a requirement that $\int_t^{t+\frac{1}{\nu}} [c_s + I_s]\, ds \leq m_t$ formalizes the liquidity constraint. A Taylor series expansion gives $\int_t^{t+\frac{1}{\nu}} [c_s + I_s]\, ds = \frac{1}{\nu}[c_t + I_t] + \frac{1}{2}\left(\frac{1}{\nu}\right)^2 [\dot{c}_t + \dot{I}_t] + \cdots$ and $\frac{1}{\nu}[c_t + I_t] \leq m_t$ can be interpreted as a first-order approximation.

$$c_t + I_t + \dot{b}_t + \dot{m}_t = (R_t - \pi_t)b_t - \pi_t m_t + (1 - \tau_t)f(k_t) + T_t \tag{3}$$

where b_t is a real measure of the stock of interest bearing government bonds, R_t is the nominal rate of interest, π_t is the rate of inflation, T_t are real lump-sum transfers, and $\tau_t \in [0, 1]$ is the income tax rate. Altogether, the household's lifetime maximization problem becomes

$$V\left[b_t, m_t, k_t\right] = Max_{\{c_s, I_s, x_s\}_{s=t}^{\infty}} \int_t^{\infty} e^{-\rho s} u(c_s) ds \tag{4}$$

$$s.t.$$

$$\dot{b}_s = (R_s - \pi_s)b_s - \pi_s m_s + (1 - \tau_s)f(k_s) + T_s - I_s - c_s - x_s$$

$$\dot{m}_s = x_s$$

$$\dot{k}_s = I_s - \delta k_s$$

$$c_s + I_s \leq \nu m_s$$

$$a_s, k_s \geq 0$$

With a borrowing constraint such that $lim_{t \to \infty} a_t e^{-\int_0^t [R_s - \pi_s] ds} \geq 0$ where $a_t \equiv b_t + m_t$ denotes the representative household's non capital wealth.

2.1 The optimal program

Each household chooses sequences of $\{c_t, I_t, x_t\}$ so as to maximize lifetime utility, taking as given the initial stock of capital k_0, and the time path $\{\tau_t, T_t, R_t, \pi_t\}_{t=0}^{\infty}$ which is exogenous from the view point of a household. The necessary conditions for an interior maximum are

$$u'(c_t) = \lambda_t(1 + \frac{1}{\nu}R_t) \tag{5a}$$

$$\mu_t = u'(c_t) \tag{5b}$$

$$\zeta_t = \frac{1}{\nu}R_t\lambda_t \tag{5c}$$

$$\zeta_t(\nu m_t - c_t - I_t) = 0; \zeta_t \geq 0 \tag{5d}$$

Where λ_t, μ_t are time-dependent co-state variables interpreted as the marginal valuation of financial wealth and capital, respectively; ζ_t is a time-dependent Lagrange multiplier associated with the liquidity constraint and equation (5d) is a Kuhn-Tucker condition.

Following Benhabib et. al. (2001b,2002), we restrict attention to a positive nominal interest rate. As a result, equation (5c) implies that ζ_t is positive. It then follows from (5d) that $m_t = \frac{1}{\nu}(c_t + I_t)$. The economic intuition is clear: where the nominal interest rate is positive, holding money entails opportunity costs. Thus, minimizing the opportunity cost of holding money implies that the liquidity constraint is binding. Second, and after substituting $m_t = \frac{1}{\nu}(c_t + I_t)$ and $a_t = b_t + m_t$ into equation (3), the state and co-state variables must evolve according to

$$\begin{aligned}
\dot{\lambda}_t &= \lambda_t [\rho + \pi_t - R_t] && (6)\\
\dot{\mu}_t &= -\lambda_t (1-\tau_t) f'(k_t) + (\rho+\delta)\mu_t && (7)\\
\dot{k}_t &= I_t - \delta k_t && (8)\\
\dot{a}_t &= (R_t - \pi_t) a_t + (1-\tau_t) f(k_t) + T_t - (c_t + I_t)\left(1 + \frac{1}{\nu} R_t\right) && (9)
\end{aligned}$$

Solving equation (9) yields that the household's intertemporal budget constraint is of the form

$$lim_{t\to\infty} e^{-\int_0^t [R_s - \pi_s] ds} a_t = a_{0+} \int_0^\infty e^{-\int_0^t [R_s - \pi_s] ds} \left[(1-\tau_t) f(k_t) + T_t - (c_t + I_t)\left(1 + \frac{1}{\nu} R_t\right)\right] dt \geq 0 \qquad (10)$$

and the condition that his intertemporal budget constraint holds with equality yields the usual transversality condition:

$$lim_{t\to\infty} a_t e^{-\int\limits_0^t [R_s - \pi_s]ds} = 0 \tag{11}$$

Equations (6) – (11) fully describe the optimal program of a representative household for which the time path $\{\tau_t, T_t, R_t, \pi_t\}_{t=0}^{\infty}$ is exogenously given.

2.2 The government and the evolution of government debt

The government consists of a fiscal authority and a monetary authority. We assume throughout the paper that the fiscal authority has no access to lump-sum taxation. The consolidated government prints money, M_t, issues nominal bonds, B_t, collects taxes to the amount of $\tau_t f(k_t)$ where τ_t denotes an income tax rate, and rebates to the households a lump-sum transfer T_t. Its dollar denominated budget constraint is therefore given by $R_t B_t + P_t T_t = \dot{M}_t + \dot{B}_t + P_t \tau_t f(k_t)$ where P_t is the nominal price of a consumption bundle. It is assumed that the monetary authority imposes a desired interest rate, R_t, and that the fiscal authority can continuously control the income tax rate, τ_t.

Dividing both sides of the nominal budget constraint by P_t and rearranging, yields that government liabilities, denoted by $a_t \equiv b_t + m_t$, evolve according to:

$$\dot{a}_t = (R_t - \pi_t)\, a_t - R_t m_t + T_t - \tau_t f(k_t) \tag{12}$$

where $\pi_t \equiv \widehat{\frac{\dot{P}_t}{P_t}}$ and the hatted time derivative is a right derivative, referring to expected inflation from now on. Equation (12) captures the following story: The consolidated budget is not necessarily balanced, and secondary deficits (surpluses) are financed via increments (decrements) to government debt. As a result, government liabilities increase with the primary deficit, $T_t - \tau_t f(k_t)$, and with the real interest paid over outstanding debt, $(R_t - \pi_t)\, a_t$, and decreases with seigniorage, $R_t m_t$.

2.2.1 Monetary Policy

Monetary policy follows a simple version of a Taylor rule that emphasizes inflation targeting,

$$R(\pi_t) = \rho + \pi^* + \alpha(\pi_t - \pi^*) \tag{13}$$

where $\alpha > 0$ is a constant that measures the responsiveness to increases in inflation around π^*. Following Leeper (1991) and Benhabib et. al. (2001a, 2001b) this rule underlies most of the recent monetary policy literature. When $\alpha > 1$ monetary policy is considered hawkish, responding to deviations of the expected inflation from the inflation target by sharply raising the nominal interest rate with the aim of stabilizing inflation around π^*. In Leeper's (1991) terminology this is called an "active" monetary policy whereas a weak response such that $\alpha < 1$ is called "passive" monetary policy.

2.2.2 Fiscal Policy

Previous work with distorting taxes emphasizes that tax rates adjust to stabilize government debt. Prominent papers in this literature include Bi (2012), Bi and Traum (2012) and Bi, Leeper and Leith (2012). Given that the state space is $\{k_t, a_t\}$, we assume a set of fiscal rules, $\tau(k_t, a_t)$ and T_t of the form:

$$\tau(k_t, a_t) = \tau^* + \beta \frac{f(k_t) - f(k^*)}{f(k^*)} + \gamma \frac{a_t - a^*}{a^*} \quad (14)$$

$$\forall t \quad : \quad T_t = \overline{T} \quad (15)$$

In the terminology of Davig et. al. (2011) equation (15) states that transfers are active. Specifically, we assume that lump-sum transfers are constant at their steady state level. Equation (14) exhibits the correlation of the income tax rate with the state variables. Terminology is important at this point. We define procyclical fiscal policy when tax rates are negatively associated with output. This component of fiscal policy is captured by β which also indicates the correlation between the tax rate and the primary deficit. The correlation between the tax rate and the secondary deficit is captured by γ. In the terminology of Leeper (1991) fiscal policy is considered 'active ' if it lets government liabilities grow at a rate greater than the real interest rate. In models where the government has access to lump-sum taxation, an active fiscal policy is represented by a rule where the government increases its [lump-sum] tax revenues in response to an increase in its liabilities by less than the amount $\rho(a_t - a^*)$. In our model, where lump-sum taxation is not accessible, the threshold for γ under which fiscal policy should be considered as 'active ' is addressed in detail in following sections. Timing is also important. All the debt is inherited from the past and the government is only setting the terms to repay it. Note that the fiscal policy sets the tax rate only after real output and the real value of government liabilities had been realized, whereas monetary policy responds to expected changes in nominal prices. Accordingly, we consider two stocks - the capital stock, and the real value of government liabilities - as predetermined variables.

2.2.3 Limitations to the Fiscal Rule

The Laffer Curve The fiscal rule (14) is not without limitations. Let $\varphi_{(\tau_t, y_t)} \equiv \frac{\partial \ln(\tau_t y_t)}{\partial \ln(\tau_t)} = 1 + \frac{\partial \ln(y_t)}{\partial \ln(\tau_t)}$ denote the marginal revenue generated from an increase in taxes. The second term is negative as higher taxes decrease output, so the elasticity of tax revenues with respect to tax rates is less than one. The peak of the "Laffer curve" is where the elasticity, $\varphi_{(\tau_t, y_t)}$, is zero. Similarly, the "slippery side" of the Laffer curve is where the elasticity is negative. Does the model economy exhibit a "Laffer curve" type relation? In this economy $y_t = f(k_t)$, accordingly $\varphi_{(\tau_t, y_t)} = 1 + \frac{\tau_t}{f(k_t)} \frac{\partial f(k_t)}{\partial \tau_t} = 1 + \frac{\tau_t}{f(k_t)} f'(k_t) \frac{dk_t}{d\tau_t}$. We cannot say much about the dynamic Laffer curve. However, we can characterize the Laffer curve near the steady state. It is straightforward to obtain[4] that $\frac{dk^*}{d\tau^*} = \frac{1}{1-\tau^*}\frac{f'(k^*)}{f''(k^*)}$. Thus, the slope of the Laffer curve in the steady state is:

$$\varphi_{(\tau^*)} = 1 + \frac{\tau^*}{1-\tau^*}\xi_k \tag{16}$$

where $\xi_k \equiv \frac{[f'(k^*)]^2}{f(k^*)f''(k^*)}$ .This expression implies that a unique maximum of the Laffer curve is obtained at $0 < \overline{\tau} = \frac{1}{1-\xi_k} < 1$. To get some intuition, consider a Cobb-Douglas production technology $f(k) = k^{\epsilon}$. For this type of production function, $\xi_k = \frac{\epsilon}{\epsilon-1}$ for all k, and the peak of the Laffer curve is where $\overline{\tau} = 1 - \epsilon$. Commonly used values of ϵ for developed economies imply that $\overline{\tau}$ exceeds 0.5.[5]

Political intolerance to raising tax rates Most authors believe that both economic and political factors weigh heavily in the fiscal limit calculus. Davig et. al. (2010, 2011) and Davig and Leeper (2010) posit that there is a regime in which higher debt is financed by higher marginal tax rates. As tax rates increase, political dissatisfaction increases which in turn raises the probability that the economy will hit its fiscal limit at which point tax

[4]By applying the implicit function theorem on equation (23) below.
[5]Reliable estimates are found in Trabandt and Uhlig (2010).

rates can no longer rise. This can happen much before the elasticity of tax revenues falls to zero. Although this type of fiscal limit is exogenous to our model, we can mimic political intolerance to raising tax rates by assuming a fiscal rule where $\beta = \gamma = 0$.

3 General Equilibrium

In equilibrium, the goods market clears

$$f(k_t) = c_t + I_t \tag{17}$$

assuming a positive nominal interest rate the money market clears such that

$$m_t = \frac{1}{\nu}(c_t + I_t) \tag{18}$$

and government liabilities equal household's assets.

Using the monetary policy rule, imposing market clearing conditions, and assuming that the elasticity of intertemporal substitution in consumption is constant, we arrive at the following characterization of the general equilibrium of the economy:

Proposition 1 *In equilibrium, the aggregate dynamics satisfy the following ODE system:*

$$\frac{\dot{c}_t}{c_t} = \sigma\left[\frac{1-\tau(k_t,a_t)}{1+\frac{1}{\nu}R(\pi_t)}f'(k_t)-(\rho+\delta)\right] \tag{19}$$

$$\dot{\pi}_t = \frac{\nu+R(\pi_t)}{\alpha}\left\{[R(\pi_t)-\pi_t]-\left[\frac{1-\tau(k_t,a_t)}{1+\frac{1}{\nu}R(\pi_t)}f'(k_t)-\delta\right]\right\} \tag{20}$$

$$\dot{k}_t = f(k_t)-c_t-\delta k_t \tag{21}$$

$$\dot{a}_t = [R(\pi_t)-\pi_t]\,a_t+T_t-\left[\tau(k_t,a_t)+\frac{1}{\nu}R(\pi_t)\right]f(k_t) \tag{22}$$

Equation (19) is an Euler equation, where $\sigma > 0$ denotes the elasticity of intertemporal substitution in private consumption. In our economy the marginal product of capital is distorted by the liquidity constraint and by the income tax. Notice that with no distortions equation (19) is reduced to the familiar Ramsey type Euler equation. Equation (20) was obtained by taking a time derivative from the first order condition ($5a$) and substituting in equations (6). It corresponds to a Fisher equation in which the nominal rate of interest varies with expected inflation and the real rate of interest. Note that the effect of liquidity constraints on investment are similar to those of adjustment costs. As a consequence, an instantaneous no-arbitrage condition between capital assets and financial assets does not hold. However, according to equation (20), in equilibrium an intertemporal no-arbitrage prevails. Finally, equations (21)-(22) were obtained by substituting market clearing conditions (17)-(18) into equations (8)-(9). At this point, the set of equations (19)-(22) internalizes the government's policies and market clearing and can therefore be viewed as the solution to the central planner's problem.

Definition *A perfect-foresight equilibrium is a set of sequences* $\{c_t, \pi_t, k_t, a_t, \tau_t, T_t, R_t\}$ *and an initial price level* $P_0 > 0$ *satisfying (17)-(22) given* $M_0 + B_0 > 0$ *and* $k_0 > 0$*.*

3.1 Steady-State Equilibrium

It follows from equation (19) that in a steady state

$$f'(k^*) = (\rho + \delta) \frac{1 + \frac{1}{\nu} R^*}{1 - \tau^*} \tag{23}$$

where τ^* denotes a steady-state income tax rate and R^* is a steady-state rate of interest. From equations (20) and (23), R^* must satisfy

$$R^* = \rho + \pi^* \tag{24}$$

where π^* is the steady-state rate of inflation. Equation (21) implies that the steady-state consumption is

$$c^* = f(k^*) - \delta k^* \tag{25}$$

Finally, it follows from equation (22), that in a steady-state equilibrium government liabilities must satisfy $a^* = \frac{1}{\rho}\left[f(k^*)(\tau^* + \frac{1}{\nu}R^*) - \overline{T}\right]$. Let $\widetilde{a}^* \equiv \frac{a^*}{f(k^*)}$, $\widetilde{T}^* \equiv \frac{\overline{T}}{f(k^*)}$ denote the steady state debt-to-GDP and transfers-to-GDP ratios, respectively, then we obtain that a sustainable debt level must satisfy

$$\widetilde{a}^* = \frac{1}{\rho}\left[\tau^* + \frac{1}{\nu}R^* - \widetilde{T}^*\right] \tag{26}$$

Equation (26) specifies that the sustainable debt-to-GDP level equals the present value of total surplus-to-GDP. [total surplus being tax revenues plus seigniorage revenues minus transfers] Note that an equilibrium trajectory $\{c_t, \pi_t, k_t, a_t, \tau_t, T_t, R_t\}_0^\infty$ should converge to the steady state $\{c^*, \pi^*, k^*, a^*, \tau^*, T^*, R^*\}$ whereas the aggregate dynamics in equations (19) - (22) imply that a steady-state equilibrium determines only 4 variables.[6] It is thus straightforward to show the following:

[6]Equations (23)-(26) demonstrate that we have four equations and seven variables.

Proposition 2 *A necessary condition for steady state determination is that the government proclaims three explicit targets.*

Proposition 2 follows mainly from the feasibility condition demonstrated by equation (26). The steady state is sustained only if the revenues from taxes and seigniorage equal the sum of transfers and debt service. Thus, as equation (26) links $\pi^*, \tau^*, a^*, \overline{T}$ to a balanced budget condition, three targets should be specified explicitly, and the fourth is implied by the stipulation to run a balanced budget in the steady state. Proposition 2 has the following implications:

Corollary 1 *Where government proclaims only fiscal targets such as* $\left\{\tau^*, \widetilde{a}^*, \widetilde{T}^*\right\}$, *the inflation target is implied according to* $\pi^* = \nu\left(\rho\widetilde{a}^* + \widetilde{T}^* - \tau^*\right) - \rho$ *and lump-sum transfers are then set according to* $\overline{T} = \widetilde{T}^* f(k^*)$.

According to Corollary 1 proclaiming $\left\{\tau^*, \widetilde{a}^*, \widetilde{T}^*\right\}$ implies that transfers are passive because first a sustainable inflation target is obtained according to the feasibility condition (26). Given a sustainable inflation target and a tax rate, output is determined according to equation (23) and only then fiscal policy can determine the size of lump-sum transfers according to $\overline{T} =$

$\widetilde{T}^* f(k^*)$. Thus, fiscal policy that emphasizes active transfers should proclaim its targets according to Corollary 2.

Corollary 2 *Where government proclaims that the explicit set of targets is $\left\{\tau^*, \pi^*, \overline{T}\right\}$, $\widetilde{a}^*$ is implied directly by condition (26).*

In the rest of the paper we assume that the government explicitly proclaims $\left\{\tau^*, \pi^*, \overline{T}\right\}$.

3.2 Equilibrium Dynamics

3.2.1 Price level determination

Solving equation (22), and letting $t \rightarrow \infty$ yields the well known assertion that market equilibrium requires intertemporal government budget balance:

Lemma 1 *In equilibrium*

$$0 = \lim_{t\rightarrow\infty} e^{-\int_0^t [R(\pi_s)-\pi_s]ds} a_t = a_0 - \int_0^\infty e^{-\int_0^t [R(\pi_s)-\pi_s]ds} \left\{ f(k_t)\left[\tau_t + \tfrac{1}{\nu}R(\pi_t)\right] - \overline{T} \right\} dt$$

Lemma 1 follows from: a) solving equation (22) which internalizes the idea that in equilibrium households' assets equal government's liabilities; and b) imposing conditions (10)-(11) that the households' intertemporal budget constraint holds with equality. Note that substituting the fiscal rule (14) into (22) yields that government liabilities evolve according to:

$$\dot{a}_t = \left[R(\pi_t) - \pi_t - \gamma \frac{f(k_t)}{a^*}\right] a_t - f(k_t)\left[\tau^* + \beta \frac{f(k_t) - f(k^*)}{f(k^*)} - \gamma + \frac{1}{\nu} R(\pi_t)\right] + \overline{T} \tag{27}$$

Solving equation (27) for a_t we obtain that:

$$Q_t a_t = a_0 - \int_0^t Q_s \left\{ f(k_s)\left[\tau^* + \beta \frac{f(k_s) - f(k^*)}{f(k^*)} - \gamma + \frac{1}{\nu} R(\pi_s)\right] - \overline{T} \right\} ds \tag{28}$$

where $Q_t \equiv e^{-\int_0^t \left[R(\pi_s) - \pi_s - \gamma \frac{f(k_s)}{a^*}\right] ds}$. Letting $t \to \infty$ and rearranging we obtain that:

$$\lim_{t\to\infty} e^{-\int_0^t [R(\pi_s)-\pi_s]ds} a_t \tag{29}$$

$$= \lim_{t\to\infty} \left[\frac{a_0 - \int_0^t Q_s \left\{ f(k_s) \left[\tau^* + \beta \frac{f(k_s)-f(k^*)}{f(k^*)} - \gamma + \frac{1}{\nu} R(\pi_s) \right] - \overline{T} \right\} ds}{e^{\int_0^t \gamma \frac{f(k_s)}{a^*} ds}} \right]$$

Equation (29) has several implication. First, according to Lemma 1 the left hand side of equation (29) equals zero in equilibrium. Thus, we must choose γ in a range so as to ensure that the right hand side of equation (29) equals zero. Second, we must ensure that the discount factor, Q_s, is contracting, otherwise the integral on the right hand side is not defined. Clearly, these requirements have substantial effect on the range of admissible fiscal policies. Proposition 3 specifies this range:

Proposition 3 *A monetary market equilibrium exists iff* $\gamma < \rho \widetilde{a}^*$ *and equilibrium trajectory is unique. The equilibrium price level is then determined to satisfy*

$$\frac{B_0+M_0}{P_0} = \int_0^\infty Q_s \left\{ f(k_s) \left[\tau^* + \beta \frac{f(k_s)-f(k^*)}{f(k^*)} - \gamma + \frac{1}{\nu} R(\pi_s) \right] - \overline{T} \right\} ds$$

Proposition 3 shows that in order to pin-point the level of nominal prices in a market equilibrium, the fiscal authority must let it's liabilities grow at a rate greater than the real interest rate. Consequently, the present discounted value of real government liabilities is not expected to vanish. And so, the price level must play an active role in bringing about fiscal solvency in equilibrium. This idea has been emphasized by the fiscal theory of prices and is discussed extensively in Woodford (1995) and Cochrane (2001, 2005). In this paper, unlike previous literature, the dichotomy between real determinacy and nominal determinacy does not prevail as real determinacy is a necessary condition for nominal determinacy. Thus, obtaining the initial price level depends not only on fiscal policy but on monetary policy as well. This issue receives further attention in the next section.

3.2.2 Transitional Dynamics

In this section we characterize the types of policy interactions that induce a unique trajectory. According to equations (19)-(22) and the policy rules (13)-(14), all the variables are a mapping in the (c, π, k, a) space. A linear approximation to equations (19)-(22) near the steady state is obtained through the system

$$\dot{x_t} = A \times (x_t - \overline{x}) \tag{30}$$

where

$$x_t \equiv \begin{bmatrix} c_t \\ \pi_t \\ k_t \\ a_t \end{bmatrix} \quad \overline{x} \equiv \begin{bmatrix} c^* \\ \pi^* \\ k^* \\ a^* \end{bmatrix} \tag{31}$$

$$A \equiv \begin{bmatrix} 0 & -\frac{\sigma\alpha\widetilde{c}^*(\rho+\delta)}{\nu+R^*} f^* & -\frac{\sigma\widetilde{c}^*}{1+\frac{1}{\nu}R^*} f_k^{*2}\left(\beta + \frac{\tau^*}{1-\varphi^*}\right) & -\gamma\frac{\sigma(\rho+\delta)}{1-\tau^*}\frac{\widetilde{c}^*}{\widetilde{a}^*} \\ 0 & \rho+\delta+(\nu+R^*)\frac{\alpha-1}{\alpha} & \frac{\nu}{\alpha}\frac{f_k^{*2}}{f^*}\left(\beta+\frac{\tau^*}{1-\varphi^*}\right) & \gamma\frac{\nu}{\alpha}\frac{f_k^*}{f^*}\frac{1}{\widetilde{a}^*} \\ -1 & 0 & f_k^* - \delta & 0 \\ 0 & f^*\left[(\alpha-1)\tilde{a}^* - \frac{\alpha}{\nu}\right] & -f_k^*\left[\frac{1}{\nu}R^* + \beta + \tau^*\right] & \rho - \frac{\gamma}{\widetilde{a}^*} \end{bmatrix}$$

[Asterisk denote steady state levels. $f_k^*, f^*, \tilde{a}^*, \widetilde{c}^*$ are marginal product of capital, GDP, debt-to-GDP, and consumption-to-GDP, respectively.].

and $k_t, a_t,$ are predetermined state variables.

When A has no eigenvalue with zero real part, the steady state $\overline{x}$ is a hyperbolic fixed point and the asymptotic behavior of solutions near it, and hence its stability type, is determined by the linearization[7]. Let $r_i \;\; i = 1, .., 4$ denote the eigenvalues of A, then by calculating the determinant and trace of A we obtain that:

$$r_1 r_2 r_3 r_4 = -\left[\widetilde{c}^{*} \nu \sigma \rho f_k^{*2}\right] \frac{\alpha - 1}{\alpha} \left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho \tilde{a}^*}\left(\frac{1}{\nu} R^* - \frac{\tau^* \varphi^*}{1-\varphi^*}\right)\right] \tag{32}$$

$$r_1 + r_2 + r_3 + r_4 = 2\rho + (\nu + R^*)\frac{\alpha - 1}{\alpha} + (\rho + \delta)\frac{1 + \frac{1}{\nu} R^*}{1 - \tau^*} - \frac{\gamma}{\tilde{a}^*} \tag{33}$$

and Proposition 5 follows directly from equation (32),

Proposition 5 *The steady state* $\overline{x}$ *is hyperbolic if and only if*

$\frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}\left(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}\right)\right] \neq 0.$

Also note that the dynamic system (30) has two predetermined variables. As a result, equilibrium is determinate if, and only if, the system has two stable roots. We can thus obtain a necessary condition for equilibrium de-

[7] See Guckenheimer and Holmes (1983) Theorem 1.3.1 - The Hartman-Grobman Theorem, and Theorem 1.3.2 - The Stable Manifold Theorem for a Fixed Point.

terminacy:

Proposition 6 *A necessary condition for determinacy of equilibrium is*

$\frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*})\right] < 0.$

That is, a monetary policy such that $\alpha > 1$ *must interact with a fiscal policy such that* $\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}) < 0$ *, and a monetary policy such that* $\alpha < 1$ *must interact with a fiscal policy such that* $\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}) > 0$*. Otherwise a market equilibrium with no sovereign default does not exist.*

We next discuss policy regimes from which we can derive sufficient conditions for equilibrium determinacy.

4 Stabilizing Monetary-Fiscal Interactions

In what follows we focus on a baseline regime for which it is possible to obtain sufficient conditions for equilibrium determinacy. We then perturb the baseline regime so as to approximate the general case. Consider a fiscal rule where $\gamma = 0$. According to this rule, income tax rates respond only to deviations of output from its long run level. Since we assume that lump-sum transfers are constant, this also implies that the tax rate responds to deviations of the primary deficit from its target. Note that under this fiscal rule $\gamma = 0 < \rho\widetilde{a}^*$, and according to Proposition 3 it satisfies a necessary condition for price level determination. However, fiscal solvency also depends on whether the monetary-fiscal regime induces a unique trajectory. Substituting $\gamma = 0$ into eq. (31) yields that:

$$A_{[\gamma=0]} = \begin{bmatrix} \widehat{A_1} & 0 \\ \widehat{A_2} & \rho \end{bmatrix}$$

and where $\widehat{A_1}$ is the upper left 3×3 submatrix of A, $\widehat{A_2}$ is the 1×3 row vector $\{A_{4,1}, A_{4,2}, A_{4,3}\}$ and the zero stands for a 3×1 vector of zeros. Examining $A_{[\gamma=0]}$, the dynamics of (c, π, k) are independent of government liabilities. This feature has two implications: (a) one eigenvalue of the (c, π, k, a) system is $\rho > 0$; (b) the remaining three eigenvalues are determined by $\widehat{A_1}$ so that the dynamics of (c, π, k) are completely determined by $\widehat{A_1}$. It is straightforward to show that the three remaining eigenvalues satisfy:

$$r_1 r_2 r_3 = -\left[\widetilde{c}^* \nu\sigma\rho f_k^{*2}\right] \frac{\alpha - 1}{\alpha} \left[\beta + \frac{\tau^*}{1 - \varphi^*}\right] \tag{34}$$

$$r_1 + r_2 + r_3 = \rho + (\nu + R^*)\frac{\alpha - 1}{\alpha} + (\rho + \delta)\frac{1 + \frac{1}{\nu}R^*}{1 - \tau^*} \tag{35}$$

and the main theoretical results of this paper are arrive directly from equations (34)-(35):

Proposition 7 *If fiscal policy targets only the primary deficit, then a sufficient condition for equilibrium determinacy is that the monetary-fiscal regime acts according to:* $\alpha < \frac{1}{1+\frac{\rho}{\nu+R^*}+\frac{\rho+\delta}{\nu(1-\tau^*)}} < 1$ *and* $\beta > -\frac{\tau^*}{1-\varphi^*}$.

Corrolary 3 *Consider a regime that induces equilibrium near a hyperbolic steady state* $\overline{x}$ *where fiscal policy targets the primary deficit. Assume that its policy stances are* $\overline{\alpha}$,$\overline{\beta}$ and that $\overline{\gamma} = 0$. *Then perturbations to* γ *in the neighborhood of* $\overline{\gamma} = 0$ *do not change the phase portrait of* $\overline{x}$ *as long as* γ *is not perturbed until its bifurcation point. Specifically, if* $\overline{\alpha} < \frac{1}{1+\frac{\rho}{\nu+R^*}+\frac{\rho+\delta}{\nu(1-\tau^*)}}$ *and* $\overline{\beta} > -\frac{\tau^*}{1-\varphi^*}$ *then a complex monetary fiscal regime will also induce equilibrium as long as it satisfies:*

$$\frac{\overline{\alpha}-1}{\overline{\alpha}}\left[\overline{\beta}+\frac{\tau^*}{1-\varphi^*}+\frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^*-\frac{\tau^*\varphi^*}{1-\varphi^*})\right] < 0 \tag{36}$$

Interpretation. Assuming endogenous output and no access to lump-sum taxation adds at least two qualities to the usual story: First, the fiscal authority may hit a fiscal limit when income taxes are close to the peak of their Laffer curves. Second, fiscal policy may cause arbitrage effects. Households optimize on their wealth portfolio which consists of productive capital, nominal bonds, and cash. Bonds yield a real return that equals the nominal rate of interest minus inflation whereas an increment to the stock of capital yields a real return that is distorted by an income tax and a liquidity requirement. At the private level bonds and capital are perfect substitutes. As a result, any change in the income tax rate or in the nominal rate of interest induces arbitrage effect as well as wealth effect. This complication can cause multiplicity of equilibria which, as demonstrated in Proposition 3, is detrimental to the valuation of government debt. Thus, the interaction between fiscal and monetary policies must bring about both real and nominal determinacy. To play this role, the interaction between monetary and fiscal policies must comply with the following principles. Firstly, in all regimes, monetary policy is passive. Specifically, the monetary rule must exhibit $\alpha < \frac{1}{1+\frac{\rho}{\nu+R^*}+\frac{\rho+\delta}{\nu(1-\tau^*)}}$. Secondly, in all regimes tax-rate responses to output must exhibit $\beta + \frac{\tau^*}{1-\varphi^*} > 0$. These results arrive from Proposition 7. Finally, applying Corollary 3, we arrive at three variations of tax-rate responses to deviations of government debt from its sustainable level:

Table 1 - Monetary-Fiscal Regimes and responses to government debt

Regime	Tax revenues elasticity	Response to Gov. Debt
FD		$\gamma = 0$
FC	$\frac{1}{\nu}R^* < \frac{\tau^*\varphi^*}{1-\varphi^*}$	$\gamma < Min[\rho\widetilde{a}^*, \frac{\beta+\frac{\tau^*}{1-\varphi^*}}{\frac{\tau^*\varphi^*}{1-\varphi^*}-\frac{1}{\nu}R^*}\rho\widetilde{a}^*]$
FC-Laffer	$\frac{1}{\nu}R^* > \frac{\tau^*\varphi^*}{1-\varphi^*}$	$-\frac{\beta+\frac{\tau^*}{1-\varphi^*}}{\frac{1}{\nu}R^*-\frac{\tau^*\varphi^*}{1-\varphi^*}}\rho\widetilde{a}^* < \gamma < \rho\widetilde{a}^*$

In the spirit of Proposition 3, a restriction that $\gamma < \rho\frac{a^*}{f(k^*)}$ obtains in all regimes which implies that the economy resides in regime-F. As a result, monetary policy stabilizes the economy by preventing debt service from growing too rapidly. By letting the nominal interest respond to inflation at a magnitude that is less than $\frac{1}{1+\frac{\rho}{\nu+R^*}+\frac{\rho+\delta}{\nu(1-\tau^*)}}$ the central bank keeps the real rate of interest below its long run level throughout the era of debt devaluation.

All three regimes share the principle that fiscal policy must not become a source of macroeconomic instability. This idea is demonstrated by the restriction that β should not be too negative. Let $DMKP_t \equiv \frac{1-\tau(k_t,a_t)}{1+\frac{1}{\nu}R(\pi_t)}f'(k_t) - \delta$ denote the distorted marginal product of capital net of depreciation, then near the steady state $\frac{d\ln DMKP_t}{d\ln f(k_t)} \approx -\frac{1}{1-\tau^*}\left[\beta + \frac{\tau^*}{1-\varphi^*}\right]$. This approximation indicates that the evolution of after tax marginal product of capital along an equilibrium trajectory is sensitive to fiscal responses to output. Take for example a rule that exhibits $\beta+\frac{\tau^*}{1-\varphi^*} < 0$. In this case, the after tax marginal product of capital is positively associated with output. Clearly, responses of this type are undesired and the intuition is the following: start from a steady state equilibrium, and suppose that the future return on capital is expected to increase.[8] Without distorting taxes, indeterminacy cannot occur since a higher capital stock is associated with a lower rate of return under constant returns to scale. However, where income taxes decrease when the return on capital is increasing, the after tax return on capital rises even further thus validating agents' expectations and any such trajectory is consistent with equilibrium. By contrast, a stance such that $\beta + \frac{\tau^*}{1-\varphi^*} > 0$ reduces higher anticipated returns on capital thus preventing flees from bonds to capital and expectations from becoming self fulfilling.

All three regimes have similar implication as to how should the fiscal authority respond to deviations of debt from its sustainable level. All regimes, including the case where the fiscal authority responds only to primary deficits and is silent when government debt diverges from its sustainable level, must satisfy inequality (36). It is straightforward to show that $\frac{d\ln DMKP_t}{d\ln a_t} \approx -\frac{1}{1-\tau^*}\gamma$. Having that, all the regimes in Table 1 imply that $y^* d\ln y_t(\frac{\tau^*\varphi^*}{1-\varphi^*} - \frac{1}{\nu}R^*) < \rho a^* d\ln a_t$.[9] That is, given a positive shock to government debt all regimes

[8] Higher anticipated returns on capital in this model arrive from future tax cuts. In stochastic versions of this model higher anticipated returns can also arrive from belief driven spurts or from news shock about future productivity.

[9] We obtain this inequality by starting from inequality (36). With a passive monetary

must allow government debt to grow at a rate grater than the addition to tax revenues. Any regime that is not consistent with this upshot is not consistent with equilibrium in regime-F, and this is exactly the idea of the fiscal theory of prices that corresponds to economies with distorting taxation.

5 Computation

Computing the entire equilibrium trajectory involves a simultaneous determination of the initial level of nominal prices and the entire state space. The intuition is straightforward: In a fiscal theoretic equilibrium, any shock causes a reevaluation of the existing stock of government debt so as to equate between the present value of the sum of future surpluses and the value of government debt. However, the entire equilibrium trajectory depends on the initial value of government debt. The crux of simultaneous determination is that the initial value of government debt should be obtained as a solution

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policy inequality (36) implies (I) $\overline{\beta}+\frac{\tau^*}{1-\varphi^*}+\frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^*-\frac{\tau^*\varphi^*}{1-\varphi^*}) > 0$. Note that (II) $\beta+\frac{\tau^*}{1-\varphi^*} = -(1-\tau^*)\frac{d\ln DMKP_t}{d\ln f(k_t)}$ and that (III) $\gamma = -(1-\tau^*)\frac{d\ln DMKP_t}{d\ln a_t}$. Finally note that (IV) $\mathrm{Si}\,gn[d\ln a_t] = \mathrm{Si}\,gn[d\ln f(k_t)]$ since in our model output decreases in the era of debt devaluation. Substituting (III) and (II) into (I) we arrive at $y^* d\ln y_t(\frac{\tau^*\varphi^*}{1-\varphi^*} - \frac{1}{\nu}R^*) < \rho a^* d\ln a_t$.

to a fixed-point problem.[10] After obtaining an initial value for non capital wealth and given an initial stock of productive capital, the model becomes a continuous-time variant of Blanchard and Kahn (1980). Impulse responses are then computed according to Sims (2002).

5.1 Calibration

The model is calibrated at an annual frequency because the purpose is to study the impact of monetary and fiscal policy interactions over a relatively long horizon. The annual (subjective) rate of time preference is set to $\rho = 0.03$. Preferences over consumption are logarithmic, $\sigma = 1$, the rate of capital depreciation is set to $\delta = 0.1$, and the elasticity of production technology is set to $\epsilon = 0.5$ so as to induce a steady state Laffer curve with a peak at 50 percent tax rate. We set $\nu = 2.4$ so money velocity in the steady state corresponds to the average U.S. monetary base velocity.

[10] The formalization and the resolution of this issue are discussed extensively in Gliksberg (2013). Specifically, the solution is obtained by applying the *Krasnoselski-Mann-Bailey Theorem.*

5.2 Results

5.2.1 Scenarios of lump-sum vis-a-vis distorting taxation

To understand how and why the model performs differently from the conventional models in regime-F, an exercise of policy experiments is performed. The experiments show how the economy responds to changes in the proclaimed inflation and tax rate targets under the scenarios of lump-sum and distorting taxation. The transition paths appear in detail in Appendix B. The aftermath of this experiment is the following:

(I) Increments to the tax rate target operate very differently in the two scenarios. In the lump-sum economy the sole effect is an instantaneous jump in the real value of government debt whereas other variables remain constant. In the distorting-tax economy, increments to the tax rate target send the economy to a transition towards a new steady state implied by the tax distortion. In addition, government debt is reevaluated so as to equate between its real value and the net present value of future surpluses along the new trajectory. Surprisingly, tax increases induce an increase in the real value of government debt even where the elasticity of tax revenues is zero or slightly negative [see figures B.1 and B.3]. For tax rate levels that do not exceed the post-war average U.S. Federal tax rate, the evaluation of real debt in the distorting-tax economy can be higher than the evaluation of real debt in the lump-sum tax economy. This result emerges when interest responses to inflation are low and when tax responses to the state variables are weak. However this is a short run phenomenon and in all regimes the long-run real value of government debt in the distorting-tax economy is lower than its lump-sum economy counterpart.

(II) Increments to the inflation rate target operate similarly in the two scenarios. As both economies have the same type of distortion - a liquidity constraint - an increase in the inflation target reduces the steady state stock of capital and sends the economies to transitions that are qualitatively similar. Increments to the inflation target increase the real value of government debt in both economies [see figures B.2 and B.4]. The addition to the real value of government debt arrives from an increase in seigniorage revenues along the transition path and a decrease in the discount factor of surpluses.

(III) When transfers permanently increase and tax rate targets and inflation targets are not modified, the only effect is a devaluation of debt. This effect can be interpreted as a market default. This upshot prevails under both scenarios of lump-sum and distorting taxation. [see figures B.5 and B.6].

5.2.2 Responses to a Fiscal Stress

To obtain prescriptions for debt-stabilizing regimes, an exercise of policy experiments is performed assuming a permanent increase in transfers. Two experiments are carried out. In the first, the inflation target is set to $\pi^* = 0.02$ and the tax rate target is set to $\tau^* = 0.23$. Although 23% is well above the post-war average U.S. Federal tax rate, it is within the benchmark limit of 24.25% assumed by Davig et. al. (2011). Lump-sum transfers are set so as to induce a primary surplus that equals 2.5 percent of GDP which implies that the steady-state debt-output ratio equals 1.527. This experiment is repeated for a tax rate target that is set to $\tau^* = 0.50$ which implies that the elasticity of tax revenues at the starting point of the experiment is zero.[11]

A persistent 5% increase in transfers induces a drop in sustainable debt-output ratios to 1.186 and 0.736 for the 23%-income-tax-rate economy and the 50%-income-tax-rate economy, respectively. Both exercises show impulse responses to a persistent 5% increase in transfers for various monetary-fiscal policy interactions. Figure 1 shows policy prescriptions where the starting point is a steady state with 23% tax rate.

[11] In the second experiment inflation target remains $\pi^* = 0.02$, and transfers correspond to a sustainable debt-output ratio of 1.527.

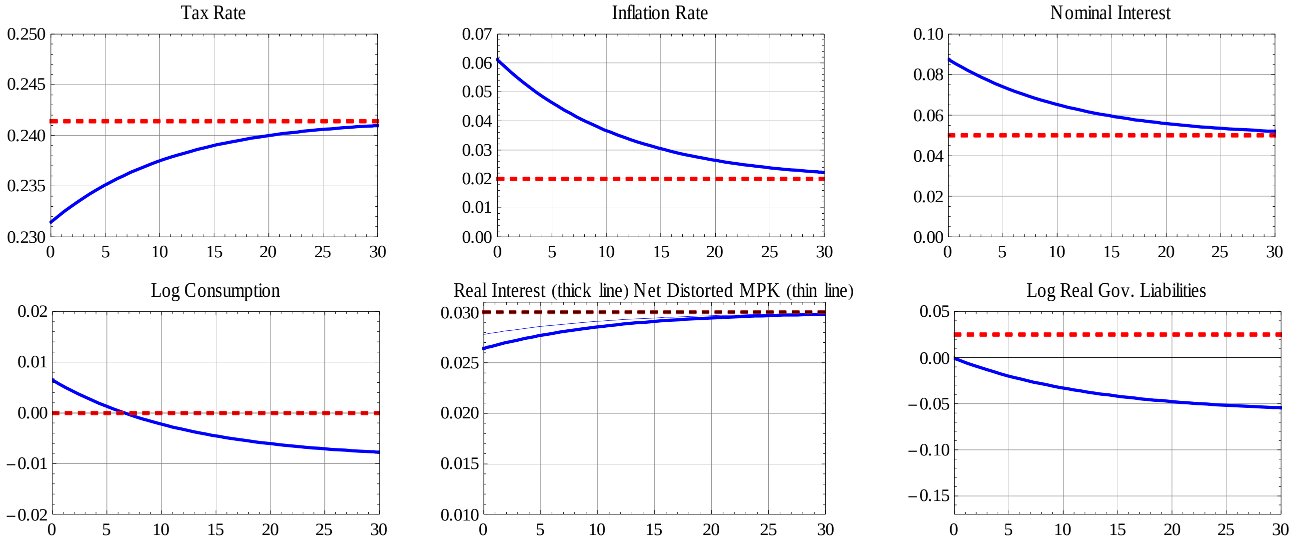

Figure 1.1: Policy parameters $\{\tau,\pi\}$={24.14%,2.%}, α=0.912809, β=-0.6586, γ=-0.00118623. Elasticity of tax revenues near target: 0.681782

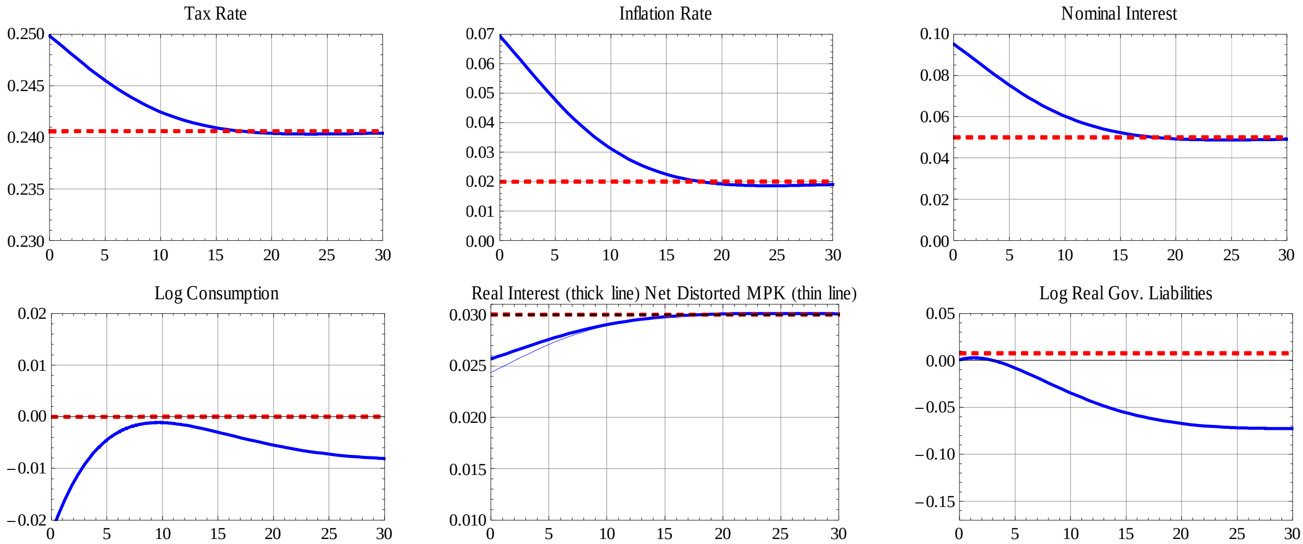

Figure 1.2: Policy parameters $\{\tau,\pi\}$={24.06%,2.%}, α=0.912873, β=0.5, γ=0.029. Elasticity of tax revenues near target: 0.683171

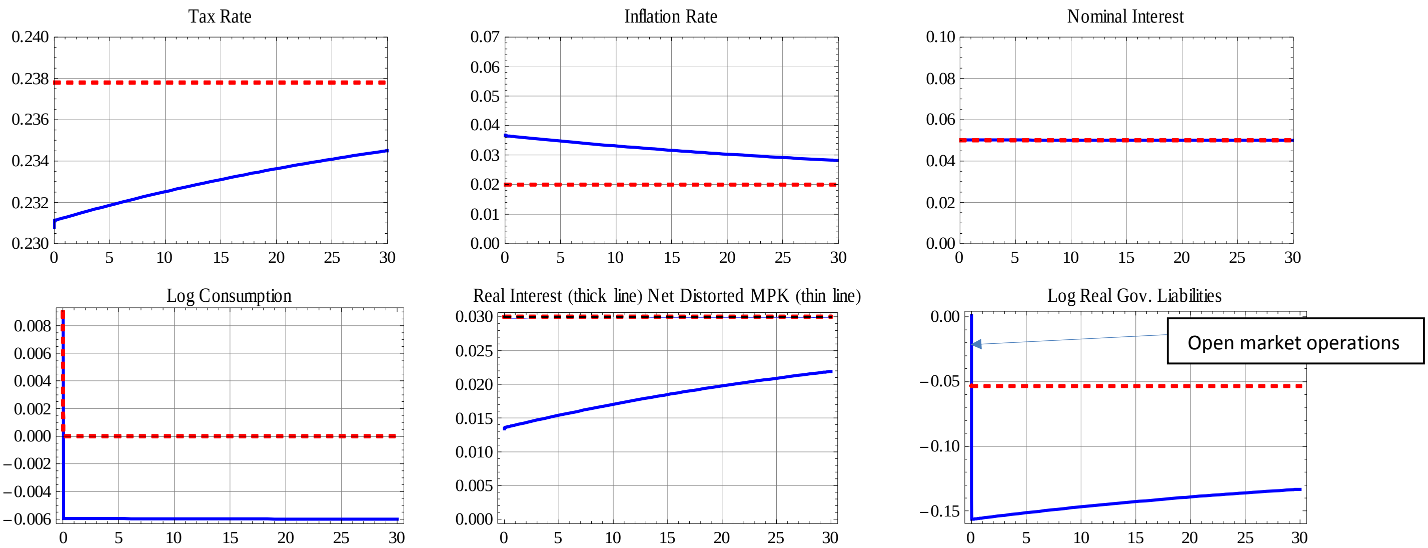

Figure 1.3: Policy parameters $\{\tau,\pi\}$={23.78%,2.%}, α=0.01, β=-0.6622, γ=-0.00182222. Elasticity of tax revenues near target: 0.688008

Figure 1: A permanent 5% increase in transfers where {Debt/GDP, τ, π}={153%,23%,2%}

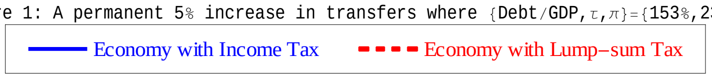

Figure 1 illustrates three policy prescriptions for the 23% tax-rate economy. All three prescriptions are successful in stabilizing the real value of government debt and letting it converge to its sustainable level. In Figures 1.1-1.2 monetary responses to inflation are at their highest admissible level under regime-F. Figure 1.2 exhibits impulse responses where the income tax rate responds more aggressively. Ceteris paribus, the economy with a more aggressive tax response exhibits higher levels of inflations. This result is consistent with the results of Davig et. al. (2011) but the mechanism is different. Higher distortionary tax reduces capital accumulation and work effort and depresses output and private consumption. Furthermore, when tax rate responds aggressively, the net distorted marginal product of capital [i.e. $\frac{1-\tau(k_t,a_t)}{1+\frac{1}{\nu}R(\pi_t)}f'(k_t)-\delta$] drops below the real rate of interest. In this case, capital is inferior to government debt, households hold on to their financial wealth, and a higher level of inflation is required to deplete government debt. This policy is not entirely disadvantageous as it shortens the process of devaluation nearly by half. Thus, where the elasticity of tax revenues is well above zero and given monetary policy, tax policy affect the extent and duration of fiscal inflation where aggressive tax responses increase inflation but shorten the process considerably. Figure 1.3 shows that given fiscal policy, passive monetary responses can further depress the rate of inflation. Specifically, under interest rate pegs the central bank purchases as much debt as needed so as to induce the market price of a government bond that implies the desired interest rate. If at the same time fiscal policy induces a net distorted marginal product of capital that is above the real rate of interest, open market purchases will successfully drain government debt. In Figure 1.3, the initial real value of government debt is unchanged relative to its value prior to the fiscal shock. However, this value drops sharply whereas inflation rates remain relatively low. This devaluation is caused by open market operations. Thus, given fiscal policy, monetary policy affects the extent and duration of fiscal inflation. Furthermore, a sensible monetary policy can reduce the long run levels of income tax required to restore solvency. In Figure 1.3 the long run tax rate target is increased by 0.78% whereas in Figure 1.1 - Figure 1.2 long run tax rate must increase by more than one percent.

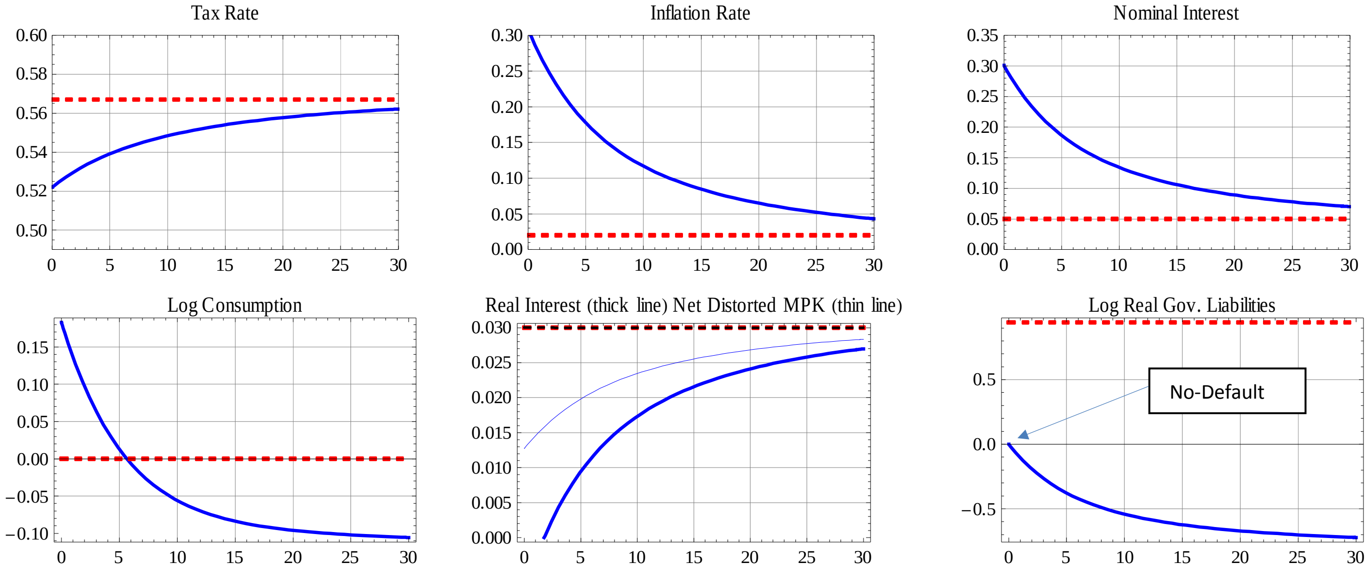

Figure 2.1: Policy parameters $\{\tau,\pi\}=\{56.7\%,2.\%\}$, $\alpha=0.869244$, $\beta=-0.333$, $\gamma=0.00190955$. Elasticity of tax revenues near target: -0.309469

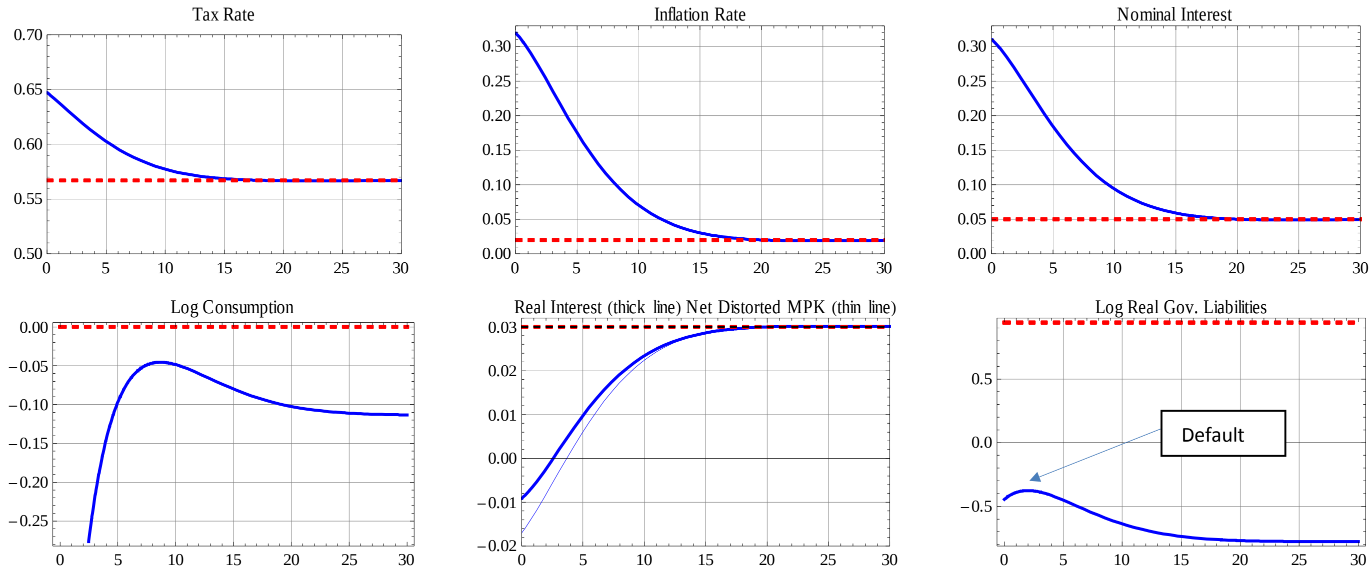

Figure 2.2: Policy parameters $\{\tau,\pi\}=\{56.7\%,2.\%\}$, $\alpha=0.869244$, $\beta=0.5$, $\gamma=0.00190955$. Elasticity of tax revenues near target: -0.309469

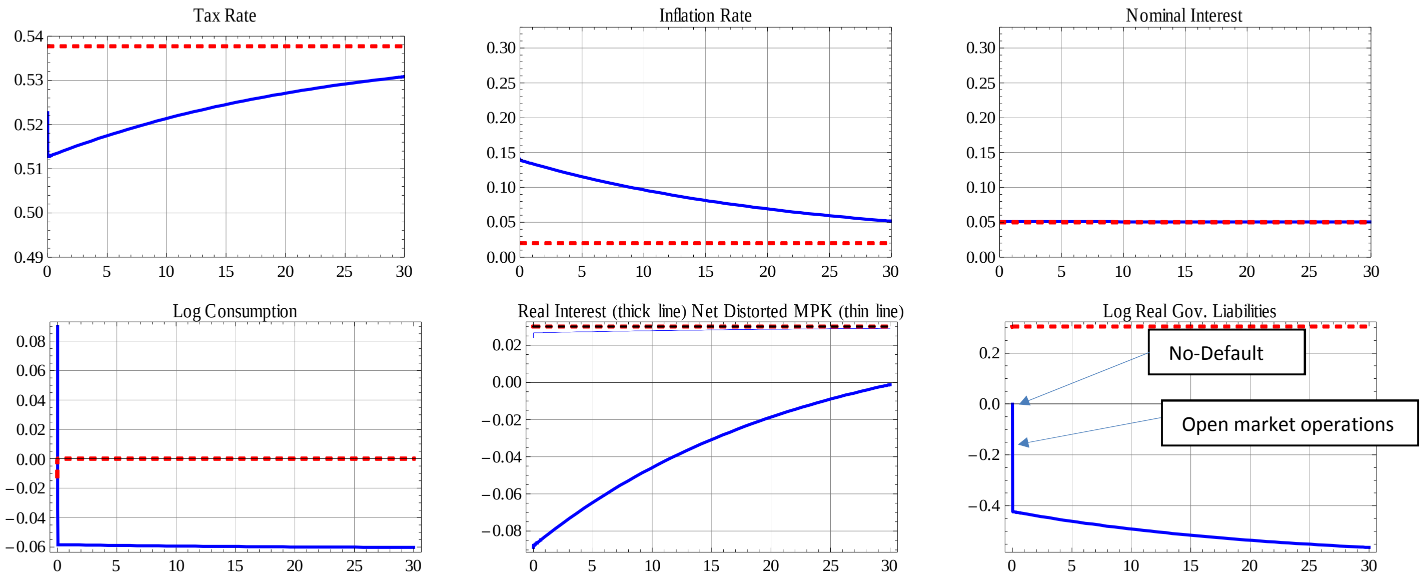

Figure 2.3: Policy parameters $\{\tau,\pi\}=\{53.77\%,2.\%\}$, $\alpha=0.01$, $\beta=-0.3623$, $\gamma=0.00911088$. Elasticity of tax revenues near target: -0.163098

Figure 2: A permanent 5% increase in transfers where $\{\text{Debt}/\text{GDP},\tau,\pi\}=\{153\%,50\%,2\%\}$

Economy with Income Tax — Economy with Lump–sum Tax

Figure 2 illustrates three policy prescriptions for the 50% tax-rate economy. Note that at the starting point of this experiment the economy resides at the peak of its Laffer curve. In general, there are three means to manipulate the value of government debt. These include changes in tax revenues which affect the stream of primary surpluses, inflation which reduces the real value of nominal liabilities, and open market operations that change the nominal amount of liabilities. Where elasticities of tax revenues are negative, tax rate responses are ineffective in increasing the flow of primary surpluses. However, tax rate responses can still interact with monetary policy so as to operate the two mechanisms at hand. In Figures 2.1-2.2 monetary responses to inflation are at their highest admissible level under regime-F. Figure 2.2 exhibits impulse responses where tax responds more aggressively. This regime leads to a default. In particular, we were not able to obtain any stabilizing policy interaction with aggressive tax responses where elasticities of tax revenues are zero or negative. As long as the fiscal rule induces a net distorted marginal product of capital that is less than the real rate of interest, open market operations are ineffective in reducing the amount of debt held by the private sector, and since tax revenues cannot increase the real value of government debt must drop - hence a default. In Figures 2.1 and 2.3 the real rate of interest is on the right side of the distorted marginal product of capital so fiscal inflation suffices to bring about fiscal solvency though at high levels. Figure 2.3 exhibits the crucial role of monetary policy. An interest rate peg cuts inflation levels nearly by half. Furthermore, the tax rate target is lower than its counterpart in Figure 2.1. Thus, a quick drain of debile government debt is imperative both for a substantial reduction in future inflation as well as a substantial reduction in future tax rates.

6 Conclusion

We construct a fiscal theoretic framework, with tax distortions, capital accumulation, and liquidity constraints, to examine policy regimes that stabilize government debt on the impact of a fiscal stress. In this model, the government has three degrees of freedom in choosing its long run targets. Other aggregates must adjust so as to balance the consolidated budget in the steady state. As a result, limitations to fiscal policy - such as a ceiling on transfers, a minimum tax rate, or a ceiling on debt to output ratio - emerge endogenously. Any shock to long run levels may render the existing debt unsustainable. Restoring fiscal solvency requires some degree of fiscal consolidation. However, an active fiscal stance that brings about fiscal inflation can devalue a substantial portion of government debt. Three monetary-fiscal regimes achieve this goal in a distorting tax scenario: (a) a regime where the fiscal authority targets only the primary deficit, (b) a. regime where the fiscal authority targets both the primary and the secondary deficits and has the ability to increase its tax revenues, (c) finally, a regime where the fiscal authority targets both the primary and the secondary deficits but is not able to increase its tax revenues. In all regimes monetary policy is passive and fiscal policy is active in the sense that it must allow government debt to grow at a rate grater than the growth rate of tax revenues. Assuming active transfers, tax hikes are imperative to restore fiscal solvency even if the elasticity of tax revenues is negative. Debt then devalues through two mechanisms: fiscal inflation and open market operations. Fiscal inflation can be moderated to a large extent if the central bank purchases large portions of government debt in the open market. Specifically we show that the interaction of interest rate pegs with a very slow adjustment to the income tax rate are most successful in restoring fiscal solvency with moderate rates of inflation throughout the era of debt devaluation.

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A. Proofs

Proof of Proposition 3 (I) Assume that $\boldsymbol{\gamma} < \rho\frac{a^*}{f(k^*)}$. Then Q_t is contracting. Assume now that the discount factor Q_t is contracting, then there is a finite $\bar{t}$ such that $\forall t > \bar{t}\ R(\pi_t) - \pi_t - \gamma\frac{f(k_t)}{a^*} > 0$. $R_t - \pi_t$ converges to ρ and $f(k_t)$ converges to $f(k^*)$. Thus, in the limit $\rho - \gamma\frac{f(k^*)}{a^*} > 0$.

(II) Proposition 3 argues that where $\boldsymbol{\gamma} < \rho\frac{a^*}{f(k^*)}$ the right hand side of equation (29) equals zero which implies that the transversality condition $\lim_{t\to\infty} e^{-\int_0^t [R(\pi_s)-\pi_s]ds} a_t$ is not violated. Where $0 \leq \boldsymbol{\gamma} < \rho\frac{a^*}{f(k^*)}$ this is straightforward. Where $\boldsymbol{\gamma} < 0$, it can be verified using L'Hospital's rule.

Proof of Proposition 5 Assume that $\frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*})\right] \neq 0$ then the multiple of eigenvalues is non zero which indicates that there is no zero eigenvalue. Assume now that $\frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*})\right] = 0$ then either $\alpha = 1$ or $\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}) = 0$. In what follows we show that either policies induce a zero eigenvalue, i.e. that there is a bifurcation at $\alpha = 1$ and given β there is a bifurcation at $\gamma = \rho\tilde{a}^*\frac{\beta + \frac{\tau^*}{1-\varphi^*}}{\frac{\tau^*\varphi^*}{1-\varphi^*} - \frac{1}{\nu}R^*}$. Note that implementing $\alpha = 1$ and $\beta + \frac{\tau^*}{1-\varphi^*} +$

$\frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}) = 0$ simultaneously, brings about a codimension two bifurcation.

Assume $\alpha = 1$ and $\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}) \neq 0$

Substituting $\alpha = 1$ into equation (31) we obtain that

$$A_{[\alpha=1]} \equiv \begin{bmatrix} 0 & -\frac{\sigma\alpha\widetilde{c}^*(\rho+\delta)}{\nu+R^*}f^* & -\frac{\sigma\widetilde{c}^*}{1+\frac{1}{\nu}R^*}f_k^{*2}\left(\beta + \frac{\tau^*}{1-\varphi^*}\right) & -\gamma\frac{\sigma(\rho+\delta)}{1-\tau^*}\frac{\widetilde{c}^*}{\widetilde{a}^*} \\ 0 & \rho+\delta & \frac{\nu}{\alpha}\frac{f_k^{*2}}{f^*}\left(\beta + \frac{\tau^*}{1-\varphi^*}\right) & \gamma\frac{\nu}{\alpha}\frac{f_k^*}{f^*}\frac{1}{\widetilde{a}^*} \\ A_{3,1} & 0 & A_{3,3} & 0 \\ 0 & A_{4,2} & A_{4,3} & A_{4,4} \end{bmatrix}$$

where $A_{i,j}$ $i,j = 1,..4$ are components of A specified in eq. (31), respectively. Where $\alpha = 1$ the central bank holds the real rate of interest constant. This policy induces a linear dependence between the first row and the second row of $A_{[\alpha=1]}$. Specifically, the second row is a multiplication of the first row by $-\frac{\sigma\alpha\widetilde{c}^*}{\nu+R^*}f^*$. Consequently $A_{[\alpha=1]}$ is singular.

Assume $\alpha \neq 1$ and $\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*}) = 0$.

Substituting $\widetilde{\gamma} \equiv \rho\tilde{a}^*\frac{\beta+\frac{\tau^*}{1-\varphi^*}}{\frac{\tau^*\varphi^*}{1-\varphi^*}-\frac{1}{\nu}R^*}$ into equation (31) we obtain that

$$A_{[\widetilde{\gamma}]} \equiv \begin{bmatrix} 0 & A_{1,2} & A_{1,3} & \psi A_{1,3} \\ 0 & A_{2,2} & A_{2,3} & \psi A_{2,3} \\ A_{3,1} & 0 & A_{3,3} & 0 \\ 0 & A_{4,2} & A_{4,3} & \psi A_{4,3} \end{bmatrix}$$

where $\psi \equiv \frac{\rho}{f_k^*\left[\frac{\tau^*\varphi^*}{1-\varphi^*}-\frac{1}{\nu}R^*\right]}$ is a constant. It is straightforward to notice that the determinant of $A_{[\widetilde{\gamma}]}$ equals zero which implies that $A_{[\widetilde{\gamma}]}$ is singular. Thus, we showed that a monetary-fiscal regime such that $\frac{\alpha-1}{\alpha}\left[\beta+\frac{\tau^*}{1-\varphi^*}+\frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^*-\frac{\tau^*\varphi^*}{1-\varphi^*})\right]=0$ brings about a non hyperbolic equilibrium and this concludes the proof.

Proof of Proposition 6

Preliminaries According to Proposition 5 if $\frac{\alpha-1}{\alpha}\left[\beta+\frac{\tau^*}{1-\varphi^*}+\frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^*-\frac{\tau^*\varphi^*}{1-\varphi^*})\right]<0$ then the steady state is hyperbolic. The rest of the proof to proposition 6 is based on the following Theorem and Definition which focus on hyperbolic fixed points.

Definition [Hirsch and Smale (1976) Chap.16] *Let $\overline{x}$ be a hyperbolic equilibrium, that is, the eigenvalues of $Df(\overline{x})$ have nonzero real parts. In this case, the index $ind(\overline{x})$ of $\overline{x}$ is the number of* *eigenvalues (counting multiplicities) of* $Df(\overline{x})$ *having negative real parts.*

The Stable Manifold Theorem

[Guckenhaimer and Holmes (1983) Theorem 1.3.2] *Suppose that* $\dot{x} = f(x)$ *has hyperbolic fixed point* $\overline{x}$*. Then there exists local stable and unstable manifolds* $W^s_{loc}(\overline{x}), W^u_{loc}(\overline{x})$*, of the same dimensions* n_s, n_u *as those of the eigenspaces* E^s, E^u *of the linearized system, respectively, and tangent to* E^s, E^u *at* $\overline{x}$*.* $W^s_{loc}(\overline{x}), W^u_{loc}(\overline{x})$ *are as smooth as the function* f*.*

Thus, the index of a hyperbolic fixed point is the dimension of the stable manifold. Given that we have two predetermined variables we also conclude that in the model economy, equilibrium $\overline{x}$ is determinate if and only if $ind(\overline{x}) = 2$. In what follows we prove Proposition 6.

Proof Note equation (32), the structural parameters ρ, σ are positive. Hence the sign of the right hand side of equation (32) is determined by the sign of $\frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\widetilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*})\right]$.

Table A.1: Index and equilibria in a four dimensional vector space with two predetermined variables

Sign(rr_1)	Sign(rr_2)	Sign(rr_3)	Sign(rr_4)	det(A)	Trace(A)	Index	Equilibrium
$+$	$+$	$+$	$+$	> 0	> 0	0	no-equilibrium
$-$	$+$	$+$	$+$	< 0	$\gtreqless 0$	1	no-equilibrium
$-$	$-$	$+$	$+$	> 0	$\gtreqless 0$	2	unique
$-$	$-$	$-$	$+$	< 0	$\gtreqless 0$	3	multiple
$-$	$-$	$-$	$-$	> 0	< 0	4	multiple

rr_i denotes the real part of eigenvalue r_i. In our model, equilibrium is determinate only where $ind(\overline{x}) = 2$. Where $ind(\overline{x}) = 0, 1$ the system has "too many" unstable roots. That is, there are fewer stable roots than predetermined variables and no convergent solution exists for arbitrary initial value of the predetermined variable[12]. Where $ind(\overline{x}) = 3, 4$ there are more stable roots than predetermined variables. In these cases, the transversality condition that the solution be convergent no longer suffices to ensure a unique solution, and thus, with no additional linear boundary conditions equilibrium is indeterminate.

Note that $ind(\overline{x}) = 2 \Rightarrow \det(A) > 0 \Rightarrow \frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*})\right] < 0$.

Thus where $\frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*} + \frac{\gamma}{\rho\tilde{a}^*}(\frac{1}{\nu}R^* - \frac{\tau^*\varphi^*}{1-\varphi^*})\right] > 0$ either $ind(\overline{x}) = 1$ hence

[12] This issue is discussed in detail in Blanchard and Kahn (1980) and in Buiter (1984).

no-equilibrium, or $ind(\overline{x}) = 3$ and hence indeterminacy and according to Proposition 3 a market equilibrium with no default does not exist. QED.

Proof of Proposition 7

Table A.2: Index and equilibria in a three dimensional vector space with two predetermined variables

Sign(rr_1)	Sign(rr_2)	Sign(rr_3)	$\det(\widehat{A_1})$	$\mathrm{tr}(\widehat{A_1})$	Index	Equilibrium
$+$	$+$	$+$	>0	>0	0	no-equilibrium
$-$	$+$	$+$	<0	$\gtreqless 0$	1	no-equilibrium
$-$	$-$	$+$	>0	$\gtreqless 0$	2	unique
$-$	$-$	$-$	<0	<0	3	multiple

Equilibrium is determinate only where $ind(\overline{x}) = 2$. $A_{[\gamma=0]}$ is block recursive with one positive eigenvalue at the lower right 1×1 submatrix, so we obtain the dimension of the stable manifold only by examining $\widehat{A_1}$. Observe Table A.2. $\det(\widehat{A_1}) > 0$ is a necessary condition. Furthermore, we must rule out the case where $ind(\overline{x}) = 0$ by requiring that policy also induce $\mathrm{tr}(\widehat{A_1}) < 0$. To conclude, we can make sure that $ind(\overline{x}) = 2$ by implementing a policy that brings about $r_1r_2r_3 > 0$ and $r_1 + r_2 + r_3 < 0$.

It follows from equation (34) that $r_1r_2r_3 > 0 \Leftrightarrow \frac{\alpha-1}{\alpha}\left[\beta + \frac{\tau^*}{1-\varphi^*}\right] < 0$. So

$\det(\widehat{A_1}) > 0$ under two regimes: $\{\alpha > 1$ and $\beta + \frac{\tau^*}{1-\varphi^*} < 0\}$ or $\{\alpha < 1$ and $\beta + \frac{\tau^*}{1-\varphi^*} > 0\}$. Under the first regime, we obtain from equation (35) that $\mathrm{tr}(\widehat{A_1}) > 0$ and we cannot rule out the possibility of No-equilibrium. However, in the second regime where $\alpha < 1$ we can ensure that $\mathrm{tr}(\widehat{A_1}) < 0$ by requiring that $\alpha < \frac{1}{1+\frac{\rho}{\nu+R^*}+\frac{\rho+\delta}{\nu(1-\tau^*)}} < 1$. *QED.*

Proof of Corollary 3

Preliminaries The main idea of this corollary is that perturbations are in the parameter space $\{\gamma\}$. Thus, starting from a determinate equilibrium, as long as γ does not reach its bifurcation point, the phase portrait of the (unchanged) steady state should not be affected by the perturbation. The formal proof of Corollary 3 follows directly from the following Theorems. Specifically, in the terminology of the Theorem 1 and 2 we choose $\overline{y} = \overline{x}$ and $g(y)$ that differs from $f(x)$ up to the perturbation of γ.

Theorem 1 [Hirsch and Smale (1976) Chap.16] *Let $f : W \rightarrow E$ be a C^1 vector field and $\overline{x} \in W$ an equilibrium of $\dot{x} = f(x)$ such that $Df(\overline{x}) \in L(E)$ is invertible. Then there exists a neighborhood $U \subset W$ of $\overline{x}$ and a neighborhood $\Re \subset \mho(W)$ of f such that for any $g \in \Re$*

there is a unique equilibrium $\overline{y} \in U$ of $\dot{y} = g(y)$. Moreover, if E is normed, for any $\epsilon > 0$ we can choose $\Re$ so that $|\overline{y} - \overline{x}| < \epsilon$.

Theorem 2 [Hirsch and Smale (1976) Chap.16] *Suppose that $\overline{x}$ is a hyperbolic equilibrium. In Theorem 1, then, $\Re$ and U can be chosen so that if $g \in \Re$, the unique equilibrium $\overline{y} \in U$ of $\dot{y} = g(y)$ is hyperbolic and has the same index as $\overline{x}$.*

Proof Consider a complex fiscal policy where $\gamma \neq 0$. Consider the system $\dot{x}_t = g_{[\gamma]}(x_t)$ where $\gamma = 0 \pm \varepsilon$, $\varepsilon > 0$, then a linearization reads

$$\dot{x}_t = \left[A_{[\gamma=0]} + \varepsilon\Delta\right] \times (x_t - \overline{x})$$

where

$$\Delta \equiv \begin{bmatrix} 0 & 0 & 0 & -\frac{\sigma(\rho+\delta)}{1-\tau^*}\frac{\widetilde{c}^*}{\widetilde{a}^*} \\ 0 & 0 & 0 & \frac{\nu}{\alpha}\frac{f_k^*}{f^*}\frac{1}{\widetilde{a}^*} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\widetilde{a}^*} \end{bmatrix}$$

Since $A_{[\gamma=0]}$ is invertible, by implicit function theorem, $\dot{x}_t = g_{[\gamma]}(x_t)$ continues to have a unique solution $\overline{\overline{x}} = \overline{x} + O(\varepsilon)$ near $\overline{x}$ for sufficiently small ε. Moreover, since we restrict the admissible monetary-fiscal regimes to a set that satisfies proposition 6, we ensure that $A_{[\gamma=0]} + \varepsilon\Delta$ is invertible which implies that $\overline{\overline{x}} = \overline{x}$ is the unique solution to $\dot{x_t} = \left[A_{[\gamma=0]} + \varepsilon\Delta\right] \times (x_t - \overline{x})$. Furthermore, since the linearized system has eigenvalues that depend continuously on ε, no eigenvalues can cross the imaginary axis if ε remains small with respect to the magnitude of the real parts of the eigenvalues of $A_{[\gamma=0]}$. Thus, the perturbed system has a unique fixed point with eigenspaces and invariant manifolds of the same dimensions as those of the unperturbed system and which are $\varepsilon-$close locally in position and slope to the unperturbed manifolds. To conclude, since the roots of $A_{[\gamma=0]}$ and the roots of $\left[A_{[\gamma=0]} + \varepsilon\Delta\right]_{\varepsilon=0}$ are identical, letting ε grow in any direction preserves the topological equivalence between the perturbed and the unperturbed systems as long as det $\left[A_{[\gamma=0]} + \varepsilon\Delta\right]$ does not change signs.

B. Solving the model under the scenarios of lump-sum and distorting taxation

Table B.1 Model Calibrations

	Parameter	Calibration
Subjective discount factor	ρ	0.03
Rate of capital depreciation	δ	0.1
Elasticity of intertemporal substitution	σ	1
Output elasticity	ϵ	0.5
Inflation rate target (annual)	π^*	0.02
Tax rate target	τ_1^*	0.23
Tax rate target	τ_2^*	0.50

Lump-sum taxes, in the lump-sum economy, are set to as to equate between tax revenues in the two economies in the steady state.

The x-axis indicates time measured in years.

Solid lines indicate throughout the distorting-tax economy whereas dashed lines indicate the lump-sum tax economy.

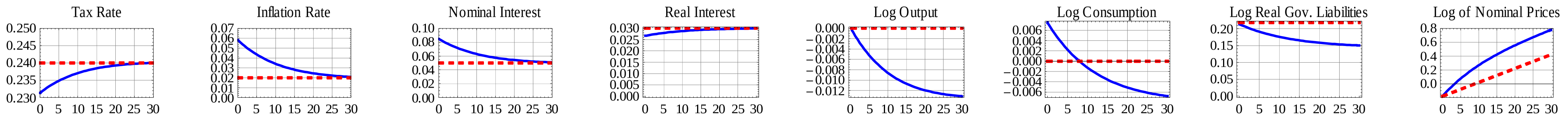
Figure B.1 The starting point is a steady state where tax rate is 23% and inflation is 2%. This panel shows transition paths to a steady state with 24% tax rate and 2% inflation. In figures B.1.1.-B.1.3 the monetary policy response parameter, α, is at the upper limit of a passive stance. Responses to output are negative, positive and mute, respectively . Given the response to output, tax rates and elasticity of tax revenues, the response to government debt is in the permissible bounds according to Corollary 3. In figures B.1.4.-B.1.6 monetary policy pegs the nominal interest rate. Figure B.1.4 shows that an increase in the tax rate from 23% to 24% induces the largest reevaluation of government debt if it interacts with an interest rate peg and a negative response to output. In this case the initial addition to the real value of government debt in the economy with distorting taxes is higher than the addition to the real value of government debt in the lump-sum tax economy.

Figure B.2 The starting point is a steady state where tax rate is 23% and inflation is 2%. This panel shows transition paths to a steady state with 23% tax rate and 3% inflation. In figures B.2.1.-B.2.3 the monetary policy response parameter, α, is at the upper limit of a passive stance. Responses to output are negative, positive and mute, respectively . Given the response to output, tax rates and elasticity of tax revenues, the response to government debt is in the permissible bounds according to Corollary 3. In figures B.2.4.-B.2.6 monetary policy pegs the nominal interest rate. In both economies investment is subject to a liquidity constraint. As a result, changes in inflation target send the two economies to similar transition paths since in both economies the policy shock is transmitted to changes in the stock of capital via the nominal rate of interest.

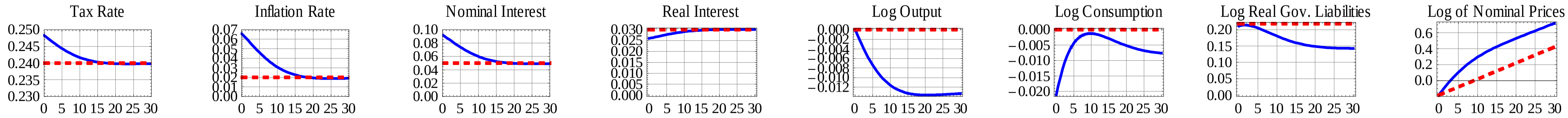
Figure B.3 The starting point is a steady state where tax rate is 50% and inflation is 2%. This panel shows transition paths to a steady state with 51% tax rate and 2% inflation. Real debt reevaluation in the distorting-tax economy is less then the reevaluation in the lump-sum tax economy. Although tax revenues elasticity is negative, an increment in the tax rate target initially increases the real value of government debt.

Figure B.4 This panel shows transition paths from a steady state with 50% tax rate and 2% inflation to a steady state with 50% tax rate and 3% inflation. As is figure B.2 dynamics are similar for the two economies.

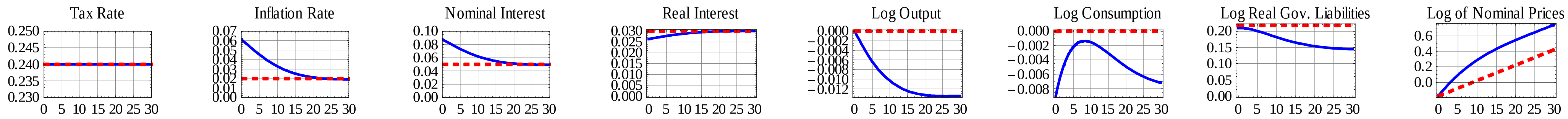
Figures B.5-B.6 These figures show the dynamics in the two economies when transfers permanently increase and policy targets remain at their levels prior to the shock. In both economies the only effect is a devaluation of debt. This result can be interpreted either as an outright default or as a jump in nominal prices that induces a market default.



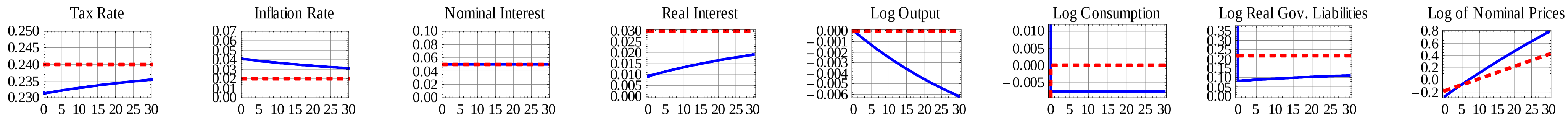
B.1.1: Policy parameters α=0.912921, β=-0.66, γ=0.000644935.



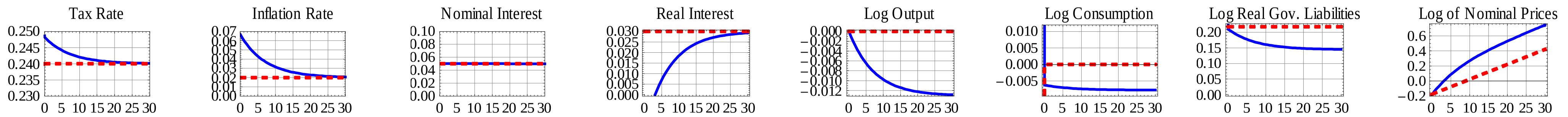
B.1.2: Policy parameters α=0.912921, β=0.5, γ=0.029.



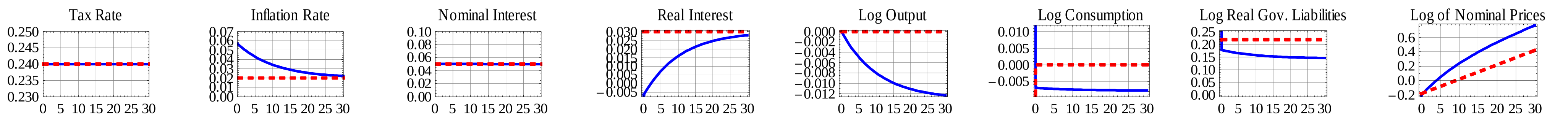
B.1.3: Policy parameters α=0.912921, β=0, γ=0.



B.1.4: Policy parameters α=0.01, β=-0.66, γ=0.000644935.



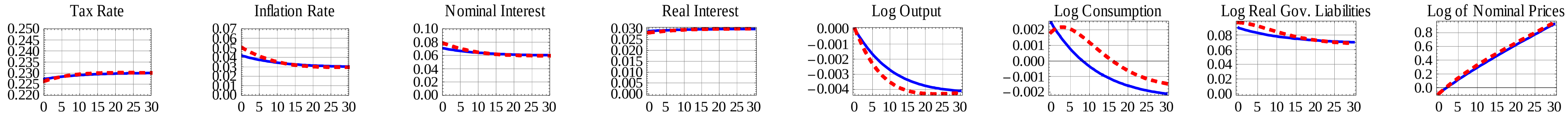
B.1.5: Policy parameters α=0.01, β=0.5, γ=0.029.



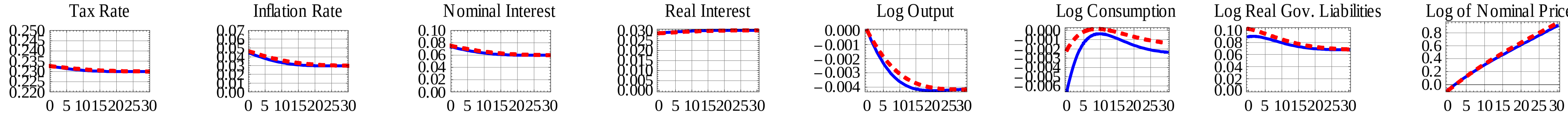
B.1.6: Policy parameters α=0.01, β=0, γ=0.

Figure B.1: Policy shock. Transition from $\{\tau,\pi\}=\{0.23,0.02\}$ to $\{\tau,\pi\}=\{0.24,0.02\}$. Elasticity of tax revenues near target: 0.684211

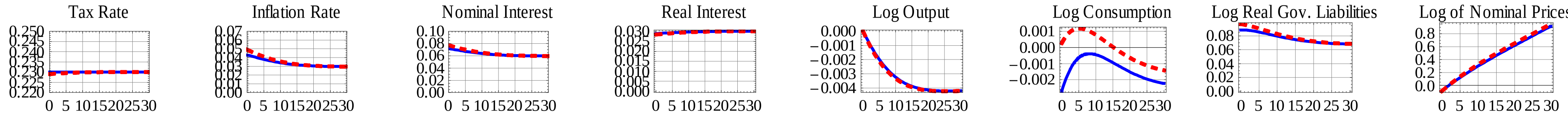
Economy with Income Tax   Economy with Lump–sum Tax



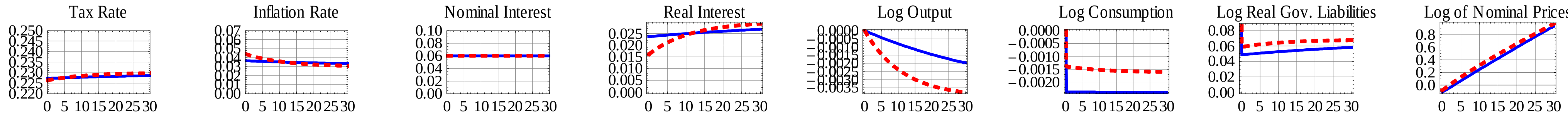
B.2.1: Policy parameters α=0.913752, β=-0.67, γ=-0.000453735.



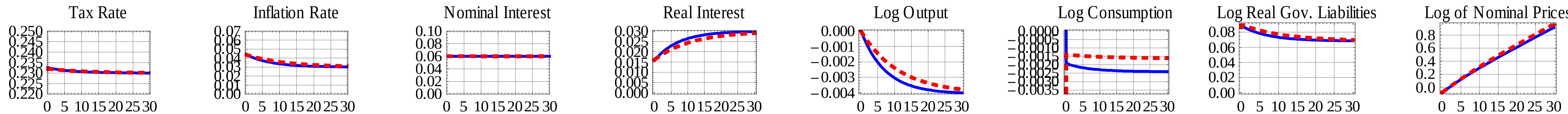
B.2.2: Policy parameters α=0.913752, β=0.5, γ=0.029.



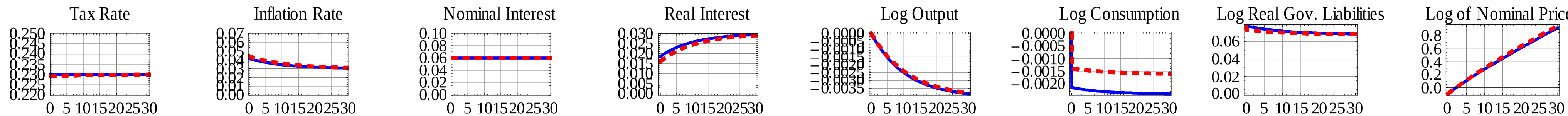
B.2.3: Policy parameters α=0.913752, β=0, γ=0.



B.2.4: Policy parameters α=0.01, β=-0.67, γ=-0.000453735.



B.2.5: Policy parameters α=0.01, β=0.5, γ=0.029.



B.2.6: Policy parameters α=0.01, β=0, γ=0.

Figure B.2: Policy shock. Transition from $\{\tau,\pi\}$={0.23,0.02} to $\{\tau,\pi\}$={0.23,0.03}. Elasticity of tax revenues near target: 0.701299

Economy with Income Tax
Economy with Lump–sum Tax

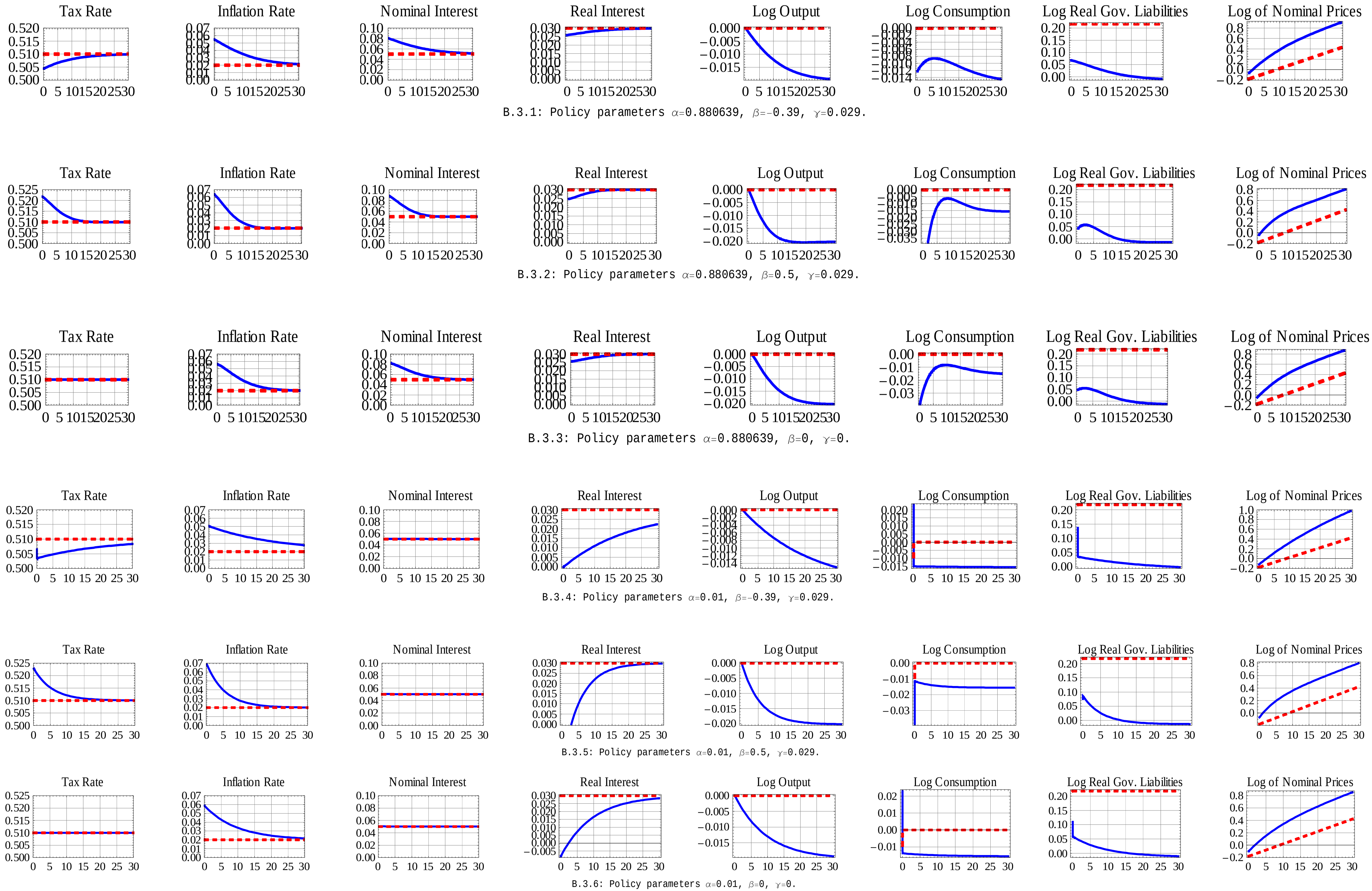
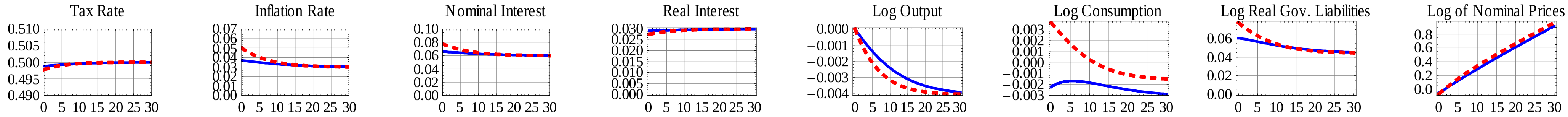
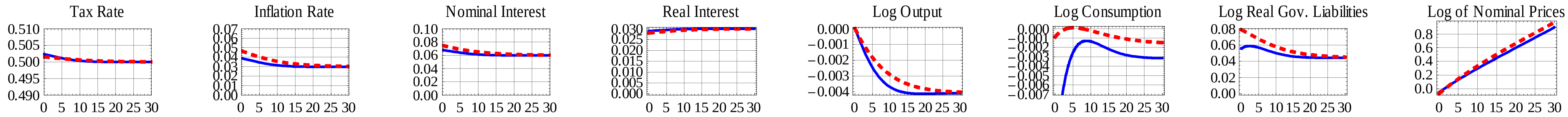


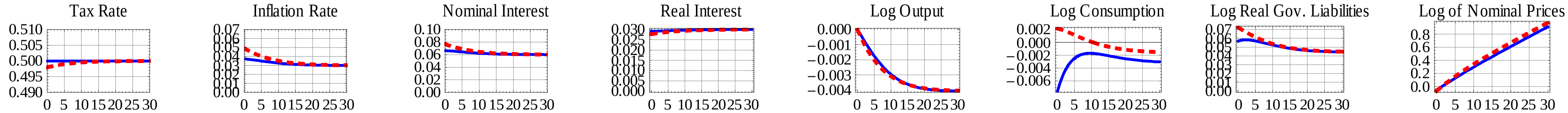
Figure B.3: Policy shock. Transition from $\{\tau,\pi\}=\{0.5,0.02\}$ to $\{\tau,\pi\}=\{0.51,0.02\}$. Elasticity of tax revenues near target: -0.0408163



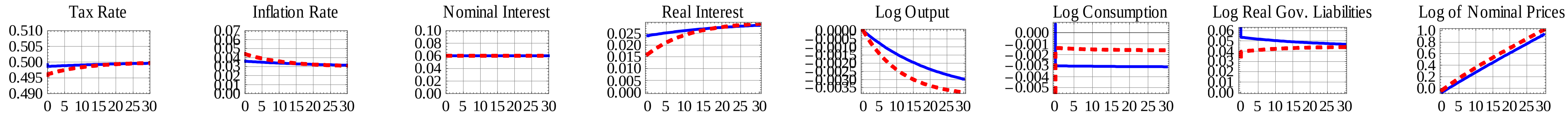
B.4.1: Policy parameters α=0.882436, β=−0.4, γ=0.029.



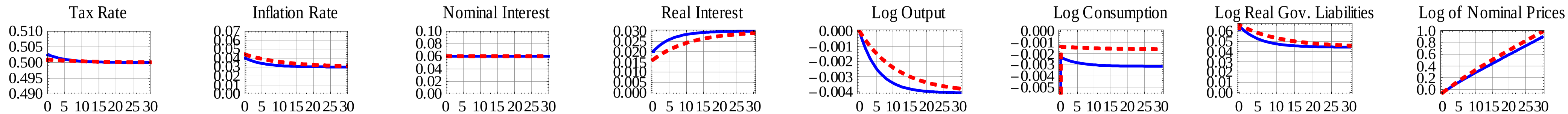
B.4.2: Policy parameters α=0.882436, β=0.5, γ=0.029.



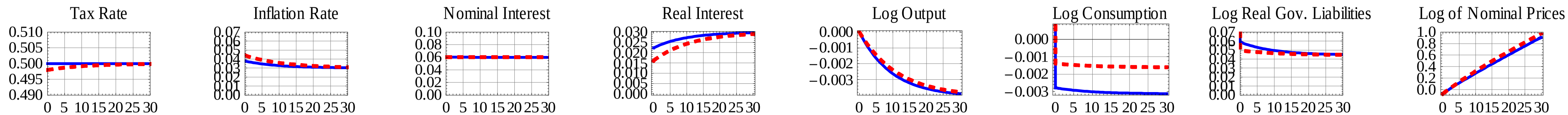
B.4.3: Policy parameters α=0.882436, β=0, γ=0.



B.4.4: Policy parameters α=0.01, β=−0.4, γ=0.029.



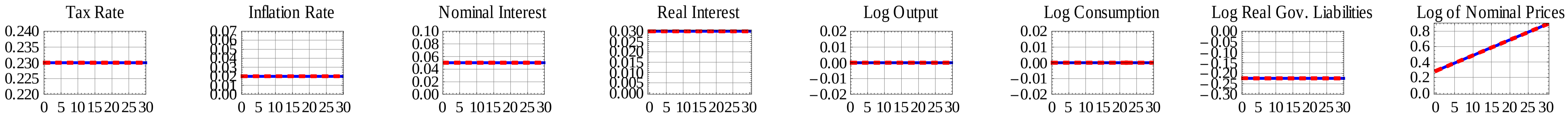
B.4.5: Policy parameters α=0.01, β=0.5, γ=0.029.



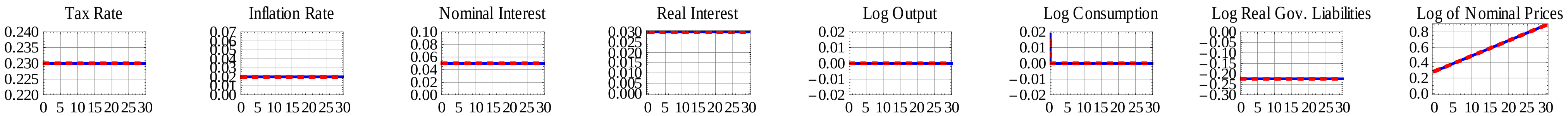
B.4.6: Policy parameters α=0.01, β=0, γ=0.

Figure B.4: Policy shock. Transition from $\{\tau,\pi\}$={0.5,0.02} to $\{\tau,\pi\}$={0.5,0.03}. Elasticity of tax revenues near target: 0.

Economy with Income Tax — Economy with Lump−sum Tax



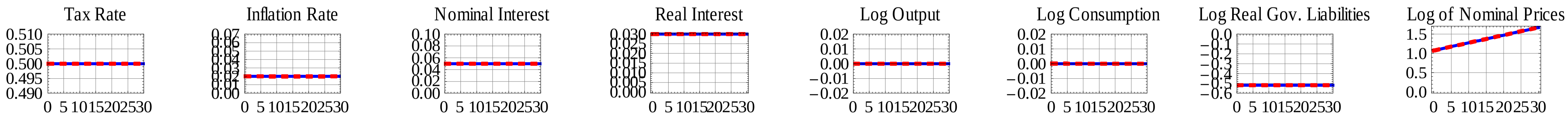
B.5.1: Policy parameters $\{\tau,\pi\}=\{23.\%,2.\%\}$, α=0.91371, β=-0.67, γ=-0.00314607. Elasticity of tax revenues near target: 0.701299



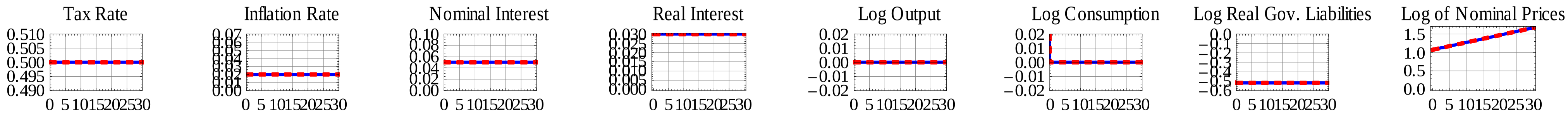
B.5.2: Policy parameters $\{\tau,\pi\}=\{23.\%,2.\%\}$, α=0.01, β=-0.67, γ=-0.00314607. Elasticity of tax revenues near target: 0.701299

Figure B.5: A permanent 5% increase in transfers where $\{\text{Debt/GDP},\tau,\pi\}=\{153\%,23\%,2\%\}$

Economy with Income Tax   Economy with Lump–sum Tax



B.6.1: Policy parameters $\{\tau,\pi\}=\{50.\%,2.\%\}$, α=0.882396, β=-0.4, γ=0.0120833. Elasticity of tax revenues near target: 0.



B.6.2: Policy parameters $\{\tau,\pi\}=\{50.\%,2.\%\}$, α=0.01, β=-0.4, γ=0.0120833. Elasticity of tax revenues near target: 0.

Figure B.6: A permanent 5% increase in transfers where $\{\text{Debt/GDP},\tau,\pi\}=\{153\%,50\%,2\%\}$

Economy with Income Tax   Economy with Lump–sum Tax