COMMUTING AND SHOPPING: DETERMINANTS OF CITY INCOME STRUCTURE

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ABSTRACT

We demonstrate how firm pricing strategy and determinants of household location can interact to determine city structure. We go beyond previous work on spatial income segregation by endogenizing the tradeoff between households' choice of location and shopping behavior, as well as solving for the firms' optimal pricing strategy in a general equilibrium framework. In this city, consumers and firms live on a continuous line interval. Our model consists of two types of firms; many high-cost perfectly competitive "Corner Stores" located in the Central Business District, and one large low-cost "Superstore", choosing its location and price strategically. We begin by considering a model with homogenous consumers in order to determine the strategy for the Superstore in a spatial model. Then we consider the impact of introducing different income classes to our city structure. We show how the shopping habits of the consumer population, as determined by the relative price of the Superstore and the Corner Stores, can contribute to the various income segregation outcomes described in previous literature. In addition we consider the impact of city income structure on the pricing decision of firms.

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1 Introduction

The purpose of this paper is to consider how the pricing decision of large retailers and Supermarkets (we group both together and refer to them as Superstores) interact with household location and shopping decisions as well as city structure. We look to build on the existing regional economics literature by extending the factors that impact the location choice of consumers to include their access to affordable shopping. Previous literature has focused on commuting costs and desire for space, access to neighborhood specific amenities, including high quality education, and the desire to locate close to job centers (see for example Guerrieri et al. (2010), Wasmer and Zenou (2002)). Though we acknowledge that all of these are very likely important factors in the decision of households on where to live, we argue that access to shopping is also an important factor, and we look to quantify the impact of shopping behavior on location while taking the impact of the above determinants as given. We go beyond previous spatial economics literature by endogenizing firms' role in the agglomeration forces within the city by considering the pricing strategy of firms when facing different city structures and a heterogeneous consumer population. Our recognition of the interdependence of firm strategy and consumer behavior in a spatial setting looks to connect the existing work in regional economics with the industrial organization literature.

One of the first attempts at understanding the economics of city structure is in the work of the pioneers of the mono-centric city model, Alonso (1964), Mills (1967) and Muth (1969), (collectively referred to as AMM). They argue that the two forces that determine the choice of location within a city are commuting costs, t, and demand for space, q. The ratio of these two factors, $\frac{t}{q}$, is said to be what determines city structure. The argument is that higher income households have a higher opportunity cost of time, and in order to cut down on commuting costs would prefer to live closer to the Central Business District (CBD), where presumably most economic and social activities are centered. At the same time higher income families are more likely to have a greater demand for space, drawing them out to the more spacious communities in the suburbs. These two opposing forces can result in different city structures depending on the extent of income inequality within the city, as well as the magnitude of demand for space across households with different levels of income. The authors also argue that the structure of the city would depend on the transportation infrastructure of the city, which can impact commuting costs and the availability of affordable space in the city center.

Though a very simple and intuitive model, the AMM explanation of city structure is not supported by the empirical data available. Wheaton (1977) tests the assumptions of the AMM model using U.S. household data and finds that the cost of transportation relative to demand for land does not vary across income, which would suggest that the AMM theory does not explain the various structures of income segregation that we observe. Wheaton's finding is corroborated by a more recent study, which finds that the elasticity of demand for land area is very low relative to the elasticity of demand for travel cost per mile, Glaeser et al. (2000), making it difficult for the AMM model to explain why in cities like Los Angeles and Detroit high-income families would choose to locate outside of the city center.

Brueckner et al. (1999) offer an alternative explanation using a model that links the location of different income groups to the spatial pattern of various types of amenities within a city. Using the assumption that the marginal valuation of amenities rises significantly with income, the authors argue that in cities like Paris where the amenities are concentrated in the city center, the rich tend to locate themselves closer to the central business district. While in cities like Detroit, where amenities are more spread out, the center of the city is mainly comprised of low-income families that cannot afford the commuting costs of living in the suburbs. In addition the paper looks to endogenize the amenities available within a city by including the neighborhood income, as well as a parameter for exogenous amenities, in the consumers' utility function. The argument is that certain types of amenities are directly related to the income of the population that live within that neighborhood, therefore wealthy consumers prefer to live in neighborhoods with high average income.

Guerrieri et al. (2010) expand on this argument through a model of "neighborhood effects". Using a continuous time model with no transportation costs they demonstrate under what conditions a segregated outcome can come about with high-income consumers located in the city center and poorer households in the periphery. Using the same argument as Brueckner et al. (1999) that all households prefer to live in richer neighborhoods, they show that poor neighborhoods located closer to the border of the high-income neighborhoods are more likely to experience price inflation during periods of economic boom. The price inflation in their model is partly driven by a process of gentrification, when higher income households expand into the low-income neighborhoods located at the border of the wealthy neighborhood in order to increase their housing consumption. Using data on intra-city housing prices in the U.S., the writers show that their results hold even after controlling for transportation costs and distance to natural amenities, factors independent of their "neighborhood effects".

This chapter adds to the above discussion by considering shopping as a factor in a household's choice of location. We approach this problem very much aware that the choice of where to live is a personal one, and is driven by a variety of factors, some of which are rational and economic, while others are more behavioral and based on personal experience. Even within the rational set of incentives there are a wide variety of important factors that can impact where families choose to live. Families with young children will very likely base their decision of location partly on availability of quality education, as is shown by Selod and Zenou (2003), and is one of the underlying concepts of the endogenous amenities of Brueckner et al. (1999). While young urban professionals will more likely be concerned with locating close to job centers, as is argued by Wasmer and Zenou (2002). Most households also consider access to local amenities, public services and the crime rate when choosing their neighborhoods. Some of these factors can be interpreted as consequences of city structure, like the crime rate at a city center, while others have been addressed in previous papers, such as Brueckner et al. (1999) use of amenities to explain segregation. The purpose of our paper is to isolate the impact of access to affordable shopping by taking these other variables as given. Clearly any real world consideration of our results would need to consider them within the context of all these other factors.

We look to contribute to the above discussion on the determinants of city structure in two ways. First we look to endogenize a cause of household location that goes beyond commuting costs by including access to affordable shopping in the consumer's decision process. Second we allow the Superstore to behave strategically, therefore we take into account the firm's side of the problem in our analysis. We attempt to address income segregation and city structure in a semi-general equilibrium framework[2]. In solving the consumer's problem, we look to determine to what extent shopping behavior drives household choice of location. We believe that since families tend to spend a significant portion of their income on household consumption, access to affordable shopping can be an important determinant of where families choose to live. We build our agglomeration model with the simplifying assumption that there are only two opposing factors that determine household choice of location, and in effect the rental rate across city neighborhoods. First is the desire to locate close to work, which in our model is the center of the city, or the Central Business District (CBD), in order to cut down on the cost of commuting. Second is the desire to live close to affordable shopping, where households will spend most of their earned income. We offer the consumer the option between convenient, but relatively expensive, shopping available at the Central Business District, and more affordable shopping in the outskirts of the city that would require additional money to be spent commuting in order to access. Households will simultaneously choose where to live and where to shop, and ultimately the tradeoff between the availability of affordable shopping with the convenience of access to the Central Business District will determine the possible equilibrium outcomes within our model.

We begin building our theoretical model with a base case that involves a homogenous consumer population living on a line interval. The Central Business District and the convenience stores (Corner Stores) are located at one end of the line, and affordable shopping (Superstore), is located at the other end. After we consider a base case with homogeneous consumers, we introduce a heterogeneous consumer population to the model and consider how are results dif-

[2]Using the term "general equilibrium" is not fully accurate since we do not consider the labor market in our model, we take the hiring activities of firms as given. We are considering equilibria involving the other three aspects of the market, consumers, producers and the rental market within the city (we are taking agricultural rent at the end of the city as fixed).

fer. Within this framework we derive the conditions under which cities become segregated or fully integrated. We consider the impact these different outcomes have on the Superstore's pricing strategy and profits, as well as considering the impact on the welfare of the consumer population.

2 The Base Case With Homogenous Consumers

The main purpose of our theoretical model is to consider the connection between the location choice of households and the pricing decision of firms within a simple city framework. In each iteration of our model below we will solve for where households choose to live and shop, taking prices in the city as given. Then we consider how the decisions made by households impacts the demand faced by firms and ultimately their choice of price. For the base case of our City Model we consider a city containing a homogenous consumer population earning wage w. At first the only option for the consumer is to perform their shopping after work at the Corner Stores located in the Central Business District (CBD). We will then introduce a Superstore into the model. The timing of the model is as follows:

1. Corner Stores choose whether or not to enter the CBD and then choose their price.
2. The Superstore decides whether or not to enter, chooses its location and its price.
3. Consumers make their location and shopping decisions.

Throughout this paper we assume that zoning laws limit firms to the Farmland and CBD and that rents go to landlords, who are absentee.

The City

The spatial representation of our city is given by a straight line interval starting from the Central Business District (CBD) and going outwards. The CBD is where all consumers work and where our Corner Stores are located. The other end of the city is Farmland, and represents the furthest out any consumer will choose to live. The city is closed such that the population size and composition are exogenously determined. We normalize the geographical and population size of the city to 1, therefore we will not include population density (population density being the ratio of population size to geographical space) in this model[3]. We use $z \in [0, 1]$ to represent the location of any given consumer or firm. The CBD will be used as the starting point of the line, $z = 0$, and the Farmland will be the outer point of the city, $z = 1$.

[3]Population density within this framework does not impact the decision of firms and consumers.

Figure 2.1: The City

The rental price at any given point in the city is determined competitively. We will normalize the rental line by taking the rental rate at the Farmland as exogenously determined by the external land market, $r(1) = a$, and allowing the rest of the market to clear.

There are two types of transportation cost in our model, the money cost of commuting, t and shopping, g. These are both per-unit costs that need to be multiplied by distance from destination to determine the full cost to the consumer.

For the sake of focusing our question and keeping our analysis simple, we make the following assumptions about our model City:

(i) The distribution of the population across the city is uniform. In other words we assume that there are no high density points in the city.

(ii) Stores have to pay rent based on their location, but do not take up any actual space, therefore a consumer can live where a firm is located.

(iii) The money cost of commuting to work is higher than the the money cost of traveling to the store, $t > g$. This is an intuitive assumption as consumers commute to work daily, while shopping might happen on a weekly basis.

(iv) Zoning laws restrict the size of stores at the CBD to 1 unit of the good. There are no size constraints for stores located at the Farmland.

(v) There are no outside options in our model. Consumers must choose a place to live and a firm to shop at given what is available to them from our setup of the model.

(vi) Firms compete on price.

As we stated above, we begin with a city where only Corner Stores exist, with a homogeneous consumer population, all earning wage w. All consumers work at the CBD and will have to commute from their homes to work, requiring them to pay a monetary cost of commuting equal to tz. Consumers shop after work at the Corner Stores located near the office, so do not have to pay any extra commuting costs related to shopping.

Consumers' Problem

Consumers spend all of their net earnings on one good, x, providing them with utility measured by $u(x)$, which we assume to be strictly increasing. They choose where to live based on the rental cost of the location and the costs of commuting to work and shopping. Therefore, the maximization problem for a representative consumer living at point z and shopping at the Corner Stores located at the CBD is given by:

$$\max_{x} \quad u(x)$$

$$s.t. \quad p_c x = w - tz - r(z) \tag{2.1}$$

Equation (2.1) is the usual budget constraint, we do not include any non-wage income, since we are only interested in how changes in wage affects consumer choice. Here p_c is the price charged by the Corner Stores and $r(z)$ is the rent paid by the consumer located at z and shopping at the CBD.

Constraint (2.1) is binding by the monotonicity of our utility function. Given that we are only concerned with the consumption of one good and since there is no uncertainty in our model, we can solve directly for the consumers' optimal choice of x. This is analogous to using a linear utility function, which, without loss of generality, is what we will work with for the rest of this paper. Therefore, the indirect utility for our representative consumer is given by:

$$v_c = \frac{w - tz - r(z)}{p_c}$$

Consumers choose z to maximize v_c. Therefore, given our homogenous consumer population, $r(z)$ must be such that the utility level for consumers is constant across the city line. We can rearrange the equation above to solve for the rental rate at any point z on the city line interval while treating v_c as fixed with respect to z.

$$r(z) = w - tz - p_c v_c \tag{2.2}$$

We can think of this equation as the "rental indifference line". Since the utility of all consumers is independent of location in equilibrium, we can fix the utility for consumers living in our model city by the utility level of the consumer living at the Farmland, $z = 1$, paying a fixed rent of $r(1) = a$:

$$v_c(z=1) = v_c = \frac{w-t-a}{p_c} \tag{2.3}$$

Combining equations (2.2) and (2.3) we can rewrite the rental formula along our city line:

$$r(z) = t(1-z) + a \tag{2.4}$$

The rental cost in our city is a decreasing function of z, as consumers move out towards the Farmland they are able to save money on rent equivalent to the extra commuting costs they must pay to get to work.

Corner Stores

All Corner Stores are located in the CBD and behave competitively without any fixed costs or other barriers to entry. As we stated above, we assume zoning restrictions in the center of the city limit store size to one unit of the good, $x = 1$. Given this market structure, the higher the demand for the Corner Stores the higher the number of stores that will be able to operate at the CBD. Throughout this paper we will assume that the commercial rental market is independent of the housing rental market, therefore the cost structure of the firms in our city is independent of the consumers' rental lines. The Corner Stores face a constant marginal cost, $\bar{c}$, and compete on price, therefore their price is equal to their constant marginal cost, $p_c = \bar{c}$.

We can plug this price into the indirect utility from equation (2.3) to determine the resulting utility level in our base case model with the Corner Stores as the only option for our consumer population:

$$v_c = \frac{w-t-a}{\bar{c}} \tag{2.5}$$

Superstore

Now we will introduce the Superstore into our city model. Similarly to the Corner Stores, the Superstore faces a constant marginal cost, $\hat{c}$. The Superstore can choose to enter at the CBD without any fixed costs, but then its marginal cost will be equivalent to that of the Corner Stores (and it will be limited to unit capacity by zoning laws). Alternatively the Superstore can pay a positive fixed cost k and locate in the Farmland. If the Superstore locates in the Farmland it is able to take advantage of its more remote location and larger size to bring down marginal costs, therefore in the Farmland $\hat{c} < \bar{c}$. As we stated in our assumptions above, we do not assume any size restriction in the Farmland, therefore the Superstore is free to choose its capacity.

Proposition 2.1: *If a Superstore chooses to enter the market, only one will enter, and that store will choose to locate out in the Farmland.*

Proof: This is a direct result of our assumption on zoning restrictions as well as the cost structure described above. If the Superstore chose to locate at the CBD its marginal cost would be equivalent to that of the Corner Stores (in addition to not having fixed costs), therefore it would price at marginal cost and earn zero profits. Given the lower marginal costs associated with locating in the Farmland, if the Superstore locates at $z = 1$ it will be able to undercut the Corner Store price and attract positive demand. As long as fixed costs are small enough this would lead to positive profits[4]. Therefore, we have that if the Superstore chooses to enter it would locate out in the Farmland[5].

As for the number of Superstores, any level of fixed costs for entering as a Superstore, combined with our zoning restrictions, assures us that only one Superstore will ever enter in the Farmland. Otherwise we would have a duopoly under Bertrand competition, resulting in negative profits.

Therefore, the Superstore enters as a monopolist facing a competitive fringe (the Corner Stores), and chooses its price strategically.

If the consumer living at point z chooses to shop at the Superstore their maximization problem becomes:

$$\max_x \quad u(x)$$
$$s.t. \quad p_s x = w - tz - g(1-z) - r_s(z)$$

Where p_s is the price charged by the Superstore and $g(1-z)$ is the commuting cost of living at point z and shopping at the Superstore. The resulting utility level is:

$$v_s = \frac{w - tz - g(1-z) - r_s(z)}{p_s}$$

[4]In the analysis that follows, and our discussion of Superstore profits, we will inherently assume that if the Superstore attracts positive demand, the profits calculated will be enough to cover the Superstore's fixed costs.

[5]The strict assumptions we make on the laws of the city lead to the specific structure we have described here. Although there are clearly examples of large discount stores located in the center of cities, Superstores are much more likely to locate in the outer suburbs. Our specific model structure allows us to focus our question on the relative abundance of different stores at either ends of the city.

Using the same argument as above we must have that the utility for consumers shopping at the Superstore is independent of location. Solving for $r_s(z)$ in the equation above we can determine the rental line faced by consumers that shop at the Superstore:

$$r_s(z) = w - tz - g(1-z) - p_s v_s \tag{2.6}$$

Now let us consider the consumer living at $z = 1$. Whether they shop at the Corner Stores or the Superstore depends on the relative price of the two. But for the Superstore to exist in our city it must at least attract the consumer living in the Farmland, otherwise it will have zero sales and, due to its fixed costs of entry, would not choose to enter the market. The utility of the consumer living in the outer part of the city is given by:

$$v_s(z=1) = v_s = v = \frac{w-t-a}{p_s} \tag{2.7}$$

As before, equilibrium requires that the utility level is constant across our city in equilibrium, independent of where consumers shop, so we have that the utility for all consumers must be equal to that of the consumer at $z = 1$ as given by equation (2.7) above.

Combining equations (2.6) and (2.7) we can solve for the rental line for consumers in our city that shop at the Superstore:

$$r_s(z) = (t-g)(1-z) + a \tag{2.8}$$

The rental line for consumers shopping at the CBD is still the same as the general form we described in equation (2.2) above. Plugging in for utility from equation (2.7) we can solve for the rental line faced by consumers shopping at the Corner Stores in terms of the price charged by the Superstore:

$$r_c(z) = w - \tfrac{\bar{c}}{p_s}(w-t-a) - tz \tag{2.9}$$

If the Superstore enters the market there are two possible structural outcomes.

1. The Superstore captures the entire market, this would mean that all consumers shop at the Farmland and the rental line for the entire city would be given by equations (2.8) above. In our analysis below we will consider under what condition on our parameters such a scenario would be an equilibrium in our model.

2. The Superstore will price in such a way as to capture a portion of the city's population

as determined by a point in the city such that above that point all consumers shop at the Farmland. This scenario is depicted in figure 2.2 below.

Though we have not yet solved for the Superstore's price, we can clearly see that the slope for this rental line relative to z is equal to $-t$, which allows us to draw the graph for the new rental lines across our city after having introduced the Superstore.

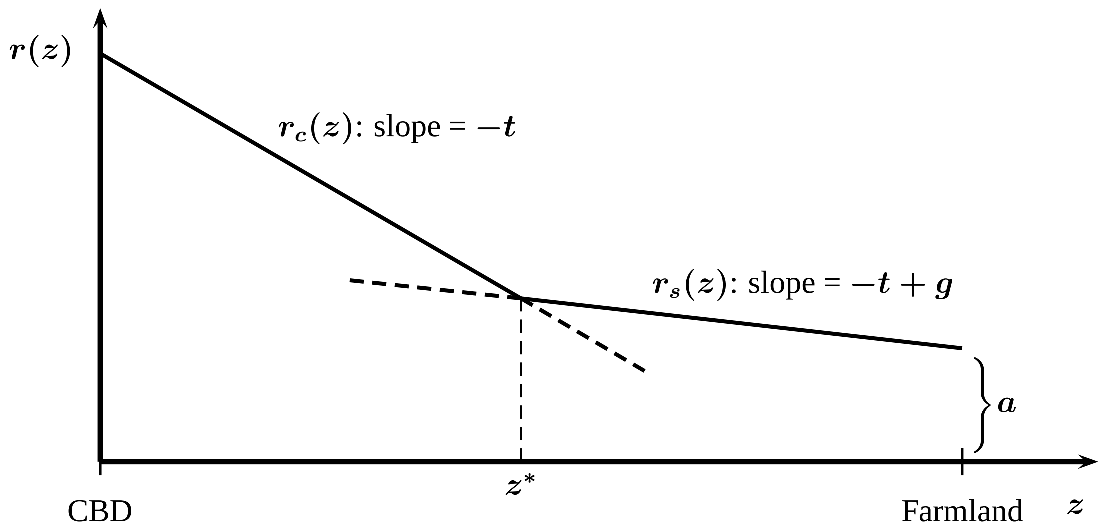

Figure 2.2: Rental Lines

Depending on the Superstore's choice of price, there is a point along the city line interval such that the consumers living to the right of that point will choose to shop at the Superstore. That point in the line, which we will call z^*, is determined by the value of z such that the two rental lines as given by equation (2.2) and (2.6) are equal, as shown in Figure 2.2 above. Setting the two equations equal to each other and solving for z we have:

$$1 - z^* = \frac{1}{\lambda}\left(\frac{\bar{c}}{p_s} - 1\right) \qquad \text{s.t.} \quad z^* \geq 0 \tag{2.10}$$

Where $\lambda = \frac{g}{w-t-a}$ is a transportation parameter that measures shopping cost as a percentage of the disposable income of the representative consumer living at $z = 1$. We define disposable income as earned wages net of the cost of commuting to work and rent. We can see from figure 2.2 and equation (2.10) that the demand faced by the Superstore is determined by the point z^*. As the Superstore increases its price, p_s, the point at which consumers switch from shopping at the Corner Stores to the Superstore increases, decreasing the number of consumers that pay the additional cost to shop at the Farmland.

Since we have assumed the Superstore has a cost advantage over the Corner Stores, $\hat{c} < \bar{c}$, if the Superstore enters the market it will always choose a price lower than $\bar{c}$ and capture a positive fraction of the market, that is $1 - z^* > 0$. If $1 - z^* = 1$ the Superstore would capture the entire market and would not have any incentive to lower its price, giving us the following lower bound for the Superstore's choice of price:

$$p_s = \frac{\bar{c}}{1+\lambda} \tag{2.11}$$

The Superstore can drive out the Corner Stores from the city by providing a significant price discount relative to the transportation cost needed to commute out to the Farmland, represented here by λ. Interestingly, in low-income communities shopping and commuting costs represent a higher percentage of disposable income, high λ. A higher lambda would impose a tighter constraint on the Superstore's price in inequality (2.11) above, making it more difficult for the Superstore to drive out the Corner Stores[6]. Though not obvious, this result is somewhat intuitive. Low-income consumers find it relatively more costly to travel to the store with the lower price, forcing those that live too far away from the Farmland to shop at the more expensive, but more convenient Corner Stores. The role of income in the condition in equation (2.11) raises the question of what happens in communities where there are both rich and poor households. We will consider that question in detail in the next section when we introduce heterogeneous consumers into our model.

For now we turn to the choice of p_s by the Superstore in our current framework.

Superstore's Problem: Plugging in our values for v and z^* into the Superstore's profit function we have the Superstore's optimization problem in terms of its choice of price and our model parameters:

$$\max_{p_s} \quad \pi_s = (1 - z^*)v[p_s - \hat{c}] - k = \frac{g}{\lambda^2}\left(\frac{\bar{c}}{p_s} - 1\right)\left(1 - \frac{\hat{c}}{p_s}\right) - k \tag{2.12}$$

$$s.t. \quad p_s \geq \frac{\bar{c}}{1+\lambda}$$

Before we go on to explicitly solve for the Superstore's price we can use our condition from equation (2.11) along with the objective function above to portray the relationship between the Superstore's choice of price and its profits. In the figure below, and in our analysis that follows, we take $k \to 0$ trivially, since any positive level of fixed costs would deter an additional Superstore entering the market.

[6] This observation is related to the results we present in the first chapter of this thesis, where now we are able to put the problem in a spatial setting and speak of the transportation costs of consumers rather than the costs of obtaining information.

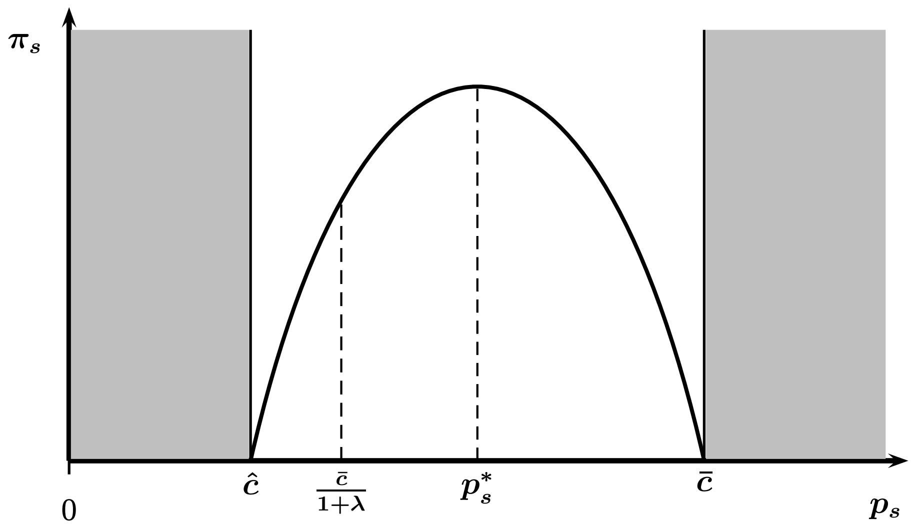

Figure 2.3: Superstore's Profit

The profit curve in figure 2.3 is drawn assuming an interior solution, the optimal price p_s^* is above the cutoff point identified in equation (2.11). If we had drawn the figure so that the cutoff point was above p_s^* then we would have had a corner solution: the Superstore would have chosen $p_s = \frac{\bar{c}}{1+\lambda}$ and captured the entire market.

In our figure above, the optimal price for the Superstore is p_s^*, which maximizes the Superstore's profit function in equation (2.12). Maximizing the objective function with respect to p_s we solve for the interior solution to the Superstore's problem. As we might expect the Superstore's choice of price is a function of the marginal costs of the two types of firms:

$$p_s^* = \bar{c}\left(\frac{2\hat{c}}{\bar{c}+\hat{c}}\right) \tag{2.13}$$

Since we have assumed that the marginal cost of the Superstore, $\hat{c}$, is lower than that of the Corner Stores, the term in the parenthesis is a positive fraction less than one. Therefore, we have that $p_s < p_c = \bar{c}$, as we expected. Interestingly, in this case of our model, consumer income, w, and the transportation costs in the city, t and g have a multiplicative effect on the Superstore's profits, and do not impact its choice of price. When we introduce consumers heterogeneous in income in the next section transportation costs as well as income impact the Superstore's choice of price as well as its profits.

Plugging in the above price into our equation for z^* above we can solve for the portion of the consumers that choose to shop at the Superstore:

$$1 - z^* = \begin{cases} \frac{1}{\lambda}\left(\frac{\bar{c}-\hat{c}}{2\hat{c}}\right) & \frac{\bar{c}}{\hat{c}} < 1 + 2\lambda \\ \\ 1 & \text{Otherwise} \end{cases} \tag{2.14}$$

Which is always positive, therefore the Superstore will enter the market and attract a proportion of the consumer population. As we argued above, whether or not it captures the entire market depends on our parameters. The condition for the existence of Corner Stores is $\frac{\bar{c}}{\hat{c}} < 1 + 2\lambda$. In order for a Corner Store to be able to survive in a market when a Superstore chooses to enter the difference between the costs of the two types of firms must be low relative to the extra commuting cost required to shop at the Superstore. This is in line with what we would expect in practice. In cities with large discount stores and low transportation costs it would be less likely that we would find a Corner Store. While in cities with more significant transportation costs and fewer large retailers, it is more likely that we would find small convenient Corner Stores.

Substituting in the resulting market price from equation (2.6) above into the Superstore's objective function in equation (2.12) we calculate the resulting level of profits for our Monopoly:

$$\pi_S = \frac{g(\bar{c} - \hat{c})^2}{4\bar{c}\hat{c}\lambda^2}$$

The profit of the Superstore is positive, and an increasing function of the marginal cost of the Corner Stores, $\frac{\partial \pi_s}{\partial \bar{c}} > 0$ (and decreasing with the marginal cost of the Superstore). As we would expect, profits for the Superstore decrease when the transportation costs in the city increase ($\frac{\partial \pi_s}{\partial t} < 0$, $\frac{\partial \pi_s}{\partial g} < 0$). An interesting point here, usually transportation costs would impact a business' profits directly through increasing their variable costs of bringing supplies to the stores. Here we have demonstrated that stores can also lose out from high transportation costs due to a decrease in demand, due to a decrease in purchasing power for the consumer. Big discount stores, or Superstores, can mitigate the effect of high transportation costs on their market share through helping reduce those costs for their customers. One example is when companies such as Tesco, Ikea and Walmart provide free buses that take customers from city centers out to their more remote store locations.

We determine the impact of the Superstore on the welfare of the consumers by comparing the indirect utility of the consumer population from equation (2.5) with the indirect utility after we introduce the Superstore in equation (2.7). Since we have shown that the price of the Superstore will be lower than that of the Corner Stores in equilibrium, we can clearly see that the indirect utility of consumers increases when the Superstore enters the market. Plugging in our calculated p_s^* into equation (2.7) we solve for the consumers' utility after the Superstore enters the market:

$$v = \frac{(\bar{c} + \hat{c})(w - t - a)}{2\bar{c}\hat{c}} = \frac{g(\bar{c} + \hat{c})}{2\bar{c}\hat{c}\lambda} \tag{2.15}$$

Subtracting the utility to consumers before and after introducing the Superstore we can quantify the change in consumer welfare due to the Superstore:

$$\Delta v = \frac{g(\bar{c} - \hat{c})}{2\bar{c}\hat{c}\lambda} \tag{2.16}$$

The positive impact of the Superstore on the consumers is an increasing function of the cost differential between the two types of firms and the purchasing power of the consumers themselves.

This positive change in consumer welfare brought about by the entry of the Superstore is in line with the empirical findings of Hausman and Leibtag (2007). In their paper they demonstrate that consumers benefit significantly from the entry of Superstores (or what they refer to as Supercenters) into a market. Though they do not consider a geographical model, they do argue that restrictive zoning laws that do not allow Superstores to enter certain markets can end up hurting consumers, an argument that our results above support.

Finally we consider the impact of the entry of the Superstore on the rental market. We found that the rental rate along the city line interval without the Superstore is given by equation (2.4). When we introduced the Superstore the rental line for consumers shopping at the Superstore is given by equation (2.8), which is clearly less than or equal to the old rental line for all values of z. We can determine the new rental line for consumers shopping at the Corner Stores by plugging in p_s from equation (2.6) and $v_c = v$ from equation (2.7) into our general rental line for shopping at the corner store in equation (2.2).

$$r_c(z) = w - \tfrac{g}{\lambda}\left(\tfrac{\bar{c}+\hat{c}}{2\hat{c}}\right) - tz \tag{2.17}$$

Which we can show to be below our old rental line for all values of z. In figure 2.4 below the grey area represents the loss to the absentee landlords due to the Superstore's entry into the city.

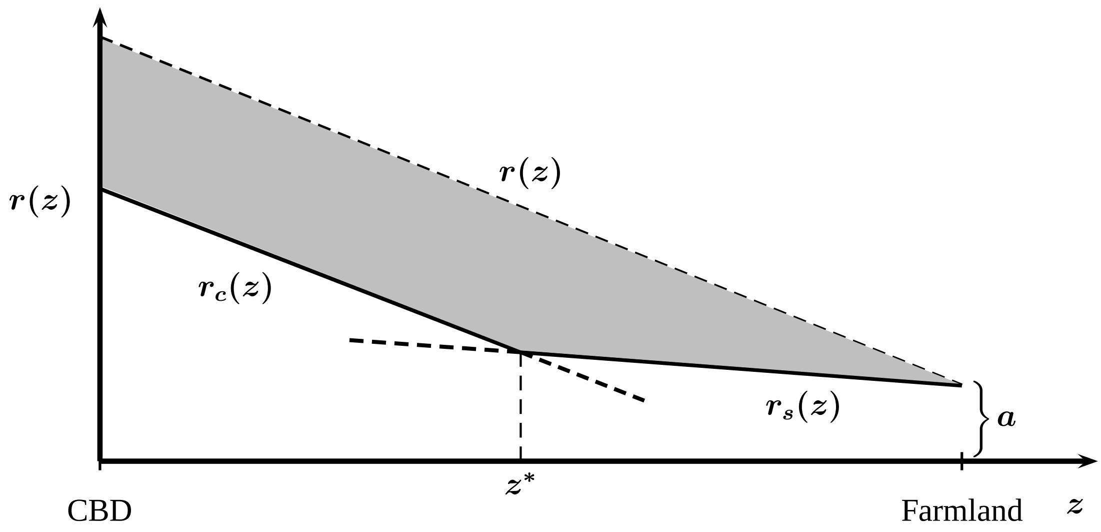

Figure 2.4: Superstore's Impact On Rental Lines

Introducing the Superstore into our simple model drives down rental costs all across our city except for at the Farmland, where the rental cost is determined by the world market. This is a very subtle but important result. Firstly, as we would expect, introducing the Superstore results in a relative increase in the rent at locations located closer to the Farmland, since the rent in the rest of the city has decreased while it has stayed the same at $z = 1$. This decrease in rent is weakly increasing as we move away from the Farmland. Secondly, it is clear from above that the landlords are the only ones who lose out from the entry of the Superstore into the city. The only exception is the landlord who rents out to the Superstore, they continue to earn $r(z = 1) =$ a. This final result is necessary for the structure we just described to be an equilibrium, if the owner of $z = 1$ would have lost out by the Superstore's entry then she would probably not have rented out to the Superstore to begin with.

As we have shown, the entry of the Superstore benefits the consumers in the form of higher utility, leaves Corner Stores indifferent earning zero profits, and leads to the Superstore earning positive profits. The only "losers" in our simple scenario are the landlords who are now earning lower rents on their property.

Now we will continue our analysis by introducing a heterogeneous consumer population to our model.

3 Consumer Demand With Households Heterogeneous In Income

In this section we extend our base case analysis by introducing two exogenously determined levels of income for our consumer population. We continue to normalize our city size and population to 1, but now our city will be made up of high-income consumers (type h) and low-income consumers (type ℓ). A proportion, α, of the consumer population are type h consumers and earn high wages, w_h, and $(1-\alpha)$ are type ℓ and earn low wages, w_ℓ. We take α as given.

All consumers work at the CBD and have to commute from their homes to work, requiring them to pay a money cost of commuting equal to tz. As before, we first consider the case where only Corner Stores exist in our city, then we introduce the Superstore and allow consumers the option of shopping after work at the grocery stores located near the office, or paying the additional transportation cost, $g(1-z)$, and shopping at the Superstore.

As we discussed above, the timing of our model is in the form of a Stackelberg leadership model, where the Superstore sets its price and consumers choose housing and shopping location in response. Therefore we must first determine how consumers would respond to different price outcomes before considering the Superstore's choice of price in the next section. In our analysis below we use the following definition:

Definition 3.1: *An integrated portion of the city is a segment of the city line such that both types of consumers would choose to live in that section of the city at the prevailing level of rent and retail prices.*

Consumers' Problem: When consumers only have the option of shopping at the Corner Stores, the city rental lines will be similar to our base case above:

$$r_i(z) = w_i - tz - \bar{c}v_{i,c} \qquad \text{for } \ i = h, \ell$$

As in our base case the utility for consumers within the same income class must be constant with respect to location, $\frac{\partial v_i}{\partial z} = 0$, therefore the slope of the rental lines with respect to z is equal to $-t$ and does not depend on income. This results in rental lines for the two types of consumers that are parallel, with their relative position determined by the difference in their wages and their consumption of the good x (in effect the level of their rental line at the CBD given by: $r_i(0) = w_i - \bar{c}v_{i,c}$). Based on this structure the only possible equilibrium outcome where both consumers live within the city is where the rental lines for the two types of consumers overlap, a fully integrated city with high and low-income consumers living side by side (otherwise, if the rental line for one type of consumer is above the other, that consumer type would have incentive to lower their rental bid, consuming more of the good x, increasing their indirect utility without giving up their choice of location).

If a type i consumer lives at the outer limit of the city, $z = 1$, then the indirect utility for all type i consumers will be equal to that representative consumer. In an integrated city structure both types of consumers are indifferent between living at the Farmland or anywhere else in the city, therefore we can represent the indirect utility of the two types of consumers similar to our base case.

$$v_i = \frac{w_i - t - a}{\bar{c}} = \frac{g}{\bar{c}\lambda_i} \qquad \text{for } \; i = h, \ell \tag{3.1}$$

λ_i is a parameter representing transportation costs, as we defined above, but now varies with income[7].

Plugging equation (3.1) into our general rental line above we can solve for the rental line in the city for the two types of consumers.

$$r(z) = t(1 - z) + a$$

Notice that this rental line is independent of income and is identical to the rental line in our base case model before we introduced a Superstore.

Before we move on it is important to note that this equilibrium structure does not mean we are suggesting that without a Superstore a city will be fully integrated, in our simple model we are ignoring some other important factors that contribute to segregation that has been noted by previous papers, Brueckner et al. (1999) and the AMM models among others. What we are looking to identify in this analysis is the kind of agglomeration pressures that introducing a Superstore into a city can create. What we see at first is that without a cheaper outside option shopping seems to have a neutral effect on household choice of location. Given how we have constructed our model this is as we would expect. Without a Superstore households shop after work at the CBD, so shopping has the same impact as commuting, so in effect no additional impact. Now we go on to consider what happens when we introduce a Superstore into our city with heterogeneous consumers.

Superstore: As before consumers can choose to shop after work at the Corner Stores located at the CBD, or pay the additional transportation costs and shop at the Superstore located at $z = 1$. The rental line for a type i consumer located at z and shopping at the Superstore would be given by:

$$r_{i,s}(z) = w_i - tz - g(1 - z) - p_s v_{i,s} \qquad \text{for } \; i = h, \ell \tag{3.2}$$

As with the rental line for shopping at the Corner Stores the gradient of the rental lines with respect to z for the two types of consumers are identical and equal to $-t + g$.

As before the utility for type i consumers from shopping at the Superstore with shopping at the Corner Stores must be equal, $v_{i,c} = v_{i,s} = v_i$. Setting the rent lines for shopping

[7] Recall that $\lambda_i = \frac{g}{w_i - t - a}$, therefore $\lambda_h < \lambda_\ell$ by construction. This means that $v_h > v_\ell$.

at the Corner Stores equal to that of shopping at the Superstore we can solve for the point in equilibrium where consumer of type i switches from shopping at the CBD to shopping at the Superstore[8]:

$$z_i^* = 1 - \frac{(\bar{c} - p_s)v_i}{g} \qquad \text{for } \ i = h, \ell \qquad s.t. \quad z_i^* \geq 0 \tag{3.3}$$

From equation (3.3) we can clearly see that since v_h is larger than v_ℓ by construction, we must have that the switching point for the type h consumers falls below that of the type ℓ consumers. That is, $z_h^* < z_\ell^*$.

Based on our results above we can set a restriction on the equilibrium structure of our city model with heterogeneous consumers after the entry of a Superstore.

Proposition 3.1: *Given the order of switching for the two types of consumers we must have that in equilibrium $r_{\ell,j}(0) \geq r_{h,j}(0)$ and $r_{h,j}(1) \geq r_{\ell,j}(1)$ for $j \in \{c, s\}$. In other words, high-income households will always weakly prefer to live at $z = 1$, while low-income households will always weakly prefer to live at $z = 0$.*

Proof: Let us assume the opposite is true, that there exists an equilibrium such that the rich outbid the poor near the CBD, $r_{h,j}(0) > r_{\ell,j}(0)$ for all $j \in \{c, s\}$, only the rich would live in the city center. But given the fact that the rich switch to shopping at the Superstore earlier than the poor ($z_h^* < z_\ell^*$) and that both types of consumers have the same slopes for their rental lines, that would mean the rental line for the rich would be higher than that of the poor across the entire city line, the poor would be homeless. But this would not be an equilibrium. First of all, the rich have no incentive to strictly outbid the poor across the entire city line since we have not included housing size in the consumers' utility function [9]. Secondly, by assumption (v), the poor would have incentive to bid up their rental lines until they are able to live somewhere within the city. Given the order of switching for the two types of consumers, as the poor raise their offer the rental line for the poor living near the CBD would overlap with that of the rich at least as quickly as that of the poor and rich living out in the Farmland, giving us $r_{h,j}(0) = r_{\ell,j}(0)$, violating our assumption above.

In the same way, if we assume that the poor outbid the rich at the Farmland then their rental line would be above the rich across the entire city, giving the rich incentive to raise their rental lines until $r_{h,j}(1) = r_{\ell,j}(1)$.

[8] Note that this is only true in equilibrium, where no consumers have incentive to change their location and shopping decisions.

[9] this is a simplifying assumption we make in our model in order to focus on shopping behavior, other papers dealing with city structure have considered the impact of space preference on structural outcome, see AMM models.

One of the consequences of this result is that the utility of the type h consumers does not depend on the rental lines across the city. This is due the equilibrium condition that the utility of each type must be constant with respect to their location and shopping choice. The utility of all type h consumers will be equal to the utility of the type h consumer living at the point $z = 1$, where rent is constant, regardless of the resulting city structure. We will come back to this point in the discussion of the equilibrium level of consumer utility.

The above argument means that our model does not allow for the structural outcome where high-income consumers live in the city center, and low-income families are segregated out in the periphery of the city. This is an unrealistic limitation within our model as that type of structure is commonly observed in the real world, especially in major European cities, Brueckner et al. (1999). It turns out that such a structure would be possible if we added a time component to transportation costs, which would make it more costly for high-income families to live away from their job location at the CBD. Since we have assumed $t > g$, if time costs were high enough we could show that their exists a segregated outcome with type h consumers living near the CBD. Although adding a time cost of commuting to our city model seems intuitive, it adds a large amount of complexity to the problem. We will leave the formal analysis of this extension for future work, but will briefly discuss the implications of adding time costs in our conclusions, below.

Given Proposition 2.2 we must have that in equilibrium the utility for all type h consumers is equal to that of the high-income consumer living at $z = 1$ and shopping at the Superstore:

$$v_h = v_{h,s}(z = 1) = \frac{g}{p_s \lambda_h} \tag{3.4}$$

Therefore, the rental line for the higher income consumers shopping at the Corner Stores and the Superstore are respectively given by:

$$r_{h,c} = w_h - \frac{g\bar{c}}{p_s \lambda_h} - tz \qquad\qquad r_{h,s} = (t - g)(1 - z) + a \tag{3.5}$$

From the second equation in (3.5) we have that the rental line for the rich shopping at the Superstore does not depend on the price of the Superstore. That is because we have fixed the rent at $z = 1$ to a. Changes in the Superstore's price impacts the high-income consumers through their rental line from shopping at the CBD. As the Superstore's price, p_s, increases the rent the rich are willing to pay to live at the center increases, leading to fewer high-income consumers that choose to shop out in the Farmland.

The level of utility and rental lines of the low-income consumers are less straight-forward, they depend on the structure of the city, which in turn depends on the choice of price by the Superstore.

Different Cases of Consumer Demand

As we have shown above, the location at which consumer i switches from shopping at the Corner Stores to shopping at the Superstore is determined by z_i^*. We can see from equation (3.3) that as the Superstore increases its price, p_s, z_i^* increases, consumer i has to be located further away from the city center in order for them to prefer to shop at the Superstore. We will demonstrate below that changes in z_i^* for the two types of consumers, as well as our the proportion of type-h consumers α, results in a non-continuously differentiable demand curve with two kinks.

(1) $D_s^{(1)} = (1 - z_h^*)v_h \iff p_s \in \left[\frac{\bar{c}}{1+\alpha\lambda_h}, \bar{c}\right)$

(2) $D_s^{(2)} = \alpha v_h \iff p_s \in \left[\frac{\bar{c}}{1+\alpha\lambda_\ell}, \frac{\bar{c}}{1+\alpha\lambda_h}\right]$

(3) $D_s^{(3)} = [\alpha v_h + (1 - \alpha - z_\ell^*)v_\ell] \iff p_s \in \left(\hat{c}, \frac{\bar{c}}{1+\alpha\lambda_\ell}\right)$

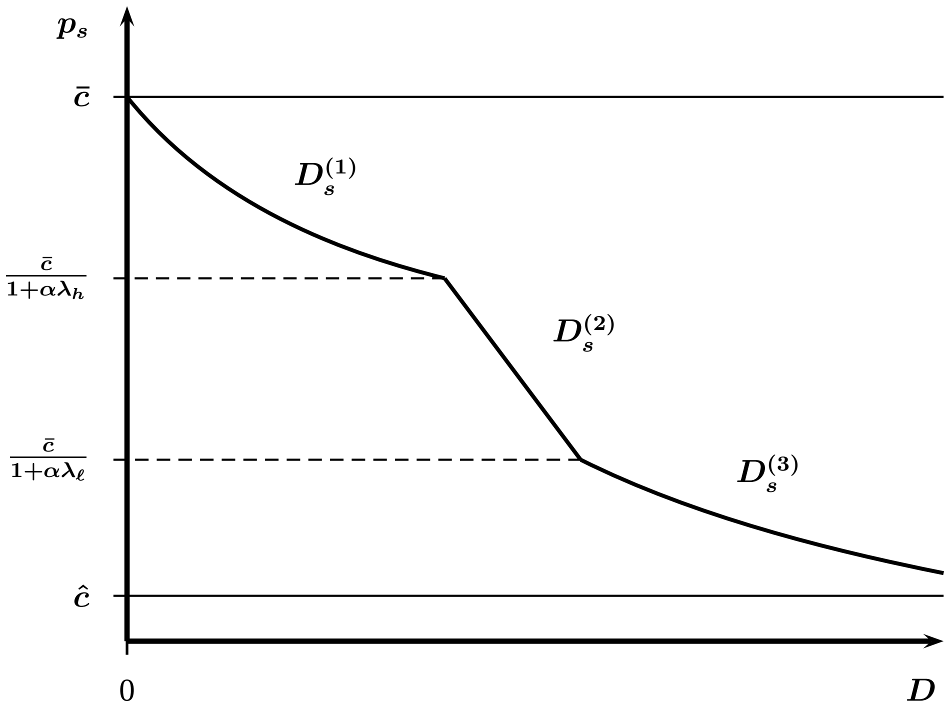

Figure 3.1: Demand Lines

In the above setup we have implicitly assumed that $\hat{c}$ is low enough relative to $\bar{c}$ to allow for the 2nd and 3rd cases of demand. In our analysis of demand below we will more formally define our assumption on the difference between the marginal cost of the two types of firms.

The demand line associated with $D_s^{(2)}$ is straight and drawn more steeply than the other two lines since in that price band demand is only increasing intensively as price decreases (no additional consumers are shopping at the Superstore, only the existing customers are buying more of the good), while in the other price ranges demand is increasing both intensively and extensively (the slopes of the lines are becoming less negative). In fact, we can show that the price elasticity of demand along $D_s^{(2)}$ is equal to -1, therefore because we assume constant marginal costs, along that section of the demand line the Superstore would increase its price until it reaches the kinked intersection between $D_s^{(1)}$ and $D_s^{(2)}$.

Since our demand line is non-continuously differentiable we will consider each of these demand lines separately. In each case of demand we will solve for the level of utility, which represents level of demand from each type of consumer, and rental bid lines in the city taking the price of the Superstore as given.

Case 1: $D_s^{(1)}$ $(\bar{c} > p_s \geq \frac{\bar{c}}{1+\alpha\lambda_h})$

In the first section of our demand line, the Superstore's price is high enough to push the switching points of the two types of consumers to a point that is further out in the city than the proportion of low-income consumers, $1-\alpha$. This means that the rental lines for the two types of consumers shopping at the CBD are overlapping, the city is integrated close to the CBD, while only high-income consumers live out in the Farmland.

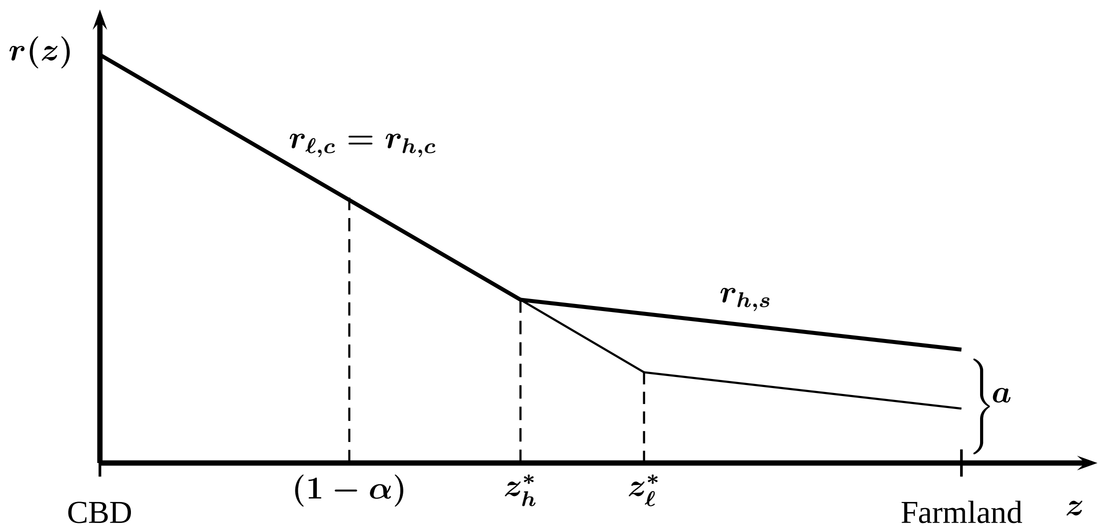

Figure 3.2: City Integrated Near the CBD ($D_s^{(1)}$)

The utility for the low-income consumers is determined by the point in the city where the rental line of the poor shopping at the CBD crosses that of the rich shopping at the Farmland, at point z_h^* in figure 3.2 above. Setting the two rental lines equal to each other we find the level of utility and the rental line for low-income consumers in terms of the Superstore's price:

$$v_{\ell}^{(1)} = \frac{g}{p_s \lambda_h} - \frac{w_h - w_\ell}{\bar{c}} \quad = \quad v_h - \frac{w_h - w_\ell}{\bar{c}} \tag{3.6}$$

$$r_{\ell,c}^{(1)} = w_h - \tfrac{g\bar{c}}{p_s \lambda_h} - tz = r_{h,c} \tag{3.7}$$

The utility of the poor is equal to the utility of the rich less the difference in the purchasing power of the two types of consumers. Interestingly, although low-income households do not shop at the Farmland, their utility is a decreasing function of the Superstore's price. In this configuration the rental lines of the poor and rich shopping at the Corner Stores are overlapping and must increase together as p_s increases, leading to a decrease in welfare for both types of consumers.

Case 2: $D_s^{(2)} \quad (\frac{\bar{c}}{1+\alpha\lambda_h} \geq p_s \geq \frac{\bar{c}}{1+\alpha\lambda_\ell})$

On the second section of the demand line the two income classes in our city are completely segregated, with the rich living out in the Farmland and shopping at the Superstore and the poor living in the city center and shopping at the CBD. At the segregation point in our city, given by $1 - \alpha$, the rental line for the low-income households shopping at the CBD equals that of the high-income households shopping at the Superstore.

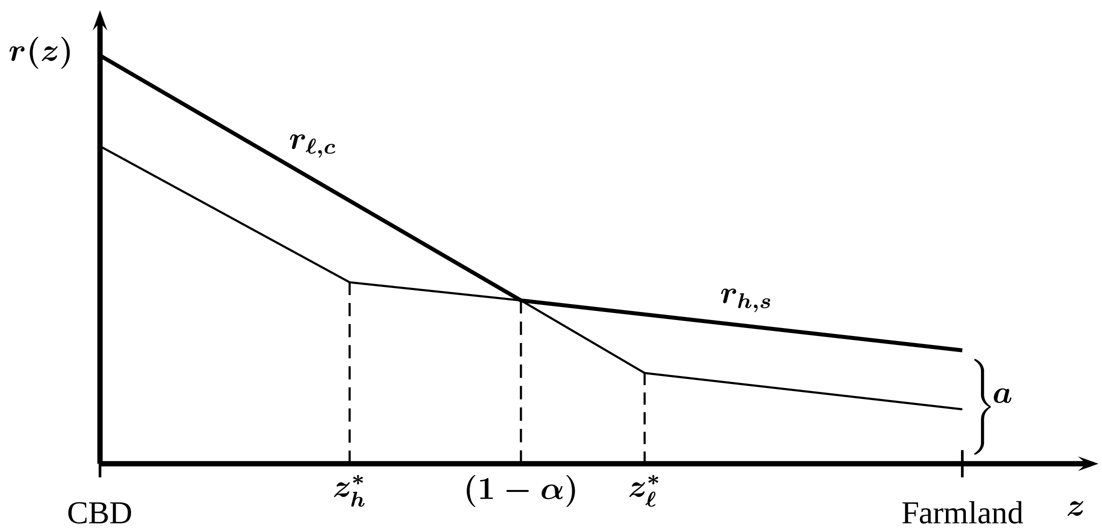

Figure 3.3: Segregated City Structure ($D_s^{(2)}$)

Evaluating the formula for $r_{h,s}$ in equation (3.5) at $z = 1 - \alpha$ and setting it equal to the rental line for the poor shopping at the CBD evaluated at the same point we can solve for the equilibrium level of utility for low-income households:

$$v_{\ell}^{*(2)} = \frac{\alpha g}{\bar{c}} + \frac{g}{\bar{c}\lambda_{\ell}} \tag{3.8}$$

We can now use our result above to get the rental line for low-income households in terms of our parameters:

$$r_{\ell,c}^{*(2)} = t(1-z) + a - \alpha g \tag{3.9}$$

From equations (3.4) and (3.8) we have that the utility of the high-income consumers depends only on transportation costs and the price of the Superstore, while the utility of the low-income households depends on the proportion of high-income households in the city, α, as well as transportation costs. In this case the utility of the poor only depends on the Superstore's price indirectly. p_s does not appear in equation (3.8) explicitly, but the Superstore's price does determine whether or not our city would be in this case of demand. The welfare of the poor increases with the percentage of the wealthy in the city because higher proportion of high-income consumers lowers the rent at the city center where the low-income households live, we can see this effect in the rental line for the low-income households in equation (3.7) above. This effect is limited in the sense that as α gets large enough it is less likely that pricing at this demand interval is optimal for the Superstore, we will get to this point below.

Case 3: $D_s^{(3)}: \quad \frac{\bar{c}}{1+\alpha\lambda_{\ell}} \geq p_s \geq \hat{c}$

In the final section of the demand line, the Superstore charges a low enough price that will push the switching points of the two types of consumers to a point that is closer to the city center than the segregation point, $1-\alpha$. This means that the rental lines of the rich and poor shopping at the Superstore are overlapping, the city is integrated out near the Farmland, while only low-income households live near the city center.

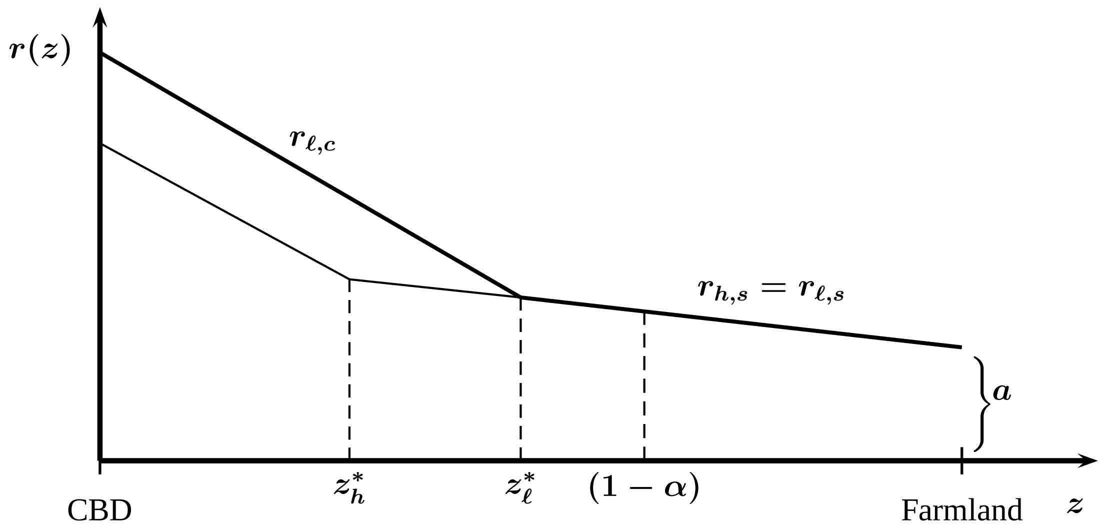

Figure 3.4: City Integrated Near the Farmland ($D_s^{(3)}$)

The Superstore captures all of the high-income consumers as well as a portion of the poor. Depending on the value of our parameters, there is also a scenario where the Superstore would choose to price low enough to capture the entire city population and push the Corner Stores out of the city. Under this scenario the city would be fully integrated with both consumer types shopping at the Superstore. We will consider under what condition such an outcome might occur below.

Since we have that the city is integrated out near the Farmland the utility for both types of consumers is given by the utility of the consumer living at $z = 1$ and shopping at the Superstore:

$$v_i = v_{i,s}(z=1) = \frac{g}{p_s \lambda_i} \tag{3.10}$$

While the two rental lines for low-income households are given by:

$$r_{\ell,c}^{(3)} = w_\ell - \tfrac{g\bar{c}}{p_s\lambda_\ell} - tz \qquad\qquad r_{\ell,s}^{(3)} = (t-g)(1-z) + a = r_{h,s} \tag{3.11}$$

The rental line for the two types of consumers shopping at the Superstore are overlapping, therefore they are identical.

Finally, although technically the Superstore might choose a price as low as its own marginal cost, $\hat{c}$, and still earn non-negative profits, we would like to consider the price at which the Superstore would capture the entire market and push out the Corner stores out of our city. Under this scenario, the rental lines for the two types of consumers shopping at the Superstore would be overlapping over the entire city line. We can solve for this lower bound on the Superstore's price by setting the switching point for the lower income consumer, z_ℓ^*, to zero. Solving for p_s we get the following additional lower bound for the Superstore's price.

$$p_s = \frac{\bar{c}}{1+\lambda_\ell} \tag{3.12}$$

Clearly this price could end up below the marginal cost of the Superstore, $\hat{c}$, but to allow for the possibility of a fully integrated city we will assume that $\hat{c} < \frac{\bar{c}}{1+\lambda_\ell}$.

4 Superstore's Problem With Households Heterogeneous In Income

Now that we have determined how the consumer population would choose their housing and shopping locations given the Superstore's price, we will look at how the Superstore will choose a price knowing the reaction of the consumer market. We will first set up the general form of the Superstore's problem, then we will solve for the optimal price within each case of demand as defined above. Finally we will identify under what conditions, if any, on our model parameters the proposed structures above constitute an equilibrium, and the resulting levels of utilities, rent and profit in each possible structure.

The Superstore's Problem: The general form of the Superstore's problem is given by:

$$\max_{p_s} \quad [\theta_h(p_s)v_h + \theta_\ell(p_s)v_\ell]\,[p_s - \hat{c}]$$

$$s.t. \quad \theta_h(p_s) \leq \alpha \quad \text{and} \quad \theta_\ell(p_s) \in [0, 1-\alpha]$$

$\theta_i(p_s)$ is the proportion of type i consumers that shop at the Superstore. We introduce this notation rather than using z_i^* since we would like to limit these proportions to correspond with the proportion of each type of consumer. We cannot do that using z_i^* since the latter can take any value between 0 and 1. The above set up is in effect a maximization problem with three inequality constraints (we know that θ_h is positive by Proposition 2.2 and the fact that it would always be optimal for the Superstore to charge a price below the Corner Stores and attract a positive number of consumers).

Whether or not the constraints in the maximization problem above are binding depends on which one of our cases of consumer demand the Superstore is facing. We will consider the Superstore's problem under each case of demand separately.

<u>The Superstore's profit in demand case $D_s^{(1)}$:</u>

$$\pi_s^{(1)} = \frac{g}{\lambda_h^2}\left(\frac{\bar{c}}{p_s} - 1\right)\left(1 - \frac{\hat{c}}{p_s}\right) \tag{4.1}$$

$$s.t. \quad p_s \geq \frac{\bar{c}}{1+\alpha\lambda_h}$$

Maximizing with respect to p_s we can solve for the Superstore's choice of price and resulting level of profit in our first case of demand:

$$p_s^{*(1)} = \frac{2\bar{c}\hat{c}}{\bar{c} + \hat{c}} \qquad \pi_s^{*(1)} = \frac{g(\bar{c} - \hat{c})^2}{4\bar{c}\hat{c}\lambda_h^2} \tag{4.2}$$

A similar result to what we calculated for the Superstore in our Base Case. This similarity is not surprising since, as in the Base Case, the Superstore is targeting a portion of one type of consumer.

Now that we have the Superstore's choice of price, we can substitute it into the levels of utility for the two types of consumers in equations (3.4) and (3.6) to determine the consumers' utility in terms of our parameters.

$$v_h^{*(1)} = \frac{g(\bar{c} + \hat{c})}{2\bar{c}\hat{c}\lambda_h} \qquad v_\ell^{*(1)} = v_h^{*(1)} - \frac{w_h - w_\ell}{\bar{c}} \tag{4.3}$$

We can do the same for our rental lines. We have already shown that the rental line for high-income consumers shopping at the Superstore does not depend on price, and is equal to the rental line from equation (3.5) above. The only binding rental line for the poor is from shopping at the CBD, and as we have previously shown, overlaps that of the rich.

$$r_{h,c}^{*(1)} = r_{\ell,c}^{*(1)} = w_h - \frac{g(\bar{c}+\hat{c})}{2\hat{c}\lambda_h} - tz \tag{4.4}$$

The Superstore's profit in demand case $D_s^{(2)}$:

This is the fully segregated outcome in our model, where the poor live in the city center and the rich live out near the Farmland. In this part of the demand graph all high-income consumers and none of the low-income consumers shop at the Superstore, therefore the Superstore's profit function is given by:

$$\pi_s^{(2)} = \frac{\alpha g(p_s - \hat{c})}{\lambda_h p_s} \tag{4.5}$$

$$s.t. \quad \frac{\bar{c}}{1+\alpha\lambda_h} \geq p_s \geq \frac{\bar{c}}{1+\alpha\lambda_\ell}$$

From figure 3.3 above, and examination of the profit function in equation (4.5), we can see that the demand faced by the Superstore is not dependent on z_i^* as long as $z_h^* \geq (1-\alpha)$, this means that any decrease in price by the Superstore does not lead to new customers, only existing customers purchasing more of the good. Since the Superstore faces constant marginal cost and demand is unit elastic as long as $p_s < \frac{\bar{c}}{1+\alpha\lambda_h}$, the Superstore would push its price up to the upper bound of the constraint above, decreasing costs without any loss to revenue. Therefore, in this case of demand we would have a corner solution with price given by:

$$p_s^{*(2)} = \frac{\bar{c}}{1+\alpha\lambda_h} \tag{4.6}$$

The price of the Superstore is an increasing function of w_h and does not depend on w_ℓ. As the income of the wealthier consumers increases, the price of the good at the Farmland approaches the price of the Corner Stores asymptotically. We can also see that p_s is a decreasing function of α, that is as the proportion of high-income families increases, it will be optimal for the Superstore to lower its price in order to prevent some wealthy consumers from switching to shopping at the Corner Stores. In the same way we can see that as the proportion of wealthy consumers decreases, the price of the Superstore approaches that of the Corner Stores, requiring higher prices from a smaller population of wealthy consumers being pushed further away from the CBD.

Substituting the price above into equation (3.4) and (4.5) we derive the utility for high-income consumers and profit of the Superstore in terms of our parameters:

$$v_h^{*(2)} = \frac{\alpha g}{\bar{c}} + \frac{g}{\bar{c}\lambda_h} \qquad \pi_s^{*(2)} = \frac{\alpha g[\bar{c} - \hat{c}(1+\alpha\lambda_h)]}{\bar{c}\lambda_h} \tag{4.7}$$

Since the utility of the poor as well as the effective rental line across the city are independent of the price of the Superstore, they are as we calculated in (3.8) and (3.9) above. We can see in this scenario of demand the utility of the rich and poor are almost exactly the same, except that the rich benefit from transportation costs being a lower percentage of their disposable income, $\lambda_h < \lambda_\ell$. It seems that the segregation of the poor into the city center does not cost the poor in terms of their level of indirect utility. That is, despite paying higher rents in the city center and being forced to shop at the more expensive Corner Stores, in our model the rental and goods markets adjust in a way to compensate the poor for their worse-off position. In real world situations there might be structural frictions that exist that would prevent such a compensation. One example would be if the poor had to pay higher commuting costs relative to wealthier households, allowing the Superstore or landlords to be less concerned with the impact of their choice of price on the less mobile poor population, LeRoy and Sonstelie (1983).

Comparing the equations above with our derived indirect utility without the Superstore in equation (3.1) we can see that under this demand scenario the utility for both types of consumers have increased by a factor of $\frac{\alpha g}{\bar{c}}$ due to the entrance of the Superstore. Clearly the positive impact of the entry of the Superstore increases as the cost of commuting to the store and the proportion of wealthy in the population increase. Therefore, from the consumers' perspective some level of transportation costs serves as a benefit by driving down rental costs.

Before we go on, it is important to note that the corner solution at the kink between $D_s^{(2)}$ and $D_s^{(3)}$ would never be an optimal outcome for the Superstore. As we argued above, once the Superstore reaches a price where all of the rich and none of the poor shop out in the Farmland, the elasticity of demand becomes -1 and the Superstore would choose to continue increasing its price until demand reaches the intersection of $D_s^{(1)}$ and $D_s^{(2)}$.

The Superstore's profit in demand case $D_s^{(3)}$:

$$\pi_s^{(3)} = g\left[\alpha\left(\frac{1}{\lambda_h} - \frac{1}{\lambda_\ell}\right) + \frac{1}{\lambda_\ell^2}\left(\frac{\bar{c}}{p_s} - 1\right)\right]\left[1 - \frac{\hat{c}}{p_s}\right]$$

$$s.t. \quad \frac{\bar{c}}{1+\alpha\lambda_\ell} \; > \; p_s \; \geq \; \frac{\bar{c}}{1+\lambda_\ell}$$

We have a strict inequality on the upper constraint from our argument above ruling out the kink between $D_s^{(2)}$ and $D_s^{(3)}$. It is possible that the Superstore will drive down its price low enough to capture the entire market. This would be a corner outcome such that the switching point of the low-income consumer will be pushed all the way back to the center of the city, we will consider this corner out come below. Now we look for an interior solution in our third case of demand.

Maximizing with respect to p_s, we get the following for the Superstore's choice of price:

$$p_s^{*(3)} = \frac{2\bar{c}}{\frac{\bar{c}+\hat{c}}{\hat{c}} - \alpha\lambda_\ell\left(\frac{\lambda_\ell - \lambda_h}{\lambda_h}\right)} \tag{4.8}$$

Resulting in consumer utility and Superstore profit of:

$$v_i^{*(3)} = \frac{g}{2\bar{c}\lambda_i}\left[\frac{\bar{c}+\hat{c}}{\hat{c}} - \alpha\lambda_\ell\left(\frac{\lambda_\ell-\lambda_h}{\lambda_h}\right)\right] \qquad \text{for } \; i = h, \ell \tag{4.9}$$

$$\pi_s^{*(3)} = \frac{g\hat{c}}{4\bar{c}\lambda_\ell^2}\left[\frac{\bar{c}-\hat{c}}{\hat{c}} + \alpha\lambda_\ell\left(\frac{\lambda_\ell-\lambda_h}{\lambda_h}\right)\right]^2 \tag{4.10}$$

The welfare of the two types of consumers is increasing with the cost advantage of the Superstore, which is not surprising since in this segment of demand they are both shopping at the Superstore. Interestingly the welfare of both types of consumers decreases as the difference in income between the two increases. This is because the Superstore's price increases with the differential in income.

The rental line for the poor and the rich shopping at the Superstore is again not impacted by the Superstore's price and is given by the second equation from (3.5) above. We can derive the rental line of the poor shopping at the CBD buy plugging in our resulting price into the rental line from equation (3.11).

In the case of a corner outcome the Superstore drives down its price to the point where all consumers in the city shop at the Farmland, in other words the Superstore would price such that $z_\ell^* = 0$:

$$p_s^{*(4)} = \frac{\bar{c}}{1+\lambda_\ell} \tag{4.11}$$

At this price the city would be fully integrated, the rental lines for the two types of consumers would be overlapping and cover the entire city line, with all consumers shopping at the Superstore. This scenario would result in consumer utility and Superstore profit equal to:

$$v_i^{*(4)} = \frac{g(1+\lambda_\ell)}{\bar{c}\lambda_i} \qquad \pi_s^{*(4)} = \frac{g[\bar{c}-\hat{c}(1+\lambda_\ell)]}{\bar{c}}\left(\frac{\alpha\lambda_\ell+(1-\alpha)\lambda_h}{\lambda_\ell\lambda_h}\right) \tag{4.12}$$

The city rental line would be determined by that of the poor and rich shopping at the Superstore and given by second equation in (3.5) above.

We will consider the implications for consumer utility for each of the outcomes described above.

5 Analysis of Equilibria

Now that we have fully described the choice of price by the Superstore under each demand scenario as well as the various city structures that result, we will consider under what conditions these different structural and price outcomes represent an equilibrium. By equilibrium we mean a case where the Superstore is maximizing profits under such a city structure, and where consumers have no incentive to move or change their shopping options. We begin by considering the constraints that we defined in the three cases of the Superstore's problem above. Comparing our derived price for the Superstore in each of the cases with the respective constraints, we obtain the following range of values for our parameters. The ranges on our parameters below signify the conditions for profit maximization for each structural outcome:

(i)	$\frac{\bar{c}-\hat{c}}{\hat{c}} < 2\alpha\lambda_h$	$\Rightarrow$	Integrated Center
(ii)	$2\alpha\lambda_h \leq \frac{\bar{c}-\hat{c}}{\hat{c}} \leq \alpha\lambda_\ell(1+\frac{\lambda_\ell}{\lambda_h})$	$\Rightarrow$	Segregated
(iii)	$\alpha\lambda_\ell(1+\frac{\lambda_\ell}{\lambda_h}) < \frac{\bar{c}-\hat{c}}{\hat{c}} < 2\lambda_\ell + \alpha\lambda_\ell(\frac{\lambda_\ell-\lambda_h}{\lambda_h})$	$\Rightarrow$	Integrated Periphery
(iv)	$2\lambda_\ell + \alpha\lambda_\ell(\frac{\lambda_\ell-\lambda_h}{\lambda_h}) \leq \frac{\bar{c}-\hat{c}}{\hat{c}}$	$\Rightarrow$	Fully Integrated

The above constraints follow the path we would expect. When income inequality in the city, measured in our model by α and $(\lambda_\ell - \lambda_h)$, is high relative to the Superstore's cost advantage over the Corner stores, $\frac{\bar{c}-\hat{c}}{\hat{c}}$, then the Superstore would be expected to charge a higher price and only target a portion of the wealthy consumers, as in our demand structure $D_s^{(1)}$. This leads to an equilibrium structure where only the rich live out in the Farmland, the area near the CBD is integrated, and only a portion of the high-income consumers shop at the Superstore.

As the cost advantage of the Superstore increases relative to the inequality within the city, we move through the different cases of city structure that we have described above. We can use the ranges of outcome on our parameters to draw figures depicting the ranges in which the four cases of demand obtain. The diagram below compares the cost advantage of the Superstore over the Corner stores, $\frac{\bar{c}-\hat{c}}{\hat{c}}$, relative to the proportion of high-income consumers in our city, α.

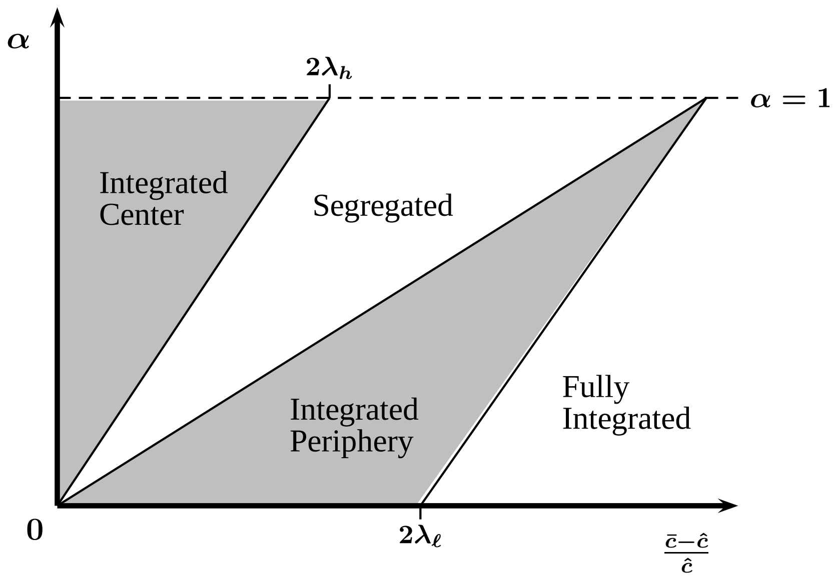

Figure 5.1: Regions of Equilibria: α

We can see that for low levels of α and a high cost advantage for the Superstore it becomes more likely that our city is fully integrated. In this structure both types of consumers are shopping at the Superstore and are indifferent as to where they live. This outcome is associated with the lowest price charged by the Superstore, and the lowest rental prices across the city. For middle-values of income inequality and Superstore cost advantage it becomes more likely that our city is either fully segregated, or integrated out near the Farmlands with only the poor living near the city center.

Similarly, for small income differentials (represented by a smaller difference between λ_h and λ_ℓ) and high cost advantage for the Superstore, the fully integrated outcome is more likely, while a middle range of the two factors leads to our segregated outcome. Interestingly, for low values of the Superstore's cost advantage, holding α constant, income differential does not impact the equilibrium outcome. Even for very small differences in income, the demand structure $D_s^{(1)}$ would be an equilibrium. We can interpret this result as a capacity requirement on stores in order for shopping options to impact consumer location decisions. For consumers to take the location of the Superstore into consideration when deciding where to live, there must be a significant difference between the price (and capacity) of the Superstore relative to the corner stores that are more readily available.

Equilibrium Utility

To see the impact of the Superstore on consumer welfare we look at the utility of each type of consumer in the different city structures. The following are the resulting utility levels from the various equilibrium outcomes described above. The first case is when we have heterogenous consumers without the Superstore. The next four outcomes are the various city structures that are possible when the Superstore enters the market.

Fully Integrated Without Superstore

$$v_i = \frac{g}{\bar{c}\lambda_i} \qquad \text{for } i = h, \ell$$

Integrated Center

$$v_h^{(1)} = \frac{g}{p_s\lambda_h} \qquad v_\ell^{(1)} = v_h^{(1)} - \frac{w_h - w_\ell}{\bar{c}}$$

Segregated

$$v_h^{(2)} = \frac{g}{p_s\lambda_h} \qquad v_\ell^{(2)} = \frac{g(1+\alpha\lambda_\ell)}{\bar{c}\lambda_\ell}$$

Integrated Periphery

$$v_h^{(3)} = \frac{g}{p_s\lambda_h} \qquad v_\ell^{(3)} = \frac{g}{p_s\lambda_\ell}$$

Fully Integrated With Superstore

$$v_i^{*(4)} = \frac{g(1+\lambda_\ell)}{\bar{c}\lambda_i} \qquad \text{for } i = h, \ell$$

The utility of type h consumers is always determined by the high-income consumer living at $z = 1$ and shopping at the Superstore. Therefore, the utility of the type h consumers is rising at an increasing rate as the Superstore lowers its price, irrespective of city structure or changes in the rental lines. The utility of the low-income consumers only increases in the semi-integrated outcomes, staying constant when the city is segregated. Figure 5.2 portrays how consumer utility changes as the Superstore's marginal cost, $\hat{c}$, changes relative to the Corner Stores', $\bar{c}$ (keeping α and λ_i constant), moving the city across the various income structures.

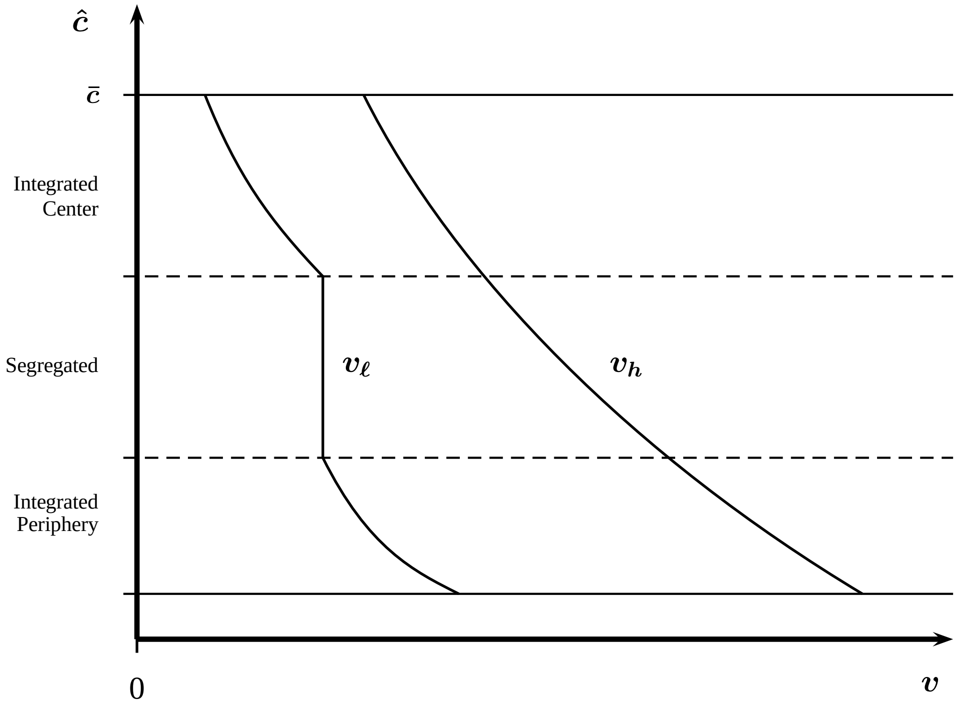

Figure 5.2: Equilibrium Utility

Interestingly, the utility of the type ℓ consumers increases at the same pace as the high types up until the price of the Superstore reaches the constraint for the segregated outcome. Although in the first case of demand low-income consumers do not shop at the Superstore, they benefit from the low price offered by the Superstore as if they were type h consumers. This is because high-income consumers live across the entire city. Therefore, the rent-savings in the city center must match the cost savings of shopping at the Superstore in order to keep the high-income consumers located at the city center indifferent to those living out near the Farmland.

In the segregated outcome the utility of the low-income consumers is fixed by the segregation point in the city. Their rent is not changing and they are continuing to pay the same price at the CBD. This is when the utility of the high-income consumers begins to increase relative to the ℓ types. In this segregated outcome h type consumers benefit from low rents and low prices in the outer part of the city. Since there are no h types living in the city center, the rent for low-income consumers does not change, leaving them relatively worse off. This is the period in the figure above where the utility of the two types of consumers diverges at the fastest pace.

Finally, when the city is integrated out in the periphery, the low-income consumers begin to benefit directly from the low price offered by the Superstore. Now the utility of both types of consumers is determined by the representative consumer living at $z = 1$ and shopping at the Superstore. Yet, the utilities of the two types of consumers continue to diverge. In this city structure rents are no longer changing across the city, only changes in Superstore's price affects consumer welfare. The greater buying power of high-income consumers allows them to benefit more from the decrease in the Superstore's price, their utility increases at a faster pace relative to that of the low-income consumers.

As we can see in the figure above, both consumers are better off from the entrance of the Superstore into the market. In addition, the welfare of both types of consumers is increasing monotonically with decreases in the price of the Superstore. These are not very controversial or surprising results. In most cases, consumer theory predicts that the entry of lower priced firms does not hurt, and in general benefits, consumers.

The interesting result in our equilibrium analysis above is that in certain forms of city income structure, the two types of consumers do not benefit equally from the Superstore's discount. When low-income consumers are segregated away from the Superstore they do not benefit from the low prices offered. This effect is mitigated by what we can call ”Neighborhood Effects”. When the city center is integrated, the rental lines across the city are interconnected, allowing low-income consumers to benefit from the Superstore's price through lower rent costs.

6 Conclusion

In this paper we have analyzed various income structures in cities and how they are related to firm pricing strategy. In our base case model with a homogeneous consumer population we showed that the Superstore's entry into the city leads to lower rental prices, allowing consumers to spend a higher portion of their income on consumption of our representative good, x. We also argued that without any income disparity, all consumers benefit equally from the entry of the Superstore into the market.

Then we introduced heterogeneous consumers into our model, with consumers earning two different exogenously determined levels of income. We showed that as the difference between the income of the rich and poor increases it becomes more likely that our city is segregated by income, with the rich consumers living in the outer part of the city. We also showed the conditions under which we would have more hybrid structural outcomes. More specifically, we showed that as the cost advantage of the Superstore increases relative to the Corner Stores, the city becomes more integrated, moving the resulting equilibrium towards the fully integrated outcome with all consumers shopping at the large discount store. At the same time we demonstrated that the two types of consumers do not benefit equally from the entry of the Superstore, especially when the city is completely segregated.

The determinants of city structure that we consider are similar to those considered in previous literature in that they are associated with the amenities available in a city. We go beyond previous literature on regional economics by focusing on the interplay of the rental market with firms' strategy, thereby looking to connect the existing work in regional economics with the industrial organization literature. We also introduce distributional considerations into the city model, demonstrating how the makeup of the consumer population can impact firm strategy and city structure. Of course households take into account many different factors when choosing where to live, and most of these issues have been dealt with in previous work. The purpose of our paper is to attempt to investigate the impact of firm pricing on household choice of location, taking the other factors analyzed in previous work as given.

The results we have presented above are an example of how spatial frictions can lead to divergence of welfare between consumers of different income levels. These results are similar to the information frictions that we discussed in the first chapter of this thesis. Whether lower income consumers are at a disadvantage because they do not have good information or physical access, the implications are similar. The existence of spatial and information frictions can lead to market outcomes that puts lower income households at an inherent disadvantage.

One natural question to ask, given our results, is what are the policy implications of such spatial frictions. Clearly the transportation costs present in our model are key factors in the outcomes we have described. But lowering transportation costs across the city line would not necessarily help remove these frictions. Our results depend mainly on $\boldsymbol{\lambda}$, transportation costs relative to consumers' disposable income. Lowering the cost of transportation would not lower the relative $\boldsymbol{\lambda}$ between the two types of consumers considered. An alternative would be to tax the high-income consumers, and/or the Superstore, and use the tax proceeds to subsidize the transportation costs of the lower income group. But these taxes might have a distortionary impact on Superstore pricing as well as our rental market. We hope to more carefully consider this question in a future paper.

Finally, one shortcoming of our model is that it does not explain a segregated city outcome with rich consumers living in the city center, a commonly observed city structure, especially in major European cities, Brueckner et al. (1999). One way to address this issue would be to include time costs of transportation into our model. The monetary costs of the time spent traveling from home to work is the loss of wages. Therefore, introducing time costs would increase the incentive for wealthier consumers to locate closer to their jobs in the city center (this is assuming that the time cost of commuting to work is greater than that of shopping at the Superstore, a reasonable assumption since most people commute to work everyday while they go shopping only one or two times a week). Adding time costs allows for a much wider set of structural outcomes in our city and would be a very interesting extension of our analysis.

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