THE EFFECT OF INCOME INEQUALITY ON PRICE DISPERSION

Author: Babak Somekh [1]

UNIVERSITY OF OXFORD AND UNIVERSITY OF HAIFA

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ABSTRACT

Using a supply/demand consumer model with search, we show under what conditions the distribution of income within a community is related to the type of firms that exist within that community, impacting the level of prices. We assume that searching for the lowest price costs both time and money to the consumer. If time and money costs are high enough low-income consumers cannot afford the monetary cost of search, while wealthy consumer are not willing to take the time to look for the lowest price. The middle class have the right balance of time and money cost of search and therefore are the most aggressive shoppers. We use a supply side model of firm output and pricing strategy to demonstrate that firms located in more informed communities are more likely to enter the market as large low-priced retailers. By connecting these two results, we show under what conditions the size of the middle class can have a negative relationship with the level of prices in a local market. Our paper goes beyond other work on causes of price dispersion by allowing consumers to purchase a continuous amount of the good, and by incorporating a distribution of search costs. Both these modifications allow us to focus more specifically on the link between income distribution and prices.

[1]Department of Economics, University of Haifa, Haifa, 31905, Israel, bsomekh@econ.haifa.ac.il

1 Introduction

Equal opportunity is one of the foundations of democratic economies. All individuals, no matter their race, gender, or economic background, are meant to be afforded the same opportunity to succeed. In general this promise is thought of in the context of access to education and the labor market. Also important is access to consumer markets. In this chapter we construct a theoretical model that describes why low-income families might have a hard time competeing in consumer markets. Given certain assumptions about market structure, we are able to demonstrate two important and connected results. First we show that when there exists a time and money cost of searching for the lowest price, the middle class have the optimum balance of cost and benefit of search, and therefore search the most intensely. Second, we connect consumer search intensity in a particular market with the types of firms that choose to compete in that market. By connecting these two results, we show that a higher proportion of middle-income households leads to more competitive firms operating in that market, leading to lower prices. The main implication of our results is that separate is not necessarily equal, income segregation can result in less competition in the segregated market, leading to higher prices for low-income families.

Whether or not the poor face higher prices in consumer markets has been the subject of extensive debate. There have been various studies done at all levels and in many countries with inconsistent results. These studies sought to connect level of income with prices paid for consumer products such as food, transportation, housing and big ticket items. Though there are certain instances where the data supports the idea of the poor paying more, there are also examples of markets where there is no evidence of price dispersion due to income. An empirical paper by Frankel and Gould (2001) offers an alternative explanation for the instances where higher prices are observed for the poor. Using a search cost function U-shaped in income, they argue that the middle class search the most intensely for the lowest price, and therefore their presence in a given market leads to lower prices.

In this paper we construct a theoretical framework to explain the relationship between the size of the middle class and market prices. First we consider some of the relevant research that has gone into studying the idea of the poor paying more. Next we begin constructing our model by setting up the consumer's problem within a model that incorporates time usage and search behavior. Consumers have the option to pay a time and monetary cost to obtain the lowest price, or randomly choose a store in the market. By comparing the utility of consumption obtained from searching versus not searching across consumers with varying levels of income, we look to determine whether or not the middle class have the highest incentive to search.

Then we consider the idea of price dispersion from the point of the view of the firm. There have been a number of theoretical papers that try to explain the presence of price dispersion in consumer products. We use a version of a model originally developed by Salop and Stiglitz (1977), where price dispersion is a result of varying levels of informed consumers in a market. We show that when the proportion of the uninformed consumer population increases, smaller, higher priced firms enter the market, leading to higher average price.

Finally, we combine these two results to show that as the size of the middle class in a market increases, the portion of the perfectly informed consumers increases, and therefore prices decrease. Our results match previous findings in theoretical and empirical search models where even small amounts of search costs can lead to an outcome where no consumers search and all firms charge the maximum price (see for example Diamond (1971) and Baye et al. (2006)). We extend this result to show that as search costs become small enough relative to the extent of price dispersion only a portion of consumers search, and this portion consists of the middle class. We show that as long as search costs exist the very rich and very poor never search, therefore they depend on the middle class to keep markets competitive.

In our model there are some key assumptions about the time and money cost of search, as well as market structure, that lead to the results described above. Future research would need to consider characteristics of specific markets and households to determine if the assumptions we have made are consistent with empirical data. If it can be shown that the poor do not search as intensely as the middle class because of time and cost constraints, then policy should be directed at alleviating these costs and allowing more information to flow to the lower income households. On the other hand, if the problem is shown to be behavioral, that the poor choose not to search for the lowest price because of a lack of motivation or desire, then the problem becomes much more complicated, and would require a deeper understanding of how the poor interact in the economy.

Now we will motivate the assumptions of our theoretical model by discussing the relevant literature.

2 Empirical Background

When the clamor that ”The Poor Pay More”, Caplovitz (1963), was first heard in the U.S. in the midst of the racial riots in the 1960s it was greeted as one of paranoia. As economists began to study the issue it became clear that there was more to the assertion than first expected. What also became clear was that this question was a complicated one and would not be answered easily. In this section we survey research focused on determining if the poor do in fact pay more. As we would expect, the issue is one of both supply and demand. The makeup of a consumer population and the amount of information available to consumers determines the characteristics of consumer demand. What types of firms enter a market and the cost structure of these firms determine the price strategy of firms in a given market. These two forces working together can help explain the disparity in prices that triggered the original uproar. What started as a racial and sociological discussion has evolved into a more analytical and economic question.

There are several quantifiable parameters that are generally discussed when trying to determine why the poor face higher prices. First, there is the issue of the lack of availability of large discount stores in predominantly poor neighborhoods, or what is called the ”store effect”, Kunreuther (1973). It is proposed that prices at smaller groceries are higher than large supermarkets and discount stores. These larger stores are said to have lower per unit fixed costs and have higher purchasing power due to their ability to buy in bulk. Fixed costs are said to be even higher for stores in poor urban neighborhoods due to higher crime rates, which tend to push up insurance and maintenance costs. Next, it is proposed that goods sold in larger packages have lower per unit costs than the same goods in smaller sized packages, or what is called the ”size effect”. Smaller groceries, due to lack of shelf space, do not provide larger sized packages, adding to the higher per unit costs of shopping at these stores relative to supermarkets. A sub theory of the size effect is that lower income families have limited liquidity and limited storage space, especially freezer and refrigerator space, not allowing them to purchase bulk items [See Attanasio and Frayne (2006), and Rao (2000)]. Finally, it is argued that low-income families find it too expensive to search for and access the lowest priced firms, Clifton (2004). In general, low-income families are less likely to own a car or have access to the internet, forcing them to rely on public transportation to perform comparative shopping, which can be prohibitively time consuming and expensive.

The book by Caplovitz (1963) was one of the first attempts at linking income level to prices in a specific retail market. He found that the poor paid significantly more for major durables such as televisions and washing machines than the average consumer. In 1971, a study into food prices in New York City found no relation between food prices and neighborhood income level, Alcaly and Klevorick (1971). In 1974 an extensive study of consumer markets in the U.K. concluded that in general the poor seemed to face higher prices, and by all accounts never faced lower prices than other income classes, Piachaud (1974). Alcaly and Klevorick and Piachaud admit that their findings have weaknesses, mainly due to lack of detailed data. Fortunately it seems the information available for analysis has improved since Piachaud conducted his study. Through the advent of scanner data, computerized operations and the initiation of government sponsored surveys, researchers have access to better analytical tools. Nonetheless, the lack of consensus on the issue remains.

In 1991, New York's Consumer Affairs Department investigated grocery store price-fixing in several neighborhoods. Their survey of sixty stores and 140 interviews throughout New York found that the poor paid more for groceries in urban areas while receiving lower quality service, Freedman (1991). Further regional studies in Pittsburgh, Austin, and Minneapolis found that there was significant evidence that the poor paid higher retail food prices than the average consumer [See Dalton et al. (2003), Clifton (2004), and Chung and Myers (1999)]. Similarly a 2007 survey of the U.K. found that, in general, low-income families faced higher prices in financial services, utilities, telecommunications and durable goods purchases, Kober and Sterlitz (2007). These tests arrived at their conclusions from different perspectives. Some attributed the discrepancy in prices to the lack of supermarkets in poor neighborhoods, or the store effect. While others cited inability to buy in bulk due to either limited budgets or lack of choice. Another group found that poor households paid more because of higher absolute prices in stores located in low-income areas.

On the other hand, a 1991 study of ten regions using data on 322 retail supermarkets found no statistically significant evidence that consumers in low-income neighborhoods paid higher food prices than consumers in high-income areas, MacDonald and Nelson (1991). In 2000, a study using unpublished CPI data on prices paid by consumers across the United States found no evidence of higher prices faced by the poor, and in fact found that in some cases the poor faced prices up to 6% lower than the average, Hayes (2000).

There are also contradictory findings in developing economies. A study of two rural towns in India found the poor paid significantly higher prices for food products, due mainly to lack of storage space and access to credit, Rao (2000). A similar study of 122 medium sized towns in Colombia found that prices decreased with bulk purchases, leading to lower income families paying higher prices due to lack of capital, Attanasio and Frayne (2006). Both of these studies found some level of coping amongst poor families through communal purchases, though limited to very specific consumer goods. A study of 256 households in rural Rwanda, using detailed consumption data collected over the course of 2 years in the early 1980s, found a significant negative relationship between the standard of living of a household and the price index faced by the household, Muller (2002). In contrast, a study of consumer prices in Brazil found little evidence of a relationship between income and prices faced. In fact the study found some evidence of a positive relationship between household income and food prices, Musgrove and Galindo (1988).

There have also been studies performed to determine if it is true that the poor do not have access to discount firms. An extensive study into the shopping habits of consumers in the U.K. found that an increasing number of large retailers are moving from town centers to out-of-town locations, Piachaud and Webb (1996). Through a survey of consumer shopping habits, the study finds that low-income families with limited mobility are left with no choice other than to shop at high-priced local stores. In another study, the concept of ”Food Deserts” in the UK, or lack of access to cheap and healthy food for low-income families, has been shown to be an increasingly common issue in cities across the country, Wrigley (2002). In the U.S., research in the late 1990s found that the number of supermarkets has dropped by 22 % from 1966 to 1993, mainly due to a major consolidation in the industry[2]. The study found that supermarkets and food retailers are not as prevalent in low-income neighborhoods[3], low-income areas in nineteen U.S. cities had 30 % fewer stores per capita compared to higher income areas. A regional study of Allegheny County, which contains the city of Pittsburgh, found similar results, Dalton et al. (2003). The study found that large supermarkets were on average a lot more accessible in middle to high-income suburban areas of the region. A Minneapolis study found that there was a significant positive relationship between income level and the number of supermarkets in a neighborhood, Chung and Myers (1999). The city of New York conducted a city wide study into the location of supermarkets. The study found that there was a significant shortage of supermarkets in the City. Lack of access to cheap and healthy food was found to be especially stark in low-income and minority neighborhoods, Gonzalez (2008).

If we accept that supermarkets are less likely to be located in poor areas, then it is important to determine if it is possible for poor families to commute to other neighborhoods to do their shopping. There are two conflicting factors affecting low-income families' ability to shop around and look beyond their own neighborhood stores. On the one hand, the poor are said to have a lower cost of time, which means it is more economic for them to comparison shop than wealthier consumers. At the same time, the poor on average tend to rely more heavily on public transportation, making it a lot harder for them to commute to the areas where lower prices might exist. Studies into the commuting habits of low-income families have found that most do not have direct access to their own cars, but are able to compensate through alternative methods of transportation, Dalton et al. (2003). A similar study in Austin, Texas, found that poor consumers are not necessarily confined to shopping within their own neighborhoods, and through the use of alternative methods of transportation are able to gain access to discount stores and supermarkets, Clifton (2004). But this access comes with a significant cost in time and money spent on transportation, which is not always economical. As a result, consumers rely heavily on local stores for their shopping needs.

Frankel and Gould (2001) attempt to overcome these conflicting findings by approaching the question from a new perspective. Using data on over 180 U.S. cities, they attempt to link the income distribution of a city with retail prices within that city. Assuming that search costs are U-shaped in income, that is, the rich have a high cost of time and the poor do not have the means, they argue that the presence of middle class consumers in a market leads to lower prices. They construct a basket of consumer goods in determining prices, including food, transportation, healthcare and other day-to-day products. After controlling for exogenous variables such as crime rate and real estate prices, they find a significant relationship between the size

[2]Economic Research Service, U.S. Department of Agriculture.

[3]Definition of what is considered a low-income neighborhood varies by research, but in general depends on data taken from the U.S. Census, and ranges from 20% to 30% of a neighborhood's population falling below the U.S. poverty line.

of the middle class and consumer prices. The study finds that with a transition of 1% from the middle-income group to the low-income group in the population, prices increased by about 0.7%, this effect increased to 1.1% when estimating using Instrumental Variables. The impact on prices was found to be slightly less, but still very significant, with transitions to the upper income groups. Either way these results would suggest that changes in the middle-income group have a significant impact on retail prices.

Though Frankel and Gould clearly demonstrate that there is a negative relationship between the size of the middle class and consumer prices, their results do not tell us much about why this relationship exists. The purpose of our paper is to construct a theoretical model that will help understand why the poor cannot drive down prices without the help of the middle class.

We begin constructing our model by looking at the choice of search from the perspective of the consumer.

3 Demand Side

In this section we look to analyze how consumers decide whether or not to search for the lowest price. Taking the behavior of firms as given, we construct the consumer's problem using a model of time allocation, first formalized in Gary Becker's "A Theory of The Allocation of Time" (1965). Becker argues that consumers spend their time either working or consuming; therefore there is a trade-off between choosing to work and dedicating your time to any other activity. This seems a natural vehicle for our analysis, as consumers must decide if it is worth their time to search for the lowest price. In our model we assume a binary search rule, either consumers search or they do not. Clearly the real world is not so simple, most consumers perform some level of search when shopping for a good. We can interpret the binary rule used in our model in the same way as if we had used a search continuum with varying levels of search. If our analysis shows that only the middle class have the incentive to search, then we can say they search the most intensely. Then by making a link between search intensity in a market and lower prices we can argue that the presence of middle-income consumers in a market leads to firms charging a lower price, the results captured in Frankel and Gould's analysis.

In this section, using some intuitive assumptions on the costs and benefit of search we determine cutoff levels of income where search occurs with the following results:

(1) If price dispersion exists and is significant, and the cost of search is not too high, consumers falling in a middle range of income will search for the lowest price.

(2) Given any positive level of fixed cost associated with search, the very poor never have an incentive to search for the lowest price.

We consider a consumer model where the number of firms and consumers are assumed to be significantly large. Taking firm pricing behavior as given, consumers determine whether to search for the lowest priced firm or to randomly choose a firm. Consumers make the decision to search period by period; we do not consider a multiple period benefit to search in our model.

The Model

There are two types of **firms**, large low-priced superstores (ℓ-types) and small high-priced stores (h-types). A proportion of firms, $\beta \in [0, 1]$, choose to be ℓ-types and charge the low price, $p_\ell = p_{\min}$, while $(1 - \beta)$ choose to be h-types and charge a high price $p_h = r$, where r is the consumers' **reservation price** above which they do not purchase the good. We can think of the reservation price as determined by their outside option, an option for the consumer away from the common market. This could be thought of as eating at home versus going out for dinner, growing vegetables in a garden versus purchasing from a grocery, or choosing to walk rather than buying a car. If a consumer encounters a firm that is charging a price above their reservation price they leave the market and consume their outside option. In this section consumers take the makeup of firms in the economy, β, as given and treat it as fixed. We will discuss the problem of the firms, and how β is determined, in more depth in our discussion of the supply side below.

All firms are assumed to be equally accessible by all consumers, therefore the average price across all firms in the market is given by[4]:

$$\bar{p} = \beta p_\ell + (1 - \beta) p_h \tag{3.1}$$

Consumers have varying levels of income, and face the choice of either purchasing the good from a randomly chosen firm, or searching for the lowest priced seller. There are two types of cost associated with searching. The first is a monetary cost that involves the cost of gathering information remotely or actually visiting each store to compare prices. We can think of this component as associated with the cost of a computer and a web connection in order to search the internet for price comparisons, the cost of a magazine subscription such as "Consumer Reports" in the U.S. or "Which" in the U.K., or the cost of a car or other mode of transportation for going out to visit different stores. There is also a variable component such as the cost of gasoline or public transportation. We will treat all of these costs as fixed and refer to them as c.

The second type of cost is the opportunity cost of searching. Here we use a concept first formalized in Becker (1965). The idea is that the time spent searching by a consumer takes

[4]We do not consider frictions related to spatial access in this model.

away from time available for working or consuming other goods. We refer to this time cost as s. As in the Becker model, the opportunity cost of the time spent searching is the foregone wage that the consumer could have earned working, w. Therefore, the time cost of searching for the lowest price is sw. The existence of a wage dependent time cost of search in effect creates a continuum of search costs across our consumer population as determined by the wage distribution. This is a different setup from previous work on price dispersion where search costs tend to be purely monetary and consumers fall into a discrete number of search cost levels. We will introduce an income distribution below so that we can more fully explore this additional dimension of our model.

Consumers will weigh the benefits and costs of search and will either search for the lowest priced firm or randomly choose a firm and purchase from that firm as long as price is below their reservation price.

Assumptions

In our model we make some simplifying assumptions that seem intuitive and help us focus on answering the question of how income affects search behavior.

(i) Consumers are risk neutral. This lets us focus on the tradeoff between the cost and benefit of search. Any degree of risk aversion would increase the incentive to search.

(ii) The monetary and time costs of search, c and s, are binary. Either a consumer searches and pays the minimum price, p_ℓ, or they do not search and randomly choose a firm. This is the ”clearinghouse” approach used in Salop and Stiglitz (1977), Varian (1980) and Carlin (2009).

(iii) c and s are independent of income. We make this assumption to simplify our analysis, but in fact there is some evidence to suggest that the poor might face higher prices for transportation or big ticket items [See Clifton (2004) and Caplovitz (1963)]. This would suggest that the cost of search decreases with income, which would give the poor even greater disincentive to search. For now we will assume constant costs.

(iv) c and s are independent of the number of low and high-priced firms in the market. We can argue that these costs should go down as the number of low-priced firms increase or decrease, since in the first case there are more of them so it is more likely a consumer will find one, while in the second case they are more unique so more easily identifiable.

(v) Reservation price, r, is independent of income. In reality a consumer’s outside option should vary with income. But the sign of variation is ambiguous, since the rich could have more outside options, while the poor need to do more with less. For the case of simplicity we will assume r to be constant.

(vi) Uninformed consumers do not have any information on prices, but their beliefs about the distribution and level of prices must be correct in equilibrium. This means that consumers will optimally choose whether or not to search taking the types of firms in equilibrium as given (in effect a simultaneous Nash equilibrium). This has become a standard assumption in consumer search models, and is a departure from Salop and Stiglitz (1977). In their model they assume that uninformed consumers know the distribution of prices and take into account possible firm deviations in their search strategy. The main difference between our assumptions about information and theirs is that in our model uninformed consumers are not able to observe when firms deviate from any proposed equilibrium strategy[5].

The Consumer's Problem

In this section we present the consumer side of our model and solve for the optimal choice of search depending on income. We solve for the consumer's expected consumption with search and without search and compare the utility achieved in each case.

Without search: If consumers do not search they will randomly choose a store and purchase the good at that store as long as the price is below their reservation price. With probability β they will enter a low-priced store charging p_ℓ, and probability $1-\beta$ they will choose a high-type charging p_h. We do not have any dynamics in the consumer's problem so once they have entered the store they do not leave and try another store (then they would in effect be searching). Given this setup the consumer's problem without search can be written as the following probabilistic maximization problem:

$$\begin{aligned} \max_x \quad & \mathbb{E}[u(x)] \\ s.t. \quad & p_i x = Lw \quad \text{for } i \in \{\ell, h\} \quad \text{where } P[p_\ell] = \beta, P[p_h] = 1-\beta \\ & tx = 1 - L \end{aligned} \tag{3.2}$$

[5] We are not straying too far from S&S in our assumption here. Uninformed consumers knowing the distribution of prices in equilibrium is a strong assumption. Without some knowledge about prices, whether in equilibrium or outside of it, making the choice between searching or not searching would require a sequential search model. For a treatment of how consumers gather information when starting out at a point of zero information on prices see Rob (1985) or Rauh (1997).

$u(x)$ is a strictly monotonic well behaved utility function, dependent only on the consumption of one good, x, this assures us that both our budget constraints are binding. These assumptions on consumer preferences limit the consumer's problem to that of maximizing x, we will adopt this approach for the rest of this section.

The first constraint in (3.2) is the usual budget constraint except that consumers do not know prices and will face a low price with probability β. We do not include any non-wage income, since we are only interested in how changes in wage affects consumer choice. The second line in (3.2) is the time constraint from Becker's model. Consumers have 1 unit of time to allocate across working and all other activities. t is the amount of time needed to consume the good, and L is the amount of time a consumer spends working, earning wage w. The parameter t takes the place of leisure in our model of time allocation[6]. Different types of goods might have different levels of t, impacting the amount of time allocated to search, we will talk more about the impact of t on search below[7]. Note the inherent tradeoff between the different activities in our time constraint. A consumer earning a wage w must choose between spending their time working, L, searching, s, or consuming, tx. Since the two constraints are binding, consumers indirectly choose L, by choosing their level of consumption, x, and by choosing whether or not to search.

We combine the two constraints and solve directly for the expected value of x purchased by uninformed consumers:

$$\mathbb{E}[x_U] \;=\; x_U(w) \;=\; \beta\left[\frac{w}{p_\ell+tw}\right]+(1-\beta)\left[\frac{w}{p_h+tw}\right] \quad \text{where} \quad \beta, t \in (0,1) \tag{3.3}$$

Next we will determine how introducing search will affect the consumer's problem.

With search:

$$\max_x \quad u(x)$$

$$s.t. \quad p_\ell x = Lw - c$$

$$tx = 1 - L - s$$

Where p_ℓ is the minimum price across all firms, c is the fixed monetary cost of search, and

[6] Note that there is no pure leisure in this model. Consumers either spend their time working, searching or consuming.

[7] Note that the number of firms in the market can have an impact on the time of consumption, t, since if there are more firms in a market consumers will have to travel a shorter distance to do their shopping, in effect spending less time consuming. But we believe this to be a second order effect in our results and choose to treat t as fixed for the purpose of our analysis.

s is the amount of time required to search for the lowest price. We solve directly for the level of x purchased by informed consumers:

$$x_I(w) = \frac{w - sw - c}{p_\ell + tw} \qquad \text{where} \quad s \in (0,1)$$

The restriction on the value of the time cost of search, s, assures us that consumers have enough time in a period to both search and consume, with some time left over to work[8]. We need to look at some comparative statics in order to determine at what levels of wage it is profitable for consumers to search. First we consider levels of x for wages close to zero and approaching infinity:

$$x_U(0) = 0 \qquad > \qquad x_I(0) = \frac{-c}{p_\ell} \tag{3.4}$$

$$\lim_{w \to \infty} x_U(w) = \frac{1}{t} \qquad > \qquad \lim_{w \to \infty} x_I(w) = \frac{1-s}{t} \tag{3.5}$$

From inequality (3.4) we can see that for any positive fixed cost of search, $c > 0$, low-income consumers with w close to zero would not choose to search[9]. Clearly if w is low enough a consumer would not have much money left to purchase the good after paying c to obtain the minimum price, therefore they would rather randomly choose a store. Similarly, from (3.5), we have that for any positive time cost of search, $s > 0$, high-income consumers would not search for the lowest price. For high levels of w the money cost of the good becomes irrelevant while time becomes increasingly valuable, so the consumer would not sacrifice any of their valuable time searching for a lower price. In fact, as wage approaches infinity the money cost of the good no longer matters, leaving consumers indifferent between the high and low price[10].

What is left to show is that there is a middle interval of income where it is optimal for consumers to search. We begin by looking at the first and second derivatives of our individual demand functions with respect to income:

[8]It is easy to show that the condition on s is sufficient to assure that $s + tx < 1$ for all consumers.

[9]Since a negative level of consumption is not very realistic, we can see the strict inequality in (3.4) as holding in the limit. That is, taking wage very close to zero, consumers who search would stop searching once their income runs out, leaving them nothing to consume, and consumers that do not search consume a small but positive expected quantity of the good. Alternatively we can see the negative level of consumption from search as the theoretical option for a consumer with no income that they would never choose to take.

[10]Note that since in our model consumption requires time, high-income consumers would never purchase more of the good than possible under the time constraint. Therefore, the ceiling for consumption of the good x when wage goes to infinity is given by $\frac{1}{t}$ where the amount of time spent working goes to zero as wage rises.

$$x_U' = \frac{\beta p_\ell}{(p_\ell+tw)^2} + \frac{(1-\beta)p_h}{(p_h+tw)^2} \quad > 0 \qquad x_I' = \frac{(1-s)p_\ell+ct}{(p_\ell+tw)^2} \quad > 0 \tag{3.6}$$

$$x_U'' = \frac{-2t\beta p_\ell}{(p_\ell+tw)^3} + \frac{-2t(1-\beta)p_h}{(p_h+tw)^3} < 0 \qquad x_I'' = \frac{-2t[p_\ell(1-s)+ct]}{(p_\ell+tw)^3} < 0 \tag{3.7}$$

The first derivatives in equations (3.6) are positive, therefore both our bundles are increasing functions of income. The second derivatives in (3.7) are negative, therefore they are both concave in income, and since w is in the denominator of both functions in (3.7), the concavity of the two functions decreases with w, approaching zero concavity from below.

As we showed in inequalities (3.4) and (3.5) above the amount of x consumed from not searching $x_U(w)$, is higher for wages close to zero and wages approaching infinity. For search to be optimal for at least some consumers, there must exist a middle range of income, $w \in [w_1, w_2]$, where $x_I(w) > x_U(w)$.

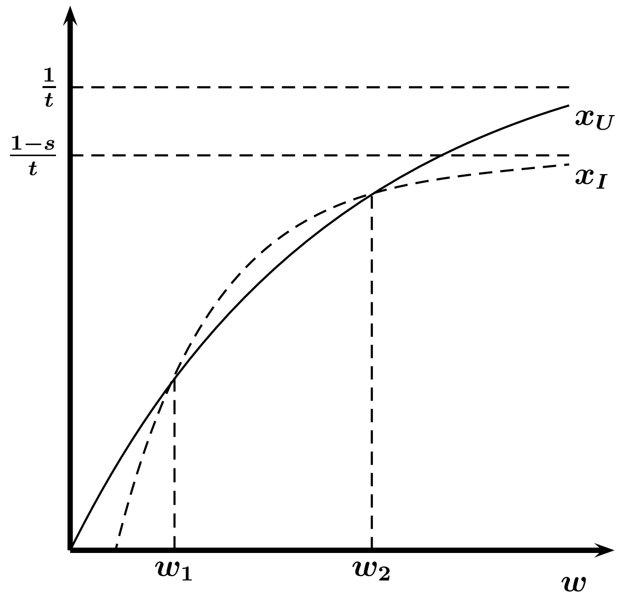

Figure 3.1: Demand as a Function of Income

We need to show under what conditions such a range of income exists. To calculate the values of w_1 and w_2, we solve for the roots of the following equation:

$$x_U(w) - x_I(w) = 0 \quad \Rightarrow \quad \beta \left[\frac{w}{p_\ell+tw}\right] + (1-\beta)\left[\frac{w}{p_h+tw}\right] - \frac{w-sw-c}{p_\ell+tw} = 0$$

$$\Rightarrow \qquad \frac{stw^2 + [sp_h + ct - (1-\beta)(p_h - p_\ell)]w + cp_h}{(p_\ell + tw)(p_h + tw)} = 0$$

The denominator of the above fraction is always positive, therefore we only need to consider the conditions under which the numerator takes negative values. Define:

$$f(w) = stw^2 + [sp_h - (1-\beta)(p_h - p_\ell) + ct]w + cp_h \tag{3.8}$$

Lemma 3.1: *The necessary and sufficient condition for there to exist a middle range of income, $[w_1, w_2]$, such that consumers within that range of income benefit from search, i.e. $x_U(w) - x_I(w) < 0 \quad \forall \quad w \in [w_1, w_2]$, is that the roots of the quadratic function $f(w)$ are positive and real.*

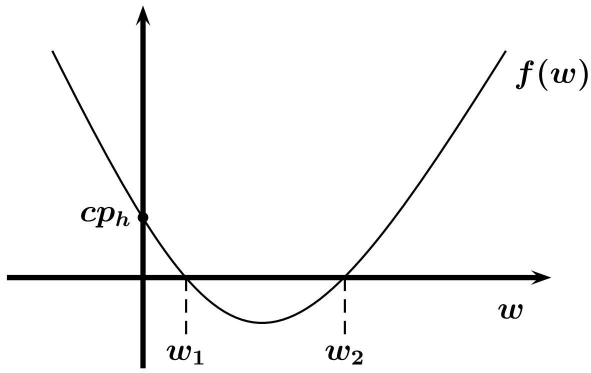

Figure 3.2: $x_U(w) - x_I(w)$

Proof: Given that $f''(w) = 2st$ and $2st > 0$, $f(w)$ is a strictly convex function of w. We also have that $f(w)$ crosses the vertical axis at $f(0) = cp_h > 0$, and that the limit of the function as w increases is positive, or more specifically: $\lim_{w\to\infty} f(w) \to \infty$. Therefore, if the roots of $f(w)$ are positive and real, then the function $f(w)$ crosses the x-axis at two points, $[w_1, w_2]$, to the right of the y-axis. Clearly $f(w)$ would be negative for all values of w between those two points, therefore we have that $x_I(w) > x_U(w) \quad \forall \quad w \in [w_1, w_2]$.

The roots for the quadratic equation $f(w)$ are given by:

$$w \;=\; \frac{(1-\beta)(p_h - p_\ell) - sp_h - ct}{2st} \quad \pm \quad \frac{\sqrt{[sp_h - (1-\beta)(p_h - p_\ell) + ct]^2 - 4sctp_h}}{2st} \tag{3.9}$$

We need the solutions to the above equation to be two **positive real roots**. A necessary, but not sufficient, condition for both roots to be positive is that the first term in equation (3.9) be positive. This gives us our first condition for the existence of a middle range of income where searching is optimal:

$$\Rightarrow \quad (1-\beta)\left(\frac{p_h - p_\ell}{p_h}\right) \;>\; s\left(1+\frac{ct}{sp_h}\right) \tag{3.10}$$

This condition says that in order for there to exist a range of consumers that choose to search the gains from searching must be greater than the costs of search. The left side of (3.10) is the expected gains from search, $(1-\beta)$ is the proportion of high-priced firms and $\left[\frac{p_h - p_\ell}{p_h}\right]$ is the relative price difference between the two types of firms. While the right side is the costs of searching.

For the roots of $f(w)$ to be real we would require that the term underneath the square root in equation (3.9) be positive:

$$[sp_h - (1-\beta)(p_h - p_\ell) + ct]^2 - 4sctp_h > 0$$

$$\Rightarrow \quad (1-\beta)\left[\frac{p_h - p_\ell}{p_h}\right] > s + \frac{ct}{p_h} + 2\sqrt{\frac{sct}{p_h}}$$

Where the last step comes from the fact that the condition from (3.10) holds, therefore the argument inside the bracket on the left is negative. This gives us the second condition for existence. Rearranging the above inequality we have our necessary and sufficient condition for the existence of a middle income of consumers that search.

$$\Rightarrow \quad \textbf{Search Condition:} \quad (1-\beta)\left(\frac{p_h - p_\ell}{p_h}\right) \;>\; s\left(1+\sqrt{\frac{ct}{sp_h}}\right)^2 \tag{3.11}$$

It is straightforward to show that if the requirement in equation (3.11) holds, then (3.10) will hold as well. To check the dimensionality of our condition we note that s and t are pure numbers and so are the ratio of prices and c to p_h. Therefore, our condition is valid and represents a relationship between values of pure numbers.

Since we have that the roots are real, then we know that w_1 and w_2 exist. To show that they are both positive it is sufficient to show that the smaller root is positive, which we can see is true from our demand curve in figure 3.1. Therefore, given our linear demand function, our requirement for positive roots as given by our "Search Condition" is the only condition necessary and sufficient for a middle range of income $[w_1, w_2]$ to exist where consumers search.

From the right hand side of the inequality we can see that s has to be quite small for the Search Condition to hold. This result is in line with existing literature on price dispersion and search costs. Diamond (1971) shows that as long as search costs exist, no matter how small, consumers will not be informed and firms will charge the monopoly price. Baye et al. (2006) show that Diamond's results seem to hold for a wide range of theoretical and empirical approaches, and that price dispersion continues to exist in both online and offline markets. Our result above differs from this previous work in the sense that even if the Search Condition holds, as long as search costs are positive, it will only hold for a middle income portion of the consumer population. Therefore, low-income consumers (as well as wealthy consumer) will always have to rely on the middle class in the market to keep prices competitive.

The extent of time costs, s, is a key feature of our model. The more readily consumers can access pricing information, the lower the time required to search for the lowest price. Improvements in information technology have gone a long way towards reducing the time cost of information for those consumers with access to the newest technology. Our model would suggest that as the speed of search increases it becomes more likely that consumers are informed about retail pricing. Here we have assumed that s is constant across the consumer population. This is very likely not to be the case. Poorer households are less likely to have access to the same technology and resources as higher income households, making it more difficult for them to search for prices. We will come back to this point below.

The left-hand side of of inequality (3.11) is decreasing with the proportion of firms charging the low price, β. This means that the incentive for middle-income consumers to search is decreasing with the proportion of low-priced firms. As the proportion of low-priced firms increases, the probability that an uninformed consumer happens upon a low-priced firm increases, therefore the benefit of paying the cost of search in order to find the lowest price decreases. Similarly the probability that there exists a middle-income group that searches is decreasing with the monetary cost of search relative to the monopoly price, $\frac{c}{p_h}$, which follows from the same intuition as above.

Interestingly as the time spent on consumption, t, increases, it becomes less likely that the Search Condition will hold. We can interpret this in two ways. Since we have defined t as our measure of time spent on leisure in our model, then it seems intuitive that as the time desired or required for leisure activities increases, it becomes more costly for consumers to dedicate their free time for searching for the lowest price. This in effect creates an inherent tradeoff between income and time spent on any other activity except consumption and labor. In addition, t encapsulates the time spent by the consumer taking part in activities other than working or searching. It is understandable that as these constraints on the consumer or household's time increases they will be less likely to spend time searching.

Finally, we have that the incentive for middle-income consumers to search is increasing with the discount offered by the low-priced firms, $(\frac{p_h - p_\ell}{p_h})$. As the percentage price difference increases the monetary benefit to search increases, giving more incentive for consumers to spend money and time to find the lowest price. This is also a result that we can directly observe in the real world: it is much more likely that consumers will take the time to shop around in a market where large discount superstores exist rather than a market with firms having similar economies of scale. This result has strong implications for households isolated in mainly poor communities. As we argued in the literature discussion above, one is much less likely to find large discount stores in low-income communities, making it more expensive for households located in these communities to access these low-priced stores.

These are all important considerations for the possibility of policy intervention. If we can show through more direct empirical analysis that the phenomenon described in our model does exist, then directed policy can target the relevant parameters above, increasing the incentive for the consumer population to search for the lowest price. As we will show below and as has been argued by Salop and Stiglitz (1977), a higher proportion of consumers that search will lead to a more competitive market environment.

Now we would like to consider the overall demand in our market, to do so we need to formally introduce an income distribution to our model.

Income Distribution

There are m consumers in our market. We consider values of X (total demand across the consumer population) as determined by the distribution of income in the market, continuing to take the firm side of the market as given. We begin by introducing a particular income distribution to our model, which will help us in determining aggregate demand as a function of the types of firms in the market, β.

In our analysis we consider the distribution of income in a community that lives within a larger economy. The larger economy is made up of many neighborhoods with varying number of middle-income individuals. We are interested in understanding how prices in a community vary as the size of the middle class in that community changes. We will work with a Uniform Income distribution, $G(w)$, defined over a range of income $[a, b]$, where the probability of a consumer earning wage $w \in [a, b]$ is given by $g(w) = \frac{1}{b-a}$. Though the Uniform Distribution is not necessarily an accurate representation of reality, it is mathematically more easy to deal with, and will allow us to more directly focus on the question at hand.

A possible extension of our results below would be to consider the impact on our analysis of introducing unbounded distribution functions with varying density across the income range (e.g. Normal or Pareto). We believe that these alternative distributions would not change our results below, but could potentially add greater depth to the analysis. Using an unbounded distribution would not impact our results directly, but would add justification for our assumption (vii) below (or even make it unnecessary in the case of distributions that are unbounded at both tails). Allowing for variation in the density function across the range of income would allow us to more carefully consider the impact of the shape of the distribution on prices, rather than being limited to using a mean preserving spread to represent inequality. Though we acknowledge the limitations imposed by using the Uniform, we find it more simple to work with, and we feel that our results below could be generalized to other types of distribution functions, including the Normal and Pareto.

We will focus our analysis in this section on mean preserving changes in the distribution of income centered around the mean of the overall economy (demonstrated by $\bar{w}$ in Figure 3.3 above), which is exogenously given. For the purpose of our theoretical model we are interested in focusing on the effect of changes in the size of the middle class on search. We do not lose any generality by centering our distribution about the mean and varying the variance of the distribution to signify changes in the concentration of the middle class.

We add the following assumption regarding our Income Distribution:

(vii) If there exists a range of income over which it is optimal for consumers to search, that range will be contained within the overall income range for the community, $[w_1, w_2] \subset [a, b]$. In other words, the range of income in any given community will never get too concentrated about the mean.

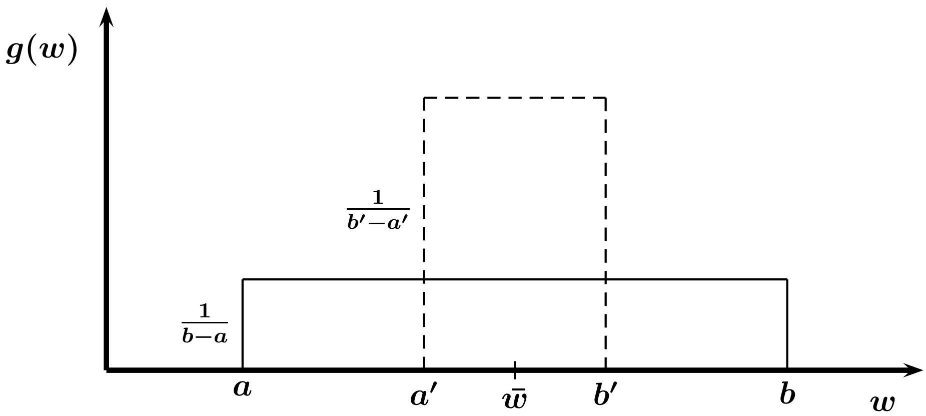

Figure 3.3: Uniform Distribution of Income

We lose very little in our analysis by using the "containment" assumption above. Our main question is whether or not increasing the size of the middle class will lead to lower prices. The income distribution we use is just a vehicle for representing changes in the size of the middle class in our model.

Aggregate Demand

In our model of Demand above we determined that as long as our Search Condition in (3.11) is satisfied with respect to our parameters there would exist a middle range of income $[w_1, w_2]$ where only consumers that fall within that range would choose to search. Now, using our income distribution $G(w)$ along with our Search Condition and our values of w_1 and w_2, we will look to calculate aggregate demand from informed and uninformed consumers as a function of our parameters and the proportion of low-priced firms, β. We begin by calculating the total demand in a given market as determined by the income distribution in that market.

As we can see in Figure 3.4 below where we have drawn consumer demand as a function of income[11], demand in a particular market is bounded by the range of income in that market, $[a, b]$. In addition, given our strictly monotonic utility function, the market demand curve is determined by the higher of the two demand curves calculated under search and no search.

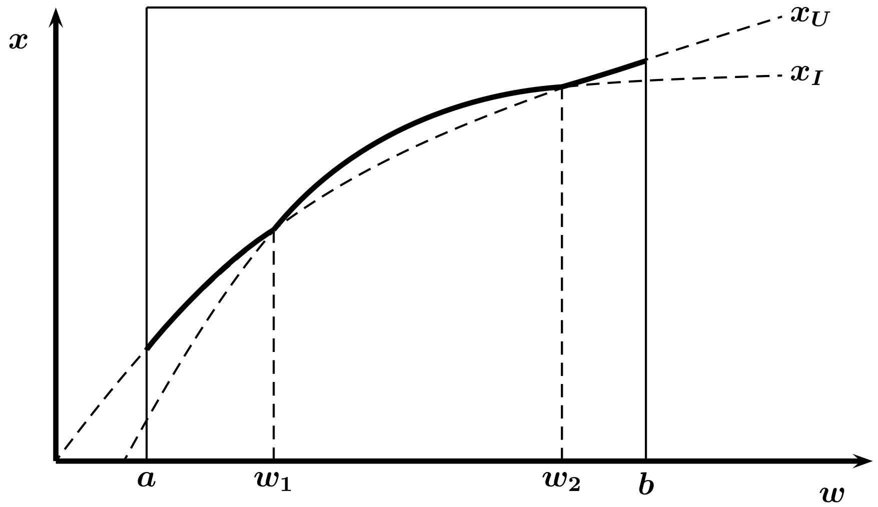

Figure 3.4: Market Demand Curve

Using our uniform distribution of income, we can calculate market demand by multiplying the total number of consumers per level of wage, $\frac{m}{b-a}$, by the integral under the curve. Total demand from consumers who search would be given by:

$$X_I \; = \; \tfrac{m}{b-a}\left(\int_{w_1}^{w_2} \frac{w - sw - c}{p_\ell + tw} \; dw\right) \tag{3.12}$$

[11]Note that in this case demand is upward sloping since higher income leads to greater demand.

Total demand from uninformed consumers is the sum of the two integrals of demand below and above our middle range of income. Rewriting equation (3.3) above we derive total demand from uninformed consumers:

$$X_U \;=\; \beta X_{U,\ell} \;+\; (1-\beta)X_{U,h} \tag{3.13}$$

$$\text{Where:} \qquad X_{U,i} \;=\; \tfrac{m}{b-a}\left(\int_a^{w_1} \frac{w}{p_i + tw}\; dw \;+\; \int_{w_2}^{b} \frac{w}{p_i + tw}\; dw\right)$$

The second line in equation (3.13) is aggregate uninformed demand if all uninformed consumers happened to walk into a type i firm. For a given distribution of firm types in the market, aggregate demand from uninformed consumers is given by the first line of equation (3.13).

Before moving on to the supply side of our model we consider the properties of the aggregate demand functions above. We would like to determine how the magnitude and composition of demand changes as we vary the proportion of low-priced firms, β. First we consider changes in demand from informed consumers with respect to the proportion of low-priced firms. Taking the derivative of equation (3.12) using Leibniz's Rule we have:

$$\frac{dX_I}{d\beta} \;=\; \frac{\partial w_2}{\partial \beta}\left(\frac{w_2 - sw_2 - c}{p_\ell + tw_2}\right) \;-\; \frac{\partial w_1}{\partial \beta}\left(\frac{w_1 - sw_1 - c}{p_\ell + tw_1}\right) \;<\; 0$$

We can show that $\frac{\partial w_2}{\partial \beta} < 0$ and $\frac{\partial w_1}{\partial \beta} > 0$, therefore we have that the above derivative is negative[12]. Demand from informed consumers is a decreasing function of the proportion of low-priced firms. This is as we would expect: a higher proportion of low-priced firms means that it is more likely that an uninformed consumer searching at random would happen across a low-priced firm, lowering the benefit of being an informed consumer.

Using the same results as well as that $X_{U,\ell} > X_{U,h}$ by construction, we can also show that the following results hold [12]:

$$\frac{dX_{U,\ell}}{d\beta} > \frac{dX_{U,h}}{d\beta} > 0 \;\Rightarrow\; \frac{dX_U}{d\beta} > 0 \quad \text{and} \quad \frac{dX}{d\beta} > 0 \tag{3.14}$$

Both demand from uninformed consumers and total demand are increasing functions of β. The first condition is the reverse of the result from the demand for informed consumers above, as the number of low-priced firms increases the incentive to search decreases, so there is more uninformed demand. Total demand increases with β, as more consumers do not pay the cost

[12] See Appendix for proof.

of search but still shop at the low-priced firms (since it is more likely they will randomly come across one) they have more money to spend on buying the good, leading to overall demand increasing. This result has interesting welfare implications. Imperfect price information costs consumers in terms of overall consumption, a portion of consumer income must be spent in gathering price information[13].

Next we look at the market from the perspective of firms. We look to determine how the presence of consumers who search and do not search effects the types of firms that enter a market.

4 Supply Side

In this section we consider how varying degrees of search by consumers can cause price dispersion in a market for a single good. Here we modify a version of a model of price dispersion first developed by Salop and Stiglitz (1977). We generalize away from most theoretical work on price dispersion by allowing consumers to buy a continuous amount of the good, $x \in \mathbb{R}_+$.

Firms

Our market consists of n firms that have identical production technology. There are different possible approaches to the cost structure of firms. Previous work on firm output and prices in a model with uninformed consumers have used various types of cost structures. Salop and Stiglitz (1977) and Braverman (1980) use a U-shaped average cost curve, Varian (1980) uses a strictly declining average cost curve, and several papers use constant marginal costs (See for example Carlin (2009)). All of these are justifiable approaches to the structure of firms in the market that we have described. Since we are interested in the choice of capacity as well as price, we use a U-shaped average cost curve characterized by a common fixed cost and increasing variable costs. This is analogous to a model of firms with soft capacity constraints, or decreasing returns to scale, allowing us to consider how firms choose their size (and in effect price) given the proportion of informed consumers that exist within that market.

Firms can choose over a continuum of prices between the consumers' reservation price r, and the minimum point on the average cost curve, p_{min}. No firm would ever choose to price below p_{min}, since it would be earning negative profits, and no consumer would ever buy the good above the price r. Given their choice of price, firms fall into two categories. If a firm is charging the lowest price in the market, then it is considered a Superstore, ℓ-type firm, characterized by high output, q_ℓ and low price, p_ℓ. These types of firms target both informed

[13] By consumption we mean consumption of this particular good. The cost of search, at least the monetary cost, does go toward another form of consumption (Transportation or information goods).

and uninformed consumers in the market. Otherwise the firm is considered a Corner store, h-type firm, with low output, q_h, and high price p_h. These types of firms only target uninformed consumers. Consumers are not able to recognize an ℓ-type or a h-type unless they are informed. This basically means that there is some level of uncertainty about what firm offers the lowest price. We can think of this as the case where a Superstore might not always charge the lowest price for all goods, and that a consumer would be able to pay a lower price for a small set of goods at a Corner store[14]. Firms compete based on price for demand from informed and uninformed consumers.

Given this setup and our assumption that all firms are equally accessible by all consumers, the firms charging the lowest price will share between them the demand from informed consumers, X_I. While both types of firms will share demand from uninformed consumers. The demand for each type of firm is given by:

$$q_\ell = \frac{X_I}{n_\ell} + \frac{X_{U,\ell}}{n_\ell + n_h} \qquad q_h = \frac{X_{U,h}}{n_\ell + n_h} \tag{4.1}$$

Where $X_{U,i}$ is as we defined in the previous section, n_h is the number of firms categorized as high-priced and n_ℓ is the number of firms categorized as low-priced. Firms must belong to only one of the two categories, so $n = n_h + n_\ell$. In this setup the proportion of low-priced firms β from the previous section is given by: $\beta = \frac{n_\ell}{n_h + n_\ell}$.

Each firm maximizes the following profit function taking the search decision of consumers and the pricing decision of other firms as given:

$$\max_{p_i} \quad \pi(p_i|p_{-i}) = p_i q_i(p_i|p_{-i}) - v[q_i(p_i|p_{-i})] - F \tag{4.2}$$

$v(q)$ is variable cost and is increasing with quantity demanded ($v'(q) > 0$). $q_i(p_i|p_{-i})$ is the demand faced by firm i charging price p_i and taking the price of all other firms as given. We assume that there are fixed costs, F, but no other barriers to entry, therefore both types of firms earn zero profits in equilibrium. The average cost function is equal to the total cost function divided by quantity:

$$AC(q_i) = \frac{v(q_i)}{q_i} + \frac{F}{q_i}$$

[14] Varian (1980) uses a model of sales to demonstrate that if firms intermittently switch from high to low price, consumers would have incentive to search and price dispersion would exist. This comes from the observation that large discount stores do not always offer the lowest price and that for a consumers to be fully informed they would have to compare prices between them and smaller retailers at the time of purchase.

Fixed costs plus increasing variable costs lead to a U-shaped average cost curve as drawn in figure 4.1 below.

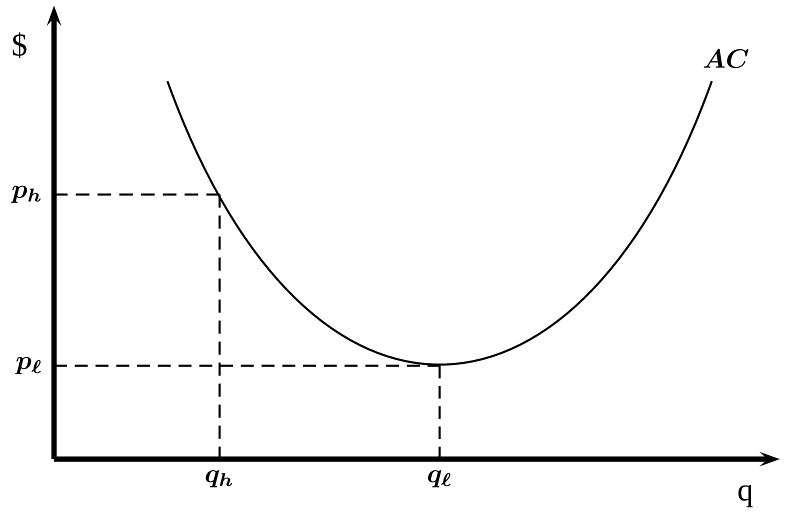

Figure 4.1: Average Cost Curve

From our zero profit condition we have that enough of both types of firms must enter to get both firms onto the average cost curve. Our assumption of profit maximization and the zero profit condition means that in equilibrium the demand curve faced by each type of firm lies below the average cost curve at each point other than that firm's choice of equilibrium price.

Using a similar setup, Salop and Stiglitz (1977) show that if search costs are high enough no consumers search and there exists a single price equilibrium (SPE) with all firms charging the reservation price. This is analogous to our Search Condition from above not holding due to high search costs. They also show that when search costs are low enough there exists a single price equilibrium with all firms charging the minimum price; we will consider this possibility in our discussion of equilibrium below. For intermediate values of search costs there exists a two price equilibrium (TPE) where some firms charge the minimum price and some charge a higher price less than or equal to the reservation price.

We differ from Salop and Stiglitz (1977) and other previous work on price dispersion in that we have consumers buying a continuous amount of the good and having a continuum of search costs. We also differ in our assumption that uninformed consumers do not know the distribution of prices. Salop and Stiglitz (1977) make the assumption that uninformed consumers know the distribution of prices in order not to stray too far from the standard competitive model, as well as to avoid the Diamond paradox, the outcome where no consumers search and all firms charge the highest price even when search costs are arbitrarily small, Diamond (1971).

As we will show below, our model still allows for a Diamond style outcome where if search costs exist, no matter how small, there exists an equilibrium where no consumers search and all firms charge the high price. We will also argue that even when there are search costs, there exists an alternative outcome where a portion of consumers search and some firms charge the low price. This is an alternative equilibrium to the Diamond no-search outcome, similar to the two price equilibrium suggested in Salop and Stiglitz (1977). Other papers have dealt with the possibility of moving away from the Diamond outcome, see for example Bagwell and Ramey (1992) and Rhodes (2011). We will focus the main part of our analysis on the possibility of the two-price outcome, and how price dispersion in such an outcome depends on the make up of the consumer population.

5 Equilibrium Analysis

As a final step in our theoretical model, we consider possible equilibria where firms and consumers act simultaneously (we are looking at Nash equilibria with respect to firm and consumer decisions). We would like to combine our results above to determine if and when there exists an ”interior” equilibrium value for our consumer and firm populations that determines search intensity and firm type, given our model parameters and income distribution. Also, if this ”interior” equilibrium does exists, we would like to determine how it changes with variations of inequality in our income distribution, as given by changes in $(b - a)$. In this final step we are able to answer the question initially set out by this chapter, ”How do changes in income distribution affect consumer prices?”

An equilibrium in our model is characterized by the following:

1. Firms maximize profits, taking the prices of other firms and consumer search as given.
2. Firms earn zero profits.
3. Consumers search optimally, taking the pricing decision of firms as given.
4. All consumer demand is met, there is no excess demand in equilibrium.

The main question we are concerned with is: if there are two firms charging different prices, in what proportion will these firms enter the market, and how will that proportion depend on the makeup of the consumer population? We can show that there exists an equilibrium such that if a firm enters as a corner store it would price at $p_h = r$, while a Superstore would choose $p_\ell = p_{\min}$[15]. From now on we will limit our analysis to firms charging these two prices. The intuition for this result comes from the price elasticity of consumer demand. Informed consumers have perfectly elastic demand with respect to a price increase from the minimum price, therefore low-priced firms would not deviate from the minimum point on the average cost

[15]See Appendix for a more formal justification for $p_h = r$ and $p_\ell = p_{\min}$ as an equilibrium price outcome.

curve. Uninformed consumer have inelastic demand by construction, therefore firms targeting these types of consumers would be best off charging the maximum price as determined by the consumers' outside option, r. From our Search Condition above it is clear that the extent of the differential between the price of the two types of firms is a key factor in whether or not the middle class search. We will consider what drives the differential between the high and low price, below.

We now look for possible equilibria in our model.

Proposition 5.1: *If search costs are zero, $c = 0$ and $s = 0$, all firms charge the low price, $\beta = 1$.*

Proof: This is just the standard perfectly competitive outcome with perfect information.

Proposition 5.2: *For any positive cost of search, $c > 0$ and/or $s > 0$, there exists an equilibrium outcome where all firms charge the high price, $\beta = 0$, and no consumers search (this is the outcome described in the Diamond Paradox).*

Proof: If all firms charge the same high price the Search Condition does not hold and consumers do not have incentive to search. If consumers do not search then they will not be able to observe price deviations, therefore a deviating firm would not induce search. Given the inelastic demand from uninformed consumers, no firm has an incentive to choose a price below the reservation price[16].

Proposition 5.3: *For any positive cost of search, $c > 0$ or $s > 0$ there does not exist an equilibrium where all firms charge the low price, $\beta = 1$.*

Proof: We can see the proof for this result from inspection of our Search Condition in equation (3.11) above. For any positive value for s or c, and a non-infinite high price, p_h, as $\beta \to 1$ our Search Condition does not hold and therefore no consumers search. Since demand from uninformed consumers is inelastic, if there are no consumers that search and all firms are charging the low price and earning zero profits, then one firm can deviate to the high price and earn positive profits.

Now we consider the conditions for an equilibrium with an intermediate proportion of the two types of firms. From our zero profit condition we have that the quantity for each firm is fixed by the firms' choice of prices, $p_{\min}$ and r, and the shape of the average cost curve.

[16]This results from our assumption that uninformed consumers do not know the distribution of prices except in equilibrium, and therefore do not observe deviations away from equilibrium. Salop and Stiglitz (1977) assume that uninformed consumers know the distribution of prices, therefore they are able to rule out the Diamond Paradox.

Using $A(q)$ to represent the downward sloping portion of the average cost curve, this condition implies:

$$p_{\min} \;=\; A(q_\ell) \;=\; \tfrac{v(q_\ell)}{q_\ell} + \tfrac{F}{q_\ell} \qquad\qquad r \;=\; A(q_h) \;=\; \tfrac{v(q_h)}{q_h} + \tfrac{F}{q_h}$$

As we can see in the figure 5.1, for a given technology of production, only one value of $\bar{q_\ell}$ and $\bar{q_h}$ can satisfy the zero profit condition (intersect the two prices on the average cost curves). From now on we refer to these fixed quantities as $\bar{q_\ell}$ and $\bar{q_h}$.

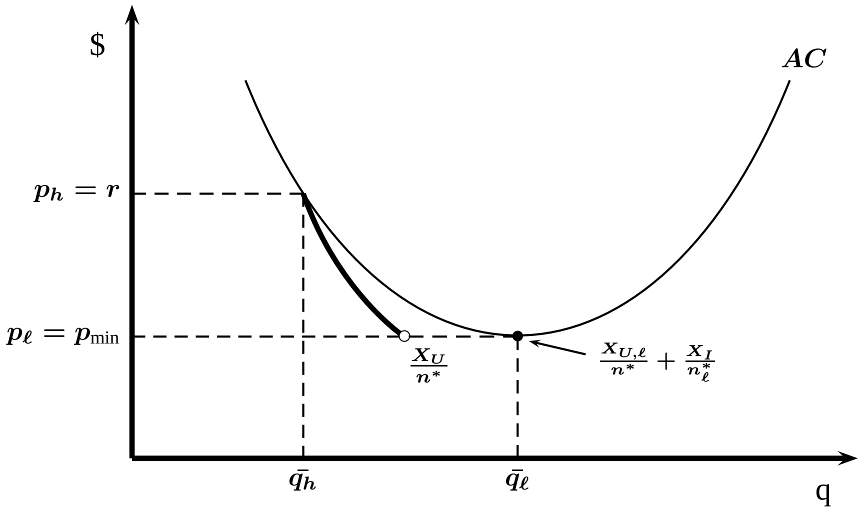

Figure 5.1: Two Price Equilibrium

Rearranging the formulas for the fixed quantities for each type of firm from (4.1) we have two equations that solve for the number of low and high-priced firms:

$$n_\ell + n_h = \frac{X_{U,h}}{\bar{q_h}} \qquad\qquad n_\ell + n_h = \frac{X_I(n_\ell + n_h)}{n_\ell \bar{q_\ell}} + \frac{X_{U,\ell}}{\bar{q_\ell}} \tag{5.1}$$

The levels of n_i (which in turn determine n and β) are determined by how many firms must be in the market at a given time for the two types of firms to be producing at their respective zero profit quantities. From Proposition 5.3 we know that the first equation must always be satisfied, while the second equation holds as long as there are some consumers that search, $X_I > 0$. Using these two equations we can solve for the number of low and high-priced firms in a two-priced equilibrium:

$$n_\ell^* = X_I \left(\tfrac{X_{U,h}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}}\right) \qquad n_h^* = \tfrac{X_{U,h}}{\bar{q_h}} \left(\tfrac{\bar{q_\ell} X_{U,h} - \bar{q_h}(X_{U,\ell} + X_I)}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}}\right) \tag{5.2}$$

From examination of the two equations above we can see that as demand from informed consumers, X_I, decreases, n_ℓ^* approaches zero and n_h^* approaches $\frac{X_{U,h}}{\bar{q_h}}$. Combining the two equations we derive an expression for the proportion of low-priced firms in the market.

$$\beta^* = \frac{n_\ell^*}{n_\ell^* + n_h^*} = X_I \left(\frac{\bar{q_h}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}} \right) \tag{5.3}$$

We can show that the denominator is positive as long as $X_I > 0$ (this is demonstrated in figure 5.1 above)[17].

Let us consider the equation for β^* in (5.3). The left and right side of the equation are both functions of β. For an equilibrium to exist we must have that there exists at least one fixed point such that $f(\beta^*) = \beta^*$, where $f(\beta)$ is the function on the right hand side of equation (5.3).

$$f(\beta) = X_I \left(\frac{\bar{q_h}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}} \right)$$

Proposition 5.4: *If the Search Condition holds for some value of $\beta \in (0, 1)$ then there exists a unique equilibrium, $(w_1^*, w_2^*, \beta^*, n^*)$ such that n^* firms enter the market, β^* of firms choose to be ℓ-types and price at the low price, p_{min}, $1 - \beta^*$ of firms choose to be h-types and price at the reservation price, r, and consumers earning wage $w \in [w_1^*, w_2^*]$ search for the lowest price.* [This result is demonstrated in figure 5.2 below.]

Proof: First we consider the extreme points of the function, $f(\beta)$. From our Search Condition we have that as β approaches 1 the condition does not hold and $X_I = 0$, which in turn means $f(\beta) = 0$. In addition, as long as the Search Condition holds for some intermediate value of β, we have that as β approaches zero $f(\beta)$ approaches some positive constant, $k > 0$[18].

$$\beta \to 1 \;\; \Rightarrow \;\; f(\beta) = 0 \qquad \text{and} \qquad \beta \to 0 \;\; \Rightarrow \;\; f(\beta) \to k > 0 \tag{5.4}$$

In our analysis of demand in section 3 we show that demand from informed consumers is a decreasing function of the proportion of low-priced firms, $\frac{dX_I}{d\beta} < 0$. We also show that uninformed demand is an increasing function of β, that is $\frac{dX_{U,\ell}}{d\beta}, \frac{dX_{U,h}}{d\beta} > 0$. Using these results it is straightforward to show that $f(\beta)$ is monotonically decreasing over $\beta \in (0, 1)$[19].

[17]See Appendix for proof.

[18]Note that when $\beta = 0$ we must have that $X_I = 0$ since no consumer will pay the cost of search if there are no low-priced firms to search for.

[19]See the Appendix for proof.

Therefore, by Brouwer's Fixed Point Theorem, we must have at least one point where $f(\beta)$ crosses the $45°$ line, that is $\exists\ \beta^*$ s.t. $f(\beta^*) = \beta^*$. From monotonicity of the function $f(\beta)$, that β^* is unique.

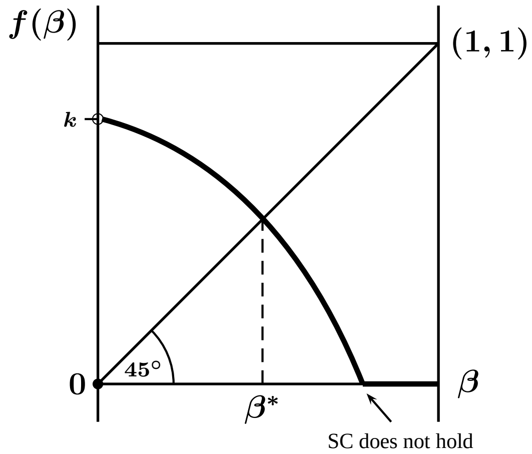

Figure 5.2: Equilibrium

We have now established that as long as the Search Condition holds for some intermediate value of β, there exists an equilibrium where some consumers search and some firms charge the low price.

Before we go on, it is interesting to note that the resulting equilibrium value of β^* depends both directly and indirectly on our Search Condition. It depends indirectly on the condition in that we need the condition to hold in order to allow for the intermediate equilibrium we have described above. The direct effect is demonstrated by the point on the graph in 5.2 where $f(\beta)$ hits the x-axis. This point is determined by the value of β where the Search Condition does not hold. A decrease in search costs, c and/or s, would cause this point to move out further on the x-axis, leading to a higher equilibrium proportion of low-priced firms. This is an intuitive result, as search costs go to zero, the intermediate outcome moves closer to the competitive outcome. Although as long as search costs are positive the alternative equilibrium with $\beta = 0$ continues to be a possibility.

Now we consider how this interior equilibrium changes with changes in our income distribution.

Variations Around Equilibrium

In this section we look to determine how our equilibrium proportion of low-priced firms, β^*, changes as we vary the range of our income distribution, $(b - a)$. Due to our use of a uniform distribution of income, an increase in the range of income is equivalent to a reduction in the size of the middle of the distribution. From our ”containment” assumption in the section on income distribution, and examination of our Search Condition, we have that the numerator of the equation for $f(\beta)$ in (5.3) is not dependent on the range of income, $(b - a)$. Using the same argument we also have that the point where the function hits the x-axis, where the Search Condition does not hold and $X_I = 0$, does not depend on the range of income. What we need to determine is how the denominator in (5.3) changes as our distribution of income becomes more spread out. In our setup such a change is equivalent to a reduction in the size of the middle range of income. The denominator in the equation for β^* is:

$$\bar{q}_\ell \left(\int_a^{w_1} \frac{w}{p_h + tw} \ dw + \int_{w_2}^{b} \frac{w}{p_h + tw} \ dw \right) \ - \ \bar{q}_h \left(\int_a^{w_1} \frac{w}{p_\ell + tw} \ dw + \int_{w_2}^{b} \frac{w}{p_\ell + tw} \ dw \right)$$

We will focus on the impact of a mean preserving spread in the distribution. A mean preserving spread is demonstrated in figure 5.3 below, where $\bar{w}$ is mean income[20].

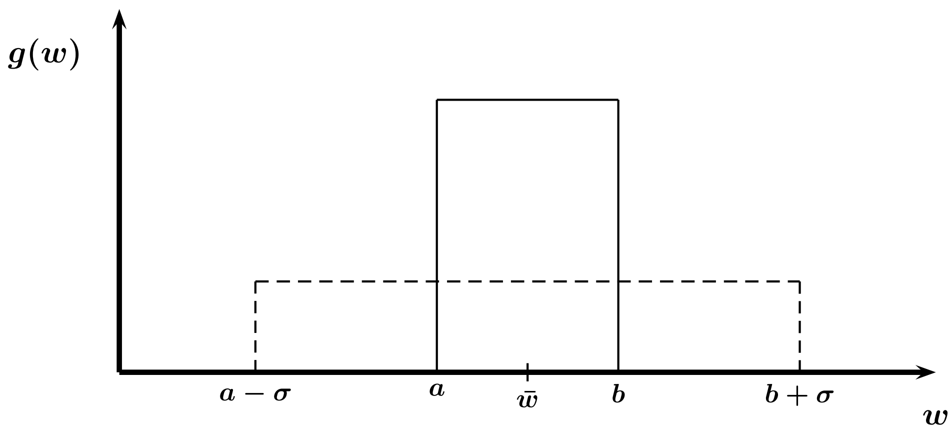

Figure 5.3: Mean Preserving Spread

Comparing the value of the denominator when the range of the distribution is $(b - a)$ with its value when the range increases to $[b + \sigma - (a - \sigma)]$, we can show that the denominator increases under the more spread out distribution[21]. Therefore, we have that $f(\beta)$ is a decreasing function of the spread of our distribution on the interior of the graph in 5.2.

[20] In a uniform distribution $\bar{w} = \frac{a+b}{2}$.
[21] See Appendix for proof.

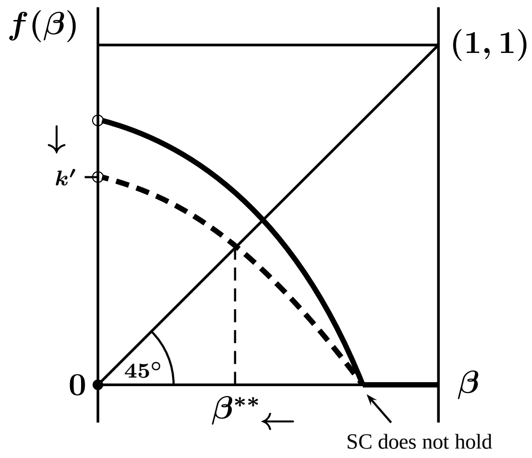

Figure 5.4: Effect of Decrease in Middle Class

As we can see in figure 5.4, when the range of income increases, $f(\beta)$ pivots down anchored at the point on the x-axis where the Search Condition is binding, leading to a lower equilibrium point, β^{**}. Therefore, a decrease in the size of the middle income group, represented by an increase in the variance in our model, leads to a lower number of firms choosing to be ℓ-types. This means that β^* decreases, which by examination of (3.1) leads to higher average prices in the market.

We have finally demonstrated that under certain market conditions we can have a negative relationship between the size of the middle class in a market and consumer prices in that market. Now we would like to explore more in-depth the conditions on our parameter values and market structure that allows this phenomenon to exist, and to discuss what types of markets would be more likely than others to allow for these conditions to hold.

Determinants Of Equilibrium

There are two main conditions on the model's parameter values that determine the intermediate equilibrium we have described. They are both represented in our Search Condition in (3.11) and the function $f(\beta)$ in (5.3). The first condition is that the costs of search, s and c, are not too high relative to consumer income. The second condition is that the difference between the two types of firms, represented by the difference between $p_{\min}$ and r, is significant enough to justify consumer search and therefore the existence of an ℓ-type firm in the market. In this final section we consider the significance of these conditions, and suggest possible consumer markets and types of cities, where these conditions are more likely to hold.

Search costs: This first condition is very intuitive, and seemingly the most relevant to policy initiatives. Clearly there are costs associated with searching for the lowest priced firm. For a household to feel that searching is optimal, they must believe that they can afford the time and money required. This is characterized by the left-hand side of the relation in our Search Condition. Lower search costs in our model would loosen the condition for search in (3.11). This would lead to a rightward shift of the $f(\beta)$ curve in figure 5.2 above, increasing the proportion of low-priced firms in the market. This result demonstrates how if we are able to lower the costs of search, we can increase the number of consumers that search, getting closer to the perfectly competitive outcome. Policy directed at lowering fixed costs through increasing availability of computers and journals, or lowering the time cost by making prices more transparent, would increase the incentive for all households to search. As we argued above, lowering search costs will bring the interior equilibrium above closer to the competitive outcome, reducing the loss to consumers of resources spent on searching.

Over the last several years, the internet has made it easier for consumers to locate the lowest priced firms in many markets, cutting down on the time cost of search. Various search engines such as ”MySimon”, ”Google”, ”Kayak”, and ”esurance” provide highly efficient venues for consumers to compare prices in a wide ranging array of products, from electronics to car insurance. But these mediums are imperfect in two main aspects:

(i) Not all consumers have internet access readily available. 26% of the U.S. population do not have access to the internet, Nielsen (2008). Access is more varied in Europe, with those lacking internet acess ranging between 20-30% in most western European countries. These percentages are significantly higher for low-income households.

(ii) Not all consumer markets provide transparent pricing on the internet. The technologies for comparative shopping are still limited, with most households doing a majority of their shopping offline.

Lack of internet access for low-income families means that though the time cost of search is decreasing, the fixed cost of search still remains an issue for many households. Policy directed at increasing internet penetration in low-income consumers could be one way of bringing down these fixed costs.

Cities: The types of cities that would support the conditions we have presented above are those with significant structural frictions that make the gathering and dissemination of information difficult. Such frictions include:

(i) High levels of income segregation across neighborhoods.

(ii) Lack of readily available and affordable modes of transportation combined with a dispersed population.

(iii) Lack of community involvement and leadership.

(iv) Ineffective news media that do not reach the majority of the population.

These factors would require greater and more costly effort from the consumer to gather information on pricing on the firms available within the city. This would translate to higher values for s, c and possibly t in our model, making it more likely that low-income families would not be able to afford to search for the lowest price. The presence of these frictions is equivalent to a leftward shift in $f(\beta)$ in 5.2 above. Such a market environment would lead to a higher portion of high-priced firms in the economy.

An interesting consequence of the first item in the list of frictions above is the very famous concept of ”Separate But Not Equal”. In our example it might be the case that segregation of low-income families from the rest of the city might lead to higher prices in the poorer neighborhoods, making their separation from the rest of the city, in effect, unequal. We consider these types of spatial access frictions in Chapter Two of this thesis.

Difference Between $p_{\min}$ and r: The second main factor in our derived equilibrium is the difference between the two types of firms in the market, as represented by the price advantage of the low-priced firm. This structure represents a world where certain firms compete through high volume and small margins, ℓ-types, and others through low volume but higher margins, h-types. h-type firms choose to forgo selling to informed consumers so that they can sell at a higher price to uninformed consumers. An h-type firm is better positioned to compete in an industry with smaller fixed costs of entry, which results in a smaller difference between the consumer's reservation price relative to the minimum point on the average cost curve.

We can see the effect of this decreasing in ”differentiation” in the left hand side of equation (3.11). As $p_{\min}$ approaches r the incentive to search decreases, reducing the demand from informed consumers and in turn reducing β^*. Therefore, we have that industries with higher fixed costs are more likely to support the type of equilibrium we have described above.

Product Markets: Product markets that are examples of industries that fit these characterisations include: Food Markets (Supermarkets versus groceries)[22], Household Goods (home product megastores versus local hardware stores), Healthcare (large hospitals versus local health centers), and Consumer Finance (large private banks versus small financial outlets).

All of the examples involve a high-fixed cost firm, such as a supermarket, that requires high volume of consumers to make up for their initial investment. If there exists a structural environment within a specific city where consumers are not perfectly informed about firm pricing, a firm can invest a small amount of fixed costs, for example a grocery, and look to attract a

[22] In the case of food markets one can argue that consumers are more aware of prices charged by different firms, since they tend to food shop more frequently relative to markets with one-off purchases. This might mitigate the impact of intermittent sales described in Varian (1980).

small volume of the uninformed consumers who will pay the higher price. These two tiers of firm types would not be sustainable in product markets with very little fixed costs, such as law firms and others in the services industry.

6 Conclusion

In our theoretical model above we have demonstrated that given a monetary and time cost of search, the middle class have the greatest incentive to search, therefore their presence in a market leads to greater competition among firms and lower prices. Frankel and Gould (2001) have shown empirically that there exists a significant relationship between the size of the middle class and consumer prices, but their findings do not tell us why such a relationship exists. Is it true that the middle class search the most intensely? And if so, what prevents the lower income class from searching, even though savings on price are a larger portion of their income relative to higher income consumers?

Through a closer analysis of specific cities and neighborhoods where these conditions are found to exist, we can attempt to see if the assumptions of our model hold. Do the poor face prohibitively high costs when shopping for the lowest priced firm? Is it a matter of search costs, or the costs of mobility that prevents low-income families from having access to the lowest price? By better understanding the link between the size of the middle class and consumer prices we can more effectively direct policy initiatives that will help the poor compete in consumer markets.

Whether or not income equality is ideal for a given society is an issue up for debate. But equal opportunity is inherent in the market ideas of free societies. Equal opportunity for low-income families to rise up out of poverty is as important a concept as that of equal opportunity based on race and gender. If we find that market frictions make it difficult for low-income families to search for the lowest price, then isolating low-income families in predominantly low-income neighborhoods can lead to them facing higher average prices. Higher prices would mean that money earned by a low-income family is worth less than that earned by the rest of society, making the climb out of poverty much steeper at the low-end of the income scale. This phenomenon, if found to exist, would be in line with an old concept usually associated with race, that ”separate is inherently not equal”.

7 Mathematical Appendix

Proof That $\frac{\partial w_2}{\partial \beta} < 0$

Differentiating the equation for w_2 (the higher value for w in equation (3.9)) we have:

$$\frac{\partial w_2}{\partial \beta} = \frac{-(p_h - p_\ell)}{2st} + \frac{[sp_h + ct - (1-\beta)(p_h - p_\ell)](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}}$$

$$= \frac{-(p_h - p_\ell)}{2st} - \frac{[(1-\beta)(p_h - p_\ell) - sp_h - ct](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}} \quad < \quad 0$$

In the second line we have negated the term in brackets in the numerator of the second term, which was negative from the Search Condition holding. Therefore, we have shown that the derivative of the upper range of income that searches, w_2, with respect to the proportion of low-priced firms β is negative.

Proof That $\frac{\partial w_1}{\partial \beta} > 0$

Differentiating the equation for w_1 (the lower value for w in equation (3.9)) we want to show that:

$$\frac{\partial w_1}{\partial \beta} = \frac{-(p_h - p_\ell)}{2st} - \frac{[sp_h + ct - (1-\beta)(p_h - p_\ell)](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}} \quad > \quad 0$$

$$= \frac{-(p_h - p_\ell)}{2st} + \frac{[(1-\beta)(p_h - p_\ell) - sp_h - ct](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}} \quad > \quad 0$$

In the second line we have negated the term in brackets in the numerator of the second term, which was negative from the Search Condition holding. Multiplying both sides by the denominator of the second term and dividing through with $(p_h - p_\ell)$ we have:

$$-\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h} \; + \; [(1-\beta)(p_h - p_\ell) - sp_h - ct] \; > \; 0$$

$$\Rightarrow \quad [(1-\beta)(p_h - p_\ell) - sp_h - ct]^2 \; > \; [sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h$$

$$\Rightarrow \quad 0 \; > \; -4sctp_h$$

Therefore, we have shown that the derivative of the lower range of income that searches, w_1, with respect to the proportion of low-priced firms β is positive.

Proof That $\frac{dX_{U,i}}{d\beta} > 0$, $\frac{dX_{U,\ell}}{d\beta} > \frac{dX_{U,h}}{d\beta}$, **and** $\frac{dX_U}{d\beta}, \frac{dX}{d\beta} > 0$

Differentiating demand from uninformed consumers from (3.13) we have:

$$\frac{dX_{U,i}}{d\beta} = \frac{m}{b-a}\left[\frac{\partial w_1}{\partial \beta}\left(\frac{w_1}{p_i+tw_1}\right) - \frac{\partial w_2}{\partial \beta}\left(\frac{w_2}{p_i+tw_2}\right)\right] \quad > \quad 0$$

Where we know that the inequality is true since $\frac{\partial w_1}{\partial \beta} > 0$ and $\frac{\partial w_2}{\partial \beta} < 0$. Therefore, demand from uninformed consumers is increasing with the fraction of low-priced firms.
It is clear from the above inequality that:

$$\frac{dX_{U,\ell}}{d\beta} > \frac{dX_{U,h}}{d\beta} \qquad \text{since} \qquad \frac{w_j}{p_\ell+tw_1} > \frac{w_j}{p_h+tw_1} \quad \text{for} \quad j \in \{1,2\}$$

We have that demand from uninformed consumers shopping at the low-priced store is increasing faster than demand from uninformed consumers shopping at high-priced stores.
To see how uninformed demand as a whole changes with respect to β we differentiate X_U to get:

$$\frac{dX_U}{d\beta} = X_{U,\ell} - X_{U,h} + \frac{dX_{U,\ell}}{d\beta}\beta + \frac{dX_{U,h}}{d\beta}(1-\beta) \quad > \quad 0$$

Which we know is positive since $X_{U,\ell} > X_{U,h}$.
Finally in order to see how total demand changes with respect to β, we differentiate $X = X_I + X_U$ to get:

$$\frac{dX}{d\beta} = X_{U,\ell} - X_{U,h} + \frac{dX_{U,\ell}}{d\beta}\beta + \frac{dX_{U,h}}{d\beta}(1-\beta) + \frac{dX_I}{d\beta} \quad > \quad 0$$

The last three terms on the left of the inequality cancel out since at w_1 and w_2 we have:

$$\frac{w_j - sw_j - c}{p_\ell + tw_j} = \beta\left(\frac{w_j}{p_\ell+tw_j}\right) + (1-\beta)\left(\frac{w_j}{p_h+tw_j}\right) \quad \text{for} \quad j \in \{1,2\}$$

As we argued above, $X_{U,\ell} > X_{U,h}$ by construction, so we have that total demand is increasing with the proportion of low-priced firms.

Proof that $p_h = r$ and $p_\ell = p_{\min}$ represent an equilibrium

In considering the choice of price by firms we consider the demand for the two types of firms and the profit function in (4.2). From examination of quantities for the two types of firms, q_ℓ and q_h, we can see that as long as the Search Condition is satisfied, consumer demand is not continuous at the point where price equals the minimum price available in the market, p_ℓ. If a firm raises its price above the minimum price they would lose their share of demand from informed consumers. Before we consider the implication of this discontinuity, we consider the decision of firms that only target uninformed consumers. Taking the derivative of the profit function above with respect to price we have:

$$\frac{\partial \pi_i}{\partial p_i} = q_i + \tfrac{\partial q_i}{\partial p_i} p_i - \tfrac{\partial v}{\partial q_i}\tfrac{\partial q_i}{\partial p_i}$$

When a firm only targets uninformed consumers, that is when they charge a price, p_i, above the minimum price, their demand from a consumer earning w is given by:

$$x = \frac{w}{p_i + tw} \qquad \text{where} \qquad \frac{\partial x}{\partial p_i} = -\frac{w}{(p_i + tw)^2}$$

Therefore, the price elasticity of demand for a consumer earning w and shopping at store charging p_i, is less than one.

$$\left|\frac{\partial x}{\partial p_i}\right| \frac{p}{x} \quad = \quad \frac{p_i}{p_i + tw} \quad < \quad 1 \qquad \forall \;\; w \in [a, b], \;\; p \in (p_\ell, r]$$

This gives us the following result regarding the pricing strategy of a firm that targets uninformed consumers.

Lemma 7.1: *If a firm does not target informed consumers, then it will charge a price equal to the consumers' reservation price, $p_h = r$.*

Proof: Let us assume the above statement to be true. Then the derivative of the profit function with respect to price from above is positive for any value of p below the reservation price. Setting the above differential to be greater than zero and dividing through by quantity we have:

$$1 + \tfrac{\partial q_i}{\partial p_i}\tfrac{p_i}{q_i} - \tfrac{1}{q_i}\tfrac{\partial v}{\partial q_i}\tfrac{\partial q_i}{\partial p_i} \quad = \quad 1 - |\epsilon| + \tfrac{1}{q_i}\tfrac{\partial v}{\partial q_i}\left|\tfrac{\partial q_i}{\partial p_i}\right| \quad > \quad 0 \qquad \forall \;\; p_i \leq r$$

Where ϵ is the price elasticity of demand. For the above relation to be true a sufficient but not necessary condition would be for the price elasticity of demand for a firm not charging the minimum price to be less than or equal to unity. Given that uninformed consumers do not know the distribution of prices, a change in the price of a high-priced firm does not impact consumer search decision, so a firm does not lose any customers by raising its price. From our examination of consumer demand we have shown that the demand for each individual uninformed consumer is inelastic for any price below the reservation price, therefore a high-priced firm faces inelastic demand up until the consumers' reservation price. Using these two results we have that if a firm chooses to price above the minimum available price, it would raise its price to the consumers' reservation price.

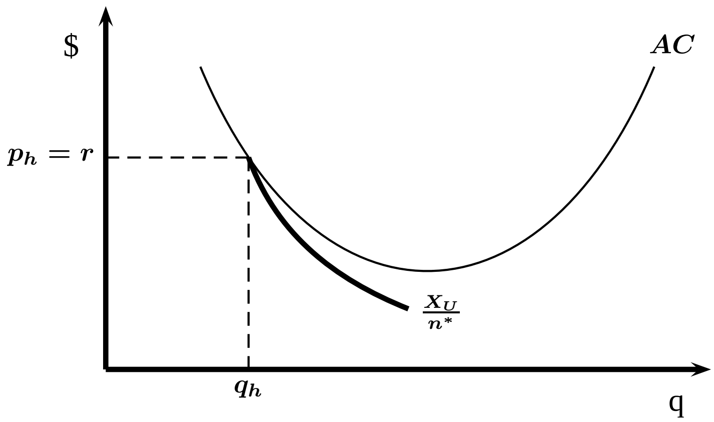

Figure 7.1: One Price Equilibrium

Lemma 7.2: *In a two-priced outcome where both types of firms are located on the average cost curve, the low-priced firms charging $p_\ell = p_{min}$ represents an equilibrium pricing strategy.*

Proof: As we argued above we must have that $p_h = r$. Now let us consider an equilibrium such that $r > p_\ell = p_{\min}$. For this to be an equilibrium it must be that there is some positive level of demand from informed consumers, $X_I > 0$. From our zero profit condition we have that both firms have to be on the downward sloping part of their average cost curve, with the low-types at the minimum point of the AC curve.

Let us consider a low-priced firm deviating to a price slightly above $p_\ell = p_{\min}$. Since we have that uninformed consumers can not observe this deviation, this change in price would not induce any additional search from consumers. In addition, since informed consumers know all prices in the market perfectly, they have perfectly elastic demand. Therefore, the deviating firm would lose all of its informed consumers and only face demand from uninformed consumers randomly choosing where to shop. As we argued above, uninformed demand for each firm is below the AC curve at all points except where $p = r$. Therefore, the deviating firm would earn negative profits.

Alternatively a deviating firm could choose to raise its price to $p = r$. Again, this does not induce any additional search by consumers. Since high-priced firms were operating on the AC curve, an additional high priced firm would shift the demand curve further to the left, earning all high priced firms negative profits, including the deviating firm.

Therefore, $p_h = r$ and $p_\ell = p_{\text{min}}$ represent an equilibrium pricing strategy in a two-priced outcome.

Proof That $\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}$ Is Positive

When $X_I > 0$ we have that the following relations must hold for any value of n^* and n_ℓ^*:

$$\bar{q_h} = \frac{X_{U,h}}{n^*} < \frac{X_{U,\ell}}{n^*} < \frac{X_{U,\ell}}{n^*} + \frac{X_I}{n_\ell^*} = \bar{q_\ell}$$

Therefore we must have that the following hold:

$$\bar{q_h}\left(\frac{X_{U,\ell}}{n^*}\right) < \bar{q_h}\bar{q_\ell} = \left(\frac{X_{U,h}}{n^*}\right)\bar{q_\ell} \quad \Rightarrow \quad \bar{q_h} X_{U,\ell} < \bar{q_\ell} X_{U,h}$$

Which proves our relation.

Proof That $f(\beta)$ Is a Decreasing Function of β

Differentiating $f(\beta)$ from equation (5.3) we want to show that the following is negative:

$$\frac{df(\beta)}{d\beta} = \frac{\bar{q_h}\frac{dX_I}{d\beta}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}}$$

$$- \frac{\bar{q_h} X_I \left[\bar{q_\ell}\left(\frac{\partial w_1}{\partial \beta} x_{U,h}(w_1) - \frac{\partial w_2}{\partial \beta} x_{U,h}(w_2)\right) - \bar{q_h}\left(\frac{\partial w_1}{\partial \beta} x_{U,\ell}(w_1) - \frac{\partial w_2}{\partial \beta} x_{U,\ell}(w_2)\right)\right]}{(\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell})^2} < 0$$

Where $x_{U,i}(w_1)$ is demand from a consumer earning w_1 and shopping at firm i, and $x_{U,i}(w_2)$ is demand from a consumer earning w_2 and shopping at firm i. We know from our discussions above that the denominators in both terms are positive and that the numerator in the first term is negative. All that is left to show is that the argument inside the brackets in the numerator of the second term is positive. Rearranging the argument inside the brackets we need that:

$$\frac{\partial w_1}{\partial \beta}\left[\bar{q}_\ell x_{U,h}(w_1) - \bar{q}_h x_{U,\ell}(w_1)\right] - \frac{\partial w_2}{\partial \beta}\left[\bar{q}_\ell x_{U,h}(w_2) - \bar{q}_h x_{U,\ell}(w_2)\right] \;\; > \;\; 0$$

We know that the terms inside the brackets are positive from our inelastic demand. Coupled with our results above that have shown $\frac{\partial w_1}{\partial \beta} > 0$ and $\frac{\partial w_2}{\partial \beta} < 0$, we have that the above inequality is true.
Therefore, $f(\beta)$ is decreasing monotonically with β.

Proof That $\bar{q}_\ell X_{U,h} - \bar{q}_h X_{U,\ell}$ Is Increasing with $(b-a)$

In showing this result we consider a change in the denominator defined above, which we call D, with a mean preserving spread of the range of income. Such a change is given by:

$$\Delta D = \bar{q}_\ell X_{U,h}(a-\sigma, b+\sigma) - \bar{q}_h X_{U,\ell}(a-\sigma, b+\sigma) - \left[\bar{q}_\ell X_{U,h}(a,b) - \bar{q}_h X_{U,\ell}(a,b)\right]$$

Rearranging the above equation and simplifying, we need the following to hold:

$$\begin{aligned}
&\bar{q}_\ell \left(\int_{a-\sigma}^{a} \frac{w}{p_h + tw}\; dw + \int_{b}^{b+\sigma} \frac{w}{p_h + tw}\; dw \right) \\
&\qquad\qquad - \;\bar{q}_h \left(\int_{a-\sigma}^{a} \frac{w}{p_\ell + tw}\; dw + \int_{b}^{b+\sigma} \frac{w}{p_\ell + tw}\; dw \right) \;\; > \;\; 0
\end{aligned}$$

Which we have shown to be true in our proof that $\bar{q}_h X_{U,\ell} < \bar{q}_\ell X_{U,h}$, above. Therefore, the denominator of $f(\beta)$ is increasing with a mean preserving spread of $(b-a)$.

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