Demand For Contract Enforcement in A Barter Environment*

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Abstract

Do greater potential gains from trade *enhance* or *erode* contracting institutions? In an anonymous exchange environment traders can sign a contract, hence agreeing to interact with the assigned partner, or wait till the next match. Any contract can be endorsed (for a pay) by the enforcement agency, which then observes the interaction with a positive probability known to the traders and punishes the detected infractors. Demand for contract enforcement is the highest amount a proposer of a contract is ready to pay to the agency (in a stationary subgame perfect equilibrium). It may be strictly positive, as we show, even when contracts are broken. Surprisingly, larger potential gains from exchange may dampen the demand, but not always: they may boost the demand for 'high quality' agencies (that oversee the interactions frequently enough).

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1 Introduction

It is a common presumption that the institutions that enforce private agreements have a positive effect on economic prosperity. However, not only the causality embedded in this argument might fail,[1] we show that the opposite effect is not necessarily positive.

In an anonymous bilateral barter environment one-time contracts between traders can be endorsed by a third party, called enforcement agency, who can credibly promise, upon observing a violation, to punish an infractor[2] and to reinstate the goods, i.e., to annul the contract. The quality of enforcement is known to all, it is the probability of being observed breaking the endorsed contract. Our results are based on a textbook definition of (inverse) demand: the highest amount that an agent (who proposes a contract) is ready to pay for a given level of enforcement, i.e., quality of service and severity of punishment. We show that the demand is non-monotonic in the immediate gains from exchange.

The supplier of enforcement — be it a government or a private firm[3] — is just an entity that offers enforcement services, and traders may obtain them (register a contract) only if it is in their interest to do so. Besides, any interaction between agents is voluntary, so the basic property rights are already in place,[4] allowing an agent to keep his good for future transactions

[1]See Acemoglu et al. (2005) for the related overview.

[2]Immediate, simple punishments characterize most past cultures and judicial systems. An early example, that had a strong influence on the following legal systems, is the Code of Hammurabi (compiled roughly 4 thousand years ago), where punishment was typically independent on the "history" of the perpetrator, although "..distinction of classes in fixing of fines and punishments is characteristic of the code throughout..." (Jastrow (1915, p. 293)). Older Babylonian codes (compiled into that of Hammurabi), simply imposed death penalty for theft, *id.* According to Foucault (1975), imprisonment did not begin to replace fines or terms of service as punishment in Europe until the 17th Century, and was the lot of few until the early 19th Century, when an elaborate prison system developed. See also Kirchheimer and Rusche (1939).

[3]See the Dixit (2004) and Greif (2005) for the survey of "privately-provided" protection. Based on this model, one could, potentially analyse a competition between the providers, following Nozick (1974).

[4]Following the classification by North (1984) and, more recently, Acemoglu and Johnson (2003): property-rights institutions protect against predation, while contracting institutions enforce private agreements. Our assumption is consistent with the empirical findings of Acemoglu and Johnson (2003) that contracting institutions do not appear to function in the absence of property rights institutions. Also, according to Jastrow (1915, p. 291),

in case he refuses to interact with the matched partner at a given time, and the value of this option (which depends on the expected behaviour of future partners), naturally, affects his decision. The level of protection (or how often theft occurs) is determined in equilibrium — for any quality and severity of enforcement, — so it is neither a matter of a societal agreement, nor can be imposed as an explicit decision of a ruler on any agent.[5] In particular, since multiple equilibria might emerge, we get multiple values for the willingness to pay (for the same parameters), in such cases we simply analyse the highest one — also across equilibria.

Leaving the choice of contracts to the agents prevents the enforcer from acting "legitimately" by default, in particular, having the right to punish the observed infractors, unless the agents give it an explicit permission to do so (pay for it), and this is a key point of our departure from the literature on the subject.[6] This choice allows to first, ask *what contracts* will be endorsed in equilibrium, and second, to determine the *demand*, given the preferences and the technology.[7] Hence we focus on the institutions that are created by "market forces".

The demand here is not driven by the "returns-to-scale" argument as is common in the property rights protection analysis:[8] whether the enforcement is provided by one or many agencies does not affect the additional value they create. Also, we do not allow agents to invest in self-defence,[9] since then the value created by the agency could stem from the elimination of excessive

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in the Code of Hammurapi "...a purchase made without witnesses or a formal contract should involve a death punishment (§7), on the assumption that a claim made under such circumstances points to fraud ... The punishment no doubt rests on a provision that every purchase must be confirmed by a contract..."

[5]As in Polishchuk and Savvateev (2004) or Sonin (2003), for example, who offer different ways to rationalise the choice of a less than "fully secured property rights regime" by a wealthy ruling elite.

[6]See e.g., Dhillon and Rigolini (2010), Boukouras (2011), where agents implicitly promise to "behave" and this promise is being enforced. In particular, in Dhillon and Rigolini (2010), due to asymmetric information, some traders are interested and have the ability to affect the frequency of observed interactions via bribes (paying to the agency), but they can not alter the terms of the implicit contract that is being enforced.

[7]We do not ask how an agency can make the agents believe that it will do what it promises: we only claim that *if* it does, it creates a value (demand), which can potentially be extracted from the agents.

[8]See e.g., Skaperdas (1992).

[9]See Hoffmann (2010) for the analysis of equilibria in an economy with conflict and mutually beneficial opportunities, as well as for the overview of the related literature.

private investments ("arms-race"). In this model the driving force for the demand depends on the contracts that the individuals choose to write: in one case endorsing a contract is a costly commitment, in the other, it buys off a better chance of staying in the marketplace for one agent at the expense of his parter.

The economy (game) is described in section 2, which also contains the definition of the equilibrium, characterized in section 3. Section 4 contains the definition and characterisation of the demand for contract enforcement services, along with its properties, i.e., the main results. Discussion and conclusions are in section 5, containing also references to additional related literature. Proofs missing in the text are in the appendix.

2 Economic Environment

2.1 Basic Model

A continuum ($[0, 1]$) of infinitely-lived, risk-neutral agents interact in the market place over time, $t \in \mathbb{N}$. Each agent holds one unit of an indivisible good that does not perish until consumed. The good comes in many varieties, and the agent herself does not directly enjoy her possession: however, other agents do.[10] For instance, the goods could be interpreted as entrepreneurial ideas that require development or financing for their execution, or simply as goods and services to be exchanged (tradeables). What is important is that agents require the cooperation of other agents to obtain any returns — and that their partners may abscond, retaining both the fruits of interaction and their investable resources.

Matching happens once a period (t) and is anonymous,[11] the probability of meeting the same partner twice is zero.[12] Each matched pair receives an

[10] This is as in the related matching literature, for example Kiyotaki and Wright (1993), though there is no problem of "double-coincidence of wants" here: probability of finding the right partner plays the same role as discount factor in this model, so we avoid the unnecessary complication.

[11] This assumption is discussed in detail in section 5: the crucial aspect is that the strategy chosen by an agent is independent of the "identity" of their matched partners, i.e., everyone is treated in the same way as an "average" partner in the population.

[12] Any procedure that achieves anonymity can work here. The simplest way to "simulate" random matching in this environment is to divide the agents into 2 groups of equal mass, form two circles (dials) out of them, insert one into the other. Every period turn "the

opportunity for a mutually beneficial project. When an agent's good is used, she leaves the market and is replaced by another agent. If after a match an agent retains her good, she is matched anew the following period, the value of the future interactions is discounted at a contant rate $\delta \in]0,1[$.[13]

Matches have two stages. In the first stage, the pair may sign a *contract*,[14] specifying one of the two possible actions to be taken by each in the second stage, and this may be used as a basis for enforcement by a third party. For example, a *trading contract*, stipulates that both should trade. We also allow for a *null contract* just expressing the agreement to interact without spelling out the choice of actions therewith. To turn this into an explicit game, assume that one agent (randomly assigned with equal probability) chooses whether to initiate a contract, and if so, he writes, signs, and offers the contract to the other, who then either signs the contract or abandons the match. Contracts are voluntary: no one can force an agent to sign a contract against her will and she can freely abandon the match before the second stage.[15]

In the second stage, each agent chooses one of two actions *trade* or *rob*. If an agent chooses *trade*, her good is used up and her partner instantaneously receives $G > 0$ units of utility, referred to as (potential) *gains from trade*. However, if she chooses *rob*, then she obtains her partner's good and receives G herself, provided her partner chose *trade*. If both agents choose *rob*, then each one succeeds in capturing the good of the other with equal probability, giving nothing in return.[16] The winner consumes the good of the loser, retaining her own resources for future transactions.[17]

inner dial" at random and match the players "facing each other."

[13]Equivalently, $0 < \delta < 1$.

[14]Contract: A mutual agreement between two or more parties that something shall be done or forborne by one or both esp. such as has legal effects [...] *Oxford English Dictionary (Second Edition, 1989)*.

[15]This would be the true for an entrepreneur-investor match, or if one is to consider marriage markets, at least in some cultures. As we mentioned in the introduction, this presumes existence of the basic property rights, and that allows us to focus on institutions supporting *exchange*.

[16]Back to the the entrepreneur-investor example (ft. 15): in this case either an entrepreneur manages to get the funds and abscond or the investor steals the idea and uses it for profit. For either of the scenarios both agents have to agree to "interact" or establish some business relationship.

[17]Limiting inventories to one good is without loss of generality, as storing the good obtained from a partner and consuming it in a later period is suboptimal due to discounting.

We say that a *breach of contract* occurs if there is a discrepancy between the actions specified in the contract and those that are actually taken in the second stage.

It is clear that in the absence of any enforcement, the contracting stage is a "cheap talk." In section 3.1, we introduce an enforcement agency, and then the payoffs will depend on the contract signed, reflecting the cost of the breach. Consequently, the nature of the contracts signed will be determined in an equilibrium.

Overall, agents view the value of their possession as the expected stream of utility for which they can exchange it in the market. It is the potential change in the value due to the presence of the enforcement agency that will be defined as the demand for enforcement below. But first, we have to define an equilibrium.

2.2 Equilibrium

We look for stationary subgame perfect Nash equilibria in pure strategies. Stationarity implies that (1) the proportion of agents choosing to sign contracts of any type is constant with time and (2) so is the fraction ($\gamma \in [0, 1]$) of agents who choose *trade* in the second stage of the interaction. In such a market-place the value of the tradeable to any agent before the match is the same (independent of time and identity), since γ is also the equilibrium expectation held by any agent that his partner in any future match is going to trade.

To be able to tackle pure strategies, we take the anonymity assumption even further, assuming that none of the traders knows the identity of the matched partner, i.e., upon a match only the aggregate equilibrium parameters (described in (1) and (2) above) are common knowledge.[18] Alternatively, one could use mixed strategies in a "discretised" version of the model.[19]

Note that for any fixed $\gamma \in]0, 1[$ one can construct multiple equilibria in pure strategies: for example, a fraction γ of the population chooses *trade* every period and the rest consistently choose *rob*; or an arbitrary fraction of the population switches the action so as to keep γ constant at each period. We will focus on the first type of equilibria, because it is simple, and can

[18] This is similar to the classical "market games" where each agent's payoff depends on own action (choice of the optimal bundle) and an aggregate variable summarizing the actions of the rest of the players (prices).

[19] See Al-Najjar (2008).

be referred to easily: sometimes, will talk about "perpetual robbers" and "fair traders", keeping in mind that this is a reference to a strategy chosen in the second stage of the match, and not to an intrinsic characteristic of the player, because all agents are identical here. Since our main results (characterisation of equilibria and willingness to pay for enforcement) depend solely on the parameters determining equilibrium γ, they are independent of the particular equilibrium played.

Now we can already establish that the value of a tradeable should be non-negative in any equilibrium for any player, since it is possible to refuse any contract and offer none at any stage of the game, and get zero as a result.

2.3 Equilibria under Anarchy

There is no enforcement agency under anarchy. This is our benchmark.

Lemma 1 *Under anarchy all agents agree to interact, any contracts can be signed, $\gamma = 0$ and the value of the tradeable, V, is $V_0 \equiv \frac{G}{2-\delta} > 0$ in any equilibrium.*

Proof. Consider the second stage of an interaction when a contract is signed. Since there is no enforcement, the payoff matrix is independent of the nature of the contract:

		Agent 2	
		Trade	Rob
Agent 1	Trade	G, G	$0, G + \delta V$
	Rob	$G + \delta V, 0$	$(G + \delta V)/2, (G + \delta V)/2$

So long as $V > 0$, this stage game is a prisoners' dilemma. Hence given the contracts are signed, and $V > 0$ both should only choose to rob, thus receiving the expected payoff of $\frac{G+\delta V}{2}$. Hence if all agents sign some contract, $V = \frac{G+\delta V}{2} = \frac{G}{2-\delta}$, which is indeed, strictly positive. Clearly, no agent should ever want to refuse signing a contract, since this will only delay receiving the value by at least a period.

It is also obvious that $V = 0$ is inconsistent with an equilibrium, since in this case the stage game above has two symmetric equilibria with payoffs of at least $G/2$ to each, hence $V \geq G/2 > 0$, a contradiction.

Remains to show that there is no equilibrium where contracts are not signed by at least one agent. Assume the second agent refuses to sign the contract offered, so that $\frac{G+\delta V}{2} \leq \delta V$, which implies $G < \frac{G}{\delta} \leq V$, which is impossible. Indeed, if the contract is ever signed by both parties the value of the tradeable can not exceed $\frac{G}{2-\delta} < G$, since both should rob, as argued above. Hence the second agent has to accept, and then the proposer, similarly, has to write and sign the contract in any equilibrium. ■

3 Equilibria with Enforcement

3.1 Enforcement Agency

Into the basic economic environment we introduce an agency that enforces contracts. A signed contract can be endorsed by the agency for a pay, which we initially set to zero. It is commonly known that, with probability $\omega \in]0, 1[$, the agency observes the second stage of the interaction for any endorsed contract. Upon detecting a violation, the agency inflicts cost C upon the actual defector and reinstates the goods to the original owners (annuls the transaction). In particular, if a contract prescribing both parties to trade was signed and subsequently *both* partners attempt to rob, only the successful robber is punished, if the interaction is observed. The presumption is that it is impossible to verify an unsuccessful attempt (or an intent) to breach the contract.

Cost C can be thought of as physical punishment, ignominy, or a claim towards a stream of goods to be owned in the future. Parameter ω may be interpreted as reflecting limitations in the technology of surveillance and forensics. ω might also depend on the structure of the internal organization of the enforcement agency, which we take as given. As mentioned in the introduction, we call ω the *quality of institutions*, and say that the agency is characterized by a punishment-quality pair. We derive the equilibria under all possible such combinations and determine the economic value generated by each. Thus, we are deriving *demand*, or highest willingness to pay, for enforcement, taking the *supply of enforcement*, parametrised by a pair ω, C (describing feasible "production plans") as given.

3.2 Payoffs with Enforcement

It is clear that if a contract is not endorsed, the payoffs are the same as if a null contract is signed, and that is what we assume happens then. Since it is impossible to detect a violation of a null contract (and signing such contract should not affect the desire of any agent to interact) whether or not the null contract is endorsed does not affect the payoffs.

So, now, the payoffs change, depending on the contract endorsed. Null contract generates symmetric payoffs (so we omit the payoffs of the second player), the same as under anarchy, only the continuation payoff might be different. Denote by w the value of winning the fight, or obtaining the good of the other, thus enjoying the immediate payoff of G and retaining own good for future interaction, $w : (G, V) \mapsto G + \delta V$. Then the payoffs are:

	Null contract endorsed	
	Trade	Rob
Trade	G	0
Rob	$w(G, V)$	$w(G, V)/2$

To express the payoffs emerging upon endorsing the symmetric trading contract (*trade, trade*), we introduce map v, denoting the payoff for the infractor who is caught: $r : (C, V) \mapsto \delta V - C$. Observe that in case both rob, only one is observed to be a violator, while the other is a victim (who then gets his good back, receiving δV.)

	Trade contract endorsed	
	Trade	Rob
Trade	G	$\omega\delta V$
Rob	$(1 - \omega)w(G, V) + \omega r(C, V)$	$\frac{1-\omega}{2}w(G, V) + \frac{\omega}{2}(r(C, V) + \delta V)$

Under a robbing contract, prescribing both agents to rob, the payoffs are symmetric too. Similarly to the previous case, here, the observed *trader* is punished, and if at least one agent trades and the enforcer observes this, the goods are reinstated (so no immediate gains are realized). That is why, in particular, the expected payoff when both rob is the same as before.

	Robbing contract endorsed	
	Trade	Rob
Trade	$(1 - \omega)G + \omega r(C, V)$	$\omega r(C, V)$
Rob	$(1 - \omega)w(G, V) + \omega\delta V$	$\frac{1-\omega}{2}w(G, V) + \frac{\omega}{2}(r(C, V) + \delta V)$

Finally, assume a rob-trade contract is endorsed, prescribing the first agent (row player) to rob and the other to trade. If the first agent trades, he is punished if observed, and the goods are reinstated, independently of the actions of the second player, whose payoff then is $(1-\omega)G + \omega\delta V$ if he follows the contract and $(1-\omega)w(G,V) + \omega r(C,V)$, if he violates it. If both follow the contract, the payoff to each is the same as under null contract for this pair of actions, whereas if both rob, it depends on the outcome of their interaction. Consider the event when the interaction is observed. If the first player succeeds to capture the good of the rival, the contract is not violated and none is punished, thus contributing $\frac{\omega}{2}w(G,V)$ to the payoff of the first player and zero to the payoff of the other. If the second player is lucky in obtaining the goods, the contract is violated by both, thus adding $\frac{\omega}{2}r(C,V)$ to the expected payoff of each. In the event the interaction is not observed, of course, the expected payoffs are the same as under null contract, thus contributing $\frac{1-\omega}{2}w(G,V)$ to the payoffs of each, so the first player gets $\frac{\omega}{2}w(G,V) + \frac{\omega}{2}r(C,V) + \frac{1-\omega}{2}w(G,V) = \frac{\omega}{2}r(C,V) + \frac{1}{2}w(G,V)$. Since the payoffs are no longer symmetric we write them both:

	Rob-trade contract endorsed	
	Trade	Rob
T	$(1-\omega)G + \omega r(V,C); (1-\omega)G + \omega\delta V$	$\omega r(V,C); (1-\omega)w(G,V) + \omega r(V,C)$
R	$w(G,V); 0$	$\frac{1}{2}\big(w(G,V) + \omega r(C,V)\big); \frac{1}{2}\big((1-\omega)w(G,V) + \omega r(C,V)\big)$

To find out what contracts will be written, we determine what happens upon endorsement for each type of contract.

3.3 Null and robbing contracts

It is clear that if only null contracts are accepted in an equilibrium, then the demand for contract enforcement is zero, since there is no reason to register contracts in this case: the equilibrium is the same as under anarchy. It will be useful to remember for later that the contract is signed by the second player (accepted) as long as $\frac{1}{2}(G + \delta V) \geq \delta V$, or equivalently,

Remark 1 *Null contracts are accepted only if $V \leq \frac{G}{\delta}$.*

Definition 1 *Contract A* dominates *contract B if the proposer gets a higher expected payoff from offering contract A than from B.*

Now we can rule out one type of equilibria.

Lemma 2 *Robbing contracts are dominated by null, if $C \geq 0$.*

3.4 Equilibria with asymmetric contracts

Intuition tells us that an asymmetric contract will be accepted if waiting is too costly for the "tame" (prescribed to trade) say, because his discount factor, δ, is low, or because the market has a lot of "frictions", which decreases his outside option, δV. This is, of course, not the whole story, since first, his expected payoff from accepting such a contract also contains a fraction of the continuation payoff δV, (recieved if he is not caught and is successful in obtaining both goods) and, second, the equilibrium value of the tradeable, V, can depend on δ as well. It so happens that if only asymmetric contracts are accepted the value is increasing in δ, so there is no opposite effect there, and the intuition goes through. However, in addition, to support such an equilibrium, the proposer should not have a profitable deviation either. In particular, rob-trade contract should dominate null. Comparing the payoff of the proposer (see section 3.2) across the two scenarios, one observes that rob-trade contract provides an additional "protection" to the proposer in case he loses the fight (hence violating the contract) and this is observed, and so the added value is $\omega r(C, V)$. But the condition $r(C, V) > 0$ translates into a lower bound on δ.

So, proposition 1 summarises the conditions assuring, first, that the tame will accept the rob-trade contract, and second, that such contract dominates null and trading ones. The latter condition requires the proposer to believe that the chance of him facing a 'fair trader' is not too high.[20]

Proposition 1 *Assume $C \geq 0$. Then then there is an equilibrium where only rob-trade contracts are endorsed iff,*

1. *in case trading contract is endorsed, a partner is expected to choose 'trade' with probability less than ρ, where ρ is a decreasing function of $c \equiv \frac{C}{G}$, and*

2. *$c \leq \frac{1-\omega}{1+\omega}$, while $\delta \in [\frac{4c}{2-\omega-c\omega+2c}, \frac{4(1-\omega-c\omega)}{4(1-\omega-c\omega)+\omega(2-\omega-c\omega)}]$.*

3.5 Equilibria with trading contracts

First, we simplify the game and assume that trading contracts have to be endorsed. The equilibria here are indexed by γ, the fraction of agents choosing 'trade' in the second stage of the interaction, as was explained in section 2.2.

[20]Of course, this belief is nothing but a part of full specification of equilibrium in this case (description of the profile of actions at each possible node in the tree).

Proposition 2 *Let $\gamma_L(c), \gamma_H(c)$ be the lower and the upper roots of $F(\gamma, c) \equiv a(c)\gamma^2 + b(c)\gamma + k(c)$, with $a(c) \equiv \delta(c\omega^2 - (1-\omega^2)), b(c) \equiv (1+\omega)(1-\delta) + c\omega, k(c) \equiv (c\omega - (1-\omega))(1-\delta\omega)$.*
Consider equilibria of the game where trading contracts have to be endorsed.

1. *Assume $\delta \leq \frac{1}{2}$. If $c < \underline{c} \equiv \frac{\delta-\omega}{\omega}$, then $\gamma = 0$; if $c = \underline{c}$ or $c = \bar{c} \equiv \frac{1-\omega}{\omega}$ then $\gamma \in \{0, 1\}$; if $c \in]\underline{c}, \bar{c}[$, then $\gamma \in \{0, \gamma_L(c), 1\}$; if $c > \bar{c}$, then $\gamma = 1$.*

2. *Otherwise, (if $\delta > \frac{1}{2}$), there is a unique $\underline{\underline{c}} < \underline{c}$ such that $0 < \gamma_L(\underline{\underline{c}}) = \gamma_H(\underline{\underline{c}}) < 1$. If $c < \underline{\underline{c}}$, then $\gamma = 0$; if $c = \underline{\underline{c}}$, then $\gamma \in \{0, \gamma_H(\underline{\underline{c}})\}$; if $c \in]\underline{\underline{c}}, \underline{c}[$, then $\gamma \in \{0, \gamma_L(c), \gamma_H(c)\}$, where $0 < \gamma_L(c) < \gamma_H(c) < 1$; if $c \in]\underline{c}, \bar{c}[$ then $\gamma \in \{0, \gamma_L(c), 1\}$; if $c = \bar{c}$, or $c = \underline{c}$ then $\gamma \in \{0, 1\}$; finally if $c > \bar{c}$, $\gamma = 1$.*

Remark 2 *Because delayed consumption constitutes an implicit punishment, there exist parameters under which there are equilibria with $\gamma > 0$ even when $C = 0$.*

The second case described in the proposition, is illustrated in figure 1.

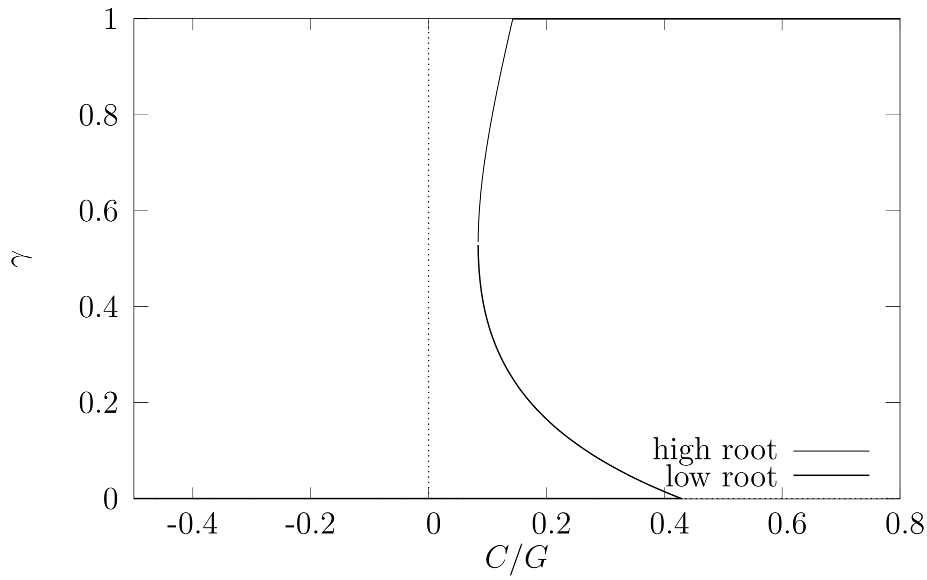

Figure 1: Equilibrium fraction γ of 'fair traders' when $\delta = .8$, $\omega = .7$.

If the punishment is small relative to the gains, then *rob* remains a dominant strategy as under anarchy and there is no trade in equilibrium. Sufficiently high relative punishment, turns *trade* into a dominant strategy and there are no breaches of contract.

For intermediate values of punishment, the difference in payoffs between robbing and trading is non-monotonic in the proportion of fair traders. If there are very few traders there is no point in trying to trade: trying to fight for goods, bearing the risk of being caught is still better than almost giving your good away (or delaying the transaction). At the other extreme, if there are sufficiently many traders then the lure of grabbing the free good is too strong even though there is a chance of being punished when caught. Thus, trading is not optimal if the expectation of there being other traders is too low (little chance of reciprocation), or too high (too many sitting ducks). This generates multiple equilibria.

Recall, our aim is to calculate the demand for enforcement, willingness to pay for endorsing a contract. By proposition 1, if there is an equilibrium where trading contracts are endorsed, it might not be uniquely defined by the parameters of the game (preferences, δ and G here; and technology, ω and C), hence the demand might be multi-valued as well. To derive definite conclusions we choose to focus on the equilibrium with the highest γ: it is clear that since the value of the tradeable increases with γ (cf. payoff tables), so will the demand. The highest equilibrium also happens to be stable in the sense suggested by DeMichelis and Germano (2000), and is also increasing in the normalized punishment c, see Lemma 7 in the appendix.

Definition 2 *1.* $\gamma^*(c) = 1$, *iff* $c \geq \underline{c}$; $\gamma^*(c) = \gamma_H(c)$, *if* $\delta > 1/2$ *and* $c \in [\underline{\underline{c}}, \underline{c}[$ *, and otherwise it is zero.*

2. The value of the tradeable in an equilibrium where only trading contracts are endorsed is the value attained in the equilibrium where the fraction of agents choosing trade ('fair traders') is γ^**. It is denoted by* V_{TT}*.*

Now we relax the assumption made at the beginning of this section and look at the initial game where any contract can be offered. It is clear that if $\gamma^* = 0$, then endorsing a trading contract is equivalent to endorsing a robbing contract, but this can not be sustained in equilibrium by lemma 2. So, for the trading contracts to appear in equilibrium, γ^* has to be strictly positive.

Using the payoff tables it is easy to see that an increase in γ^* augments the instantaneous payoff of a proposer (other things, including V, being equal), suggesting that trading contracts should dominate others if the equilibrium fraction of fair traders is high enough. Lemma 9 in the appendix extends this partial argument, identifying the threshold in γ assuring existence of equilibria where trading contracts are endorsed, the condition is also necessary. In fact, this condition is independent of our equilibrium selection. However, to provide easy necessary and sufficient conditions for existence of these equilibria in terms of relative punishment, c, as in the case of asymmetric contracts, we use monotonicity of equilibrium γ with respect to c, implied by definition 2.

Proposition 3 *Assume $C \geq 0$. Then there is a unique threshold $t_+ \in [0, \frac{1-\omega}{\omega}]$ such that there is an equilibrium where only trading contracts are endorsed iff $c \geq t_+$.*

Remark 3 *The threshold can be calculated explicitly using its characterization in lemma 9.*

4 Demand for enforcement

4.1 Definition and characterization

Now we can turn to the analysis of demand for contract enforcement.

Definition 3 Demand for enforcement D^* *is the most that a proposer of a contract is willing to pay for the third-party agency to endorse the contract in a given equilibrium.*

D^* represents the highest economic value created by endorsing a contract, and this value is positive if only one type of contract can be endorsed in equilibrium (i.e., the contract endorsed strictly dominates all others). Indeed, D^* is the gain of the proposer from not deviating to null contract at any point in time.

In an equilibrium with trading contracts (generating value V_{TT} of the tradeable), the demand is positive if and only if $V_{TT} > V_0 = \frac{G}{2-\delta}$, so, in this case, one could give an alternative specification of the game, where the players choose the location to interact: either the 'anarchy island' or the 'minimal government town;' and then the demand could be defined as the difference in payoffs between the locations.

However, if the equilibrium where only rob-trade contracts are endorsed is played, the value of the tradeable *conditional* on the agent being a proposer is strictly higher than V_{RT} so the alternative definition is not valid. In fact, it is not hard to check that $V_{RT} < V_0$, hence the presence of enforcement agency *ex-ante* is a nuisance for all agents in this case. So, we use the original definition of demand.

Having described the equilibria, we can now characterise the demand in terms of technology of enforcement (C, ω) and preferences (G, δ).

Theorem 1 *Assume* $C \geq 0$.

1. *If the equilibrium where rob-trade contracts are endorsed is played (hence conditions 1 and 2 of proposition 1 hold), then the demand for enforcement is* $\frac{\omega}{4}\frac{\delta G(1-\omega/2)-C(2-\delta(1-\omega/2))}{2-\delta(1+\omega/2)}$.

2. *If the equilibrium where only trading contracts are offered is played (hence* $c \geq t_+$*, with* t_+ *from proposition 3), then the demand for enforcement is*

   (a) $(1-\delta)\frac{G}{2}$, *if* $c \geq \underline{c}$,

   (b) $\frac{\gamma^*(c)(2-\delta-\delta\omega)+\delta\omega-1}{1-\delta\omega(1-\gamma^*(c))}\frac{G}{2}$, *if* $\delta > \frac{1}{2}$ *and* $c \in [\underline{\underline{c}}, \underline{c}[$.

Otherwise, the demand is zero.

It follows by definition of demand that in an equilibrium where both contracts are signed, the proposer should be indifferent between offering either contract, hence the demand is the same. So the theorem fully characterises the demand.

In the second case described in the theorem, *all* agents write trading contracts in equilibrium, even if $\gamma^* < 1$ (breaches of contracts occur), and since the payoffs are symmetric, naturally, both the proposer and the responder at every stage have the same willingness to pay for enforcement. The demand is also independent of the pure strategy chosen by the player upon endorsing the contract. Although they run the risk of punishment if caught, infringers enjoy being surrounded by traders whom they can defraud. In order to lure potential victims into a transaction, one has to agree to the contract, which serves as a costly commitment mechanism. Signing such a contract is worthwhile for those intending to rob, if they expect their partners to trade with high enough probability, γ^*.

In case of asymmetric contracts, of course, the willingness to pay by the proposer is larger than that of his partner (who would prefer a null contract, hence is not ready to pay for such a contract). What creates demand for enforcement here is just the asymmetry in payoffs, the ability of the proposer to choose the game where he is treated favourably: first, he is not punished when observed capturing both goods (which he could have gotten under a null contract) and, second, he gets his good re-instated *whenever* the interaction is observed (albeit then also getting humiliated in case his tame partner succeeds in capturing the possessions of both). So, in case the latter payoff is positive, so is the demand. Breaches of contracts occur, since the tame will choose to rob, hence will succeed in breaking the contract half of the time. Punishment in this case plays a somewhat obscure role of simply diminishing the value of the tradeable to each agent, rather than "preventing the crime."

4.2 Properties of the Demand for Enforcement

Now we can finally answer the question we started with. Do greater potential gains from trade *enhance* or *erode* contracting institutions? Since the demand is fully determined by the parameters of the environment, we compare across equilibria by fixing all the parameters but G.

It is clear from theorem 1 that demand is increasing in the gains from trade if rob-trade contracts are endorsed in equilibrium (case 1) or, if trading contracts are endorsed and no one robs (case 2a).

However, when the fraction of fair traders is smaller than unity, (case 2b of theorem 1), the connection between the gains from trade and the demand is less clear. As is evident from the formula for demand, there are two effects that stem from an increase in gains from trade, G. The direct effect is to boost the value of the tradeable good, holding the equilibrium fraction of fair traders constant. However, there is also an indirect effect: the equilibrium fraction of fair traders decreases, since γ^* increases in c, which falls in G. This lowers the equilibrium value of goods, and decreases the willingness to pay for contract enforcement. The second effect might be dominant, as the next result shows, when demand is close to zero.

Notation 1 *Let $\gamma^e(c)$ be the extremum of F in γ (characterized in lemma 6). Let γ_{T0} be the value of γ^* that satisfies $V_{TT} = V_0$ (characterized in lemma 9). Let α solve in c the equation $\gamma^e(c) = \gamma_{T0}$. Let β be the smallest solution* *in c of $F(\gamma_{T0}, c) = 0$.*[21]

Theorem 2 *Assume $\delta > \frac{1}{2}$ and an equilibrium where trading contracts are endorsed is played.*

1. *If $t_+ = \beta > 0$, then there is $\epsilon > 0$ such that for $c \in]\beta, \beta + \epsilon[$, demand is decreasing in G and for $c \in [\underline{\underline{c}}, \beta[$ the value generated by enforcement is zero, even though $\gamma_H(c) > 0$.*

2. *If $t_+ = \underline{\underline{c}}$ then there is $\varepsilon > 0$ such that for $c \in]\underline{\underline{c}}, \underline{\underline{c}} + \varepsilon[$, demand is decreasing in G.*

The following lemma provides sufficient conditions for the assumptions of the theorem to hold. Along with the figures (2, 3) below, mapping the two sets of conditions onto the two-dimensional space of parameters (δ, ω) it demonstrates that assumptions of the theorem are not "knife-edge", i.e., they are satisfied for an open set of parameters.

Lemma 3 *Assume $\delta > \frac{1}{2}$. Then*

1. *$t_+ = \beta > 0$, if $\frac{\delta}{2-\delta} \leq \beta < \alpha$.*

2. *$t_+ = \underline{\underline{c}} > 0$, if $\beta > \alpha$, $F(\gamma^e(\frac{\delta}{2-\delta}), \frac{\delta}{2-\delta}) \leq 0$ and $\omega < \frac{1}{2}$.*

Here we see again that relatively low quality (low ω) institutions might not "benefit" from an increase in the gains from trade, G. Taking the interpretation a bit further, more prosperous trade opportunities might not boost economic viability of enforcement institutions supporting trade, if such institutions can not assure high enough quality of enforcement (or likelihood of an effective instance of 'following up' the endorsed contract).

Finally, we ask : how does D^* vary with the other parameters – the quality of institutions and the 'patience' of the traders (or market frictions)?

Proposition 4 *Assume $\delta > \frac{1}{2}, c \geq t_+$ and the equilibrium where only trading contracts are endorsed is chosen. The demand for enforcement D^* increases with the quality of institutions ω, and falls with the discount factor δ.*

[21] By lemma 6 γ^e is strictly increasing on the the domain specified in lemma 6, and by lemma 9, $0 < \gamma_{T0} < 1$, so α is well defined as is unique, $\alpha = \frac{2\delta(1-\omega^2)(1-\delta\omega)-(1+\omega)(1-\delta)(2-\delta\omega-\delta)}{\omega(2-\delta\omega-\delta)+2\delta\omega^2(1-\delta\omega)}$, $\beta = \frac{2(\delta^2\omega^2-2\delta\omega^2+\delta^2\omega-2\delta\omega+3\omega-1)}{(2-\delta)\omega(2\delta\omega+\delta-3)}$.

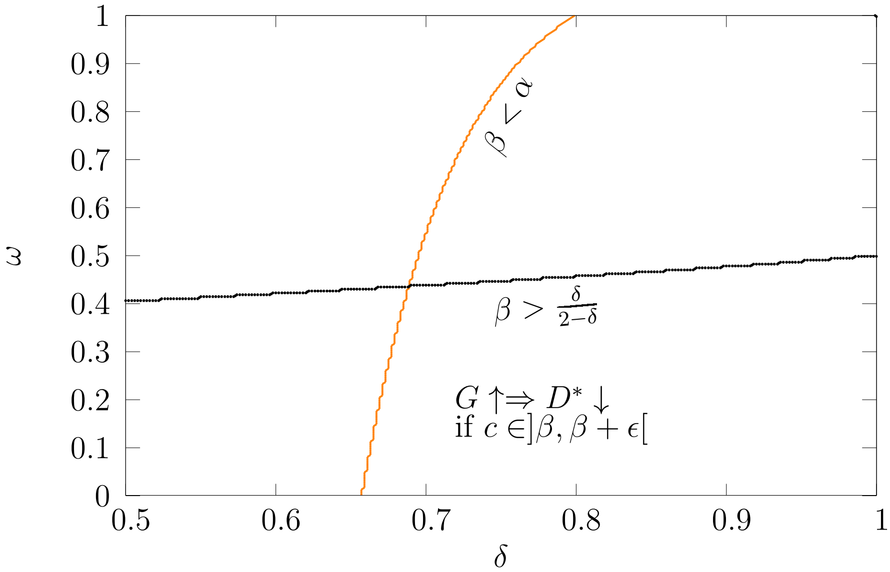

Figure 2: "Low quality" low punishment protection agencies: case 1 of theorem 2.

While the first result may seem intuitive, the second might be less so. Recall that a lower discount factor acts as a higher effective punishment in this environment, because delayed consumption is valued less. Thus, the equilibrium fraction of fair traders, or the effectiveness of enforcement, rises as the discount factor drops, increasing the willingness to pay for the agency that is capable of deterring an observed infractor from consuming for one period.

5 Discussion and Conclusions

While there is an extensive literature devoted to the determinants of property rights enforcement,[22] the focus here is on the value created by institutions supporting *exchange*.[23]

[22]See for example Umbeck (1981), Skaperdas (1992), Piccione and Rubinstein (2007), Moselle and Polak (2001), Grossman (2001), Grossman and Kim (1996), Gonzalez (2007) and Bös and Kolmar (2003).

[23]There are several related contributions on contracting institutions, e.g., Fafchamps (2002), Dixit (2003b) and most recently, Dhillon and Rigolini (2010), Boukouras (2011),

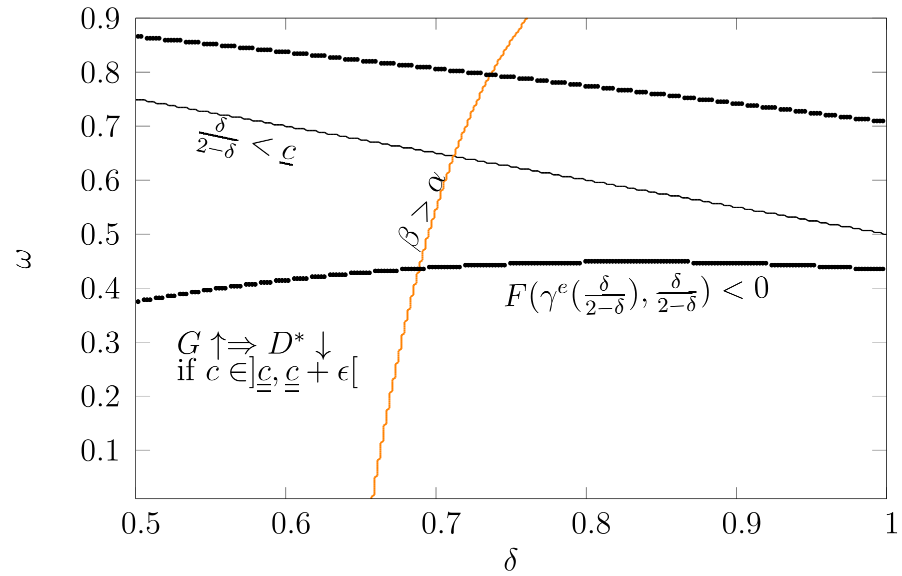

Figure 3: Case 2 of theorem 2. It is obvious that the specified region is for $\omega < 1/2$, as required.

Our goal was not to create a detailed descriptive model (though we made meaningful assumptions whenever possible), even less so did we aim at creating any prescriptions. Rather, the main objective was to highlight the most direct economic factors responsible for emergence of contract enforcement. They arise neither via social contract signed by Rousseauvian noble men, nor by the hand of Harsanyi's hypothetical individuals (whom we empathise with) — instead, they are brought by market forces, or the "invisible hand." Whether or not we (or our hypothetical counterparts) think these institutions are just or fair is immaterial: if there is a profit to be made (once the technology exists, of which we are agnostic) to meet *the demand*, such agency will show up.[24] In particular, one could use the model to analyse the hierarchies in the "informal" sector, including, what is often referred to as "organised crime."

We have shown when in a stylized anonymous stationary trading envi-

see sect. 5 for the relevant discussion

[24]Thus, our approach is in the spirit of Stigler (1992), who argues that, under a strong interpretation of the Coase (1960) theorem, if there is value to be created by the provision of a certain service, then economic incentives would lead some arrangement to arise that provides it, be it state-based or otherwise.

ronment there is a demand for a 'primitive' enforcement which consists of re-instating the good to the owners (annulling the contract) and punishing the cheater. The main question is whether the presence of potential gains from trade may generate an economic basis for contract enforcement institutions. For those endorsing trade contracts and of high quality the answer is positive: enforcement appears to be valuable, even though, as we assumed, 'fighting' (that prevails under anarchy, i.e., in the absence of the third-party enforcement) does not directly consume any resources. Perhaps surprisingly, comparing across equilibria of economies that differ in G, we find that larger gains from trade do not necessarily contribute to the economic success of contracting institutions. In particular, the demand for some low quality institutions decreases in the potential gains from trade. Although the benefit to the interacting parties when trading contracts are followed is higher when gains are so, the equilibrium rate of contract violations is higher too, and this second effect may dominate for large enough gains from trade. The second effect is neutralized if, in addition, the punishments are no less severe, also in proportion to the gains. While it might be interesting to explore whether punishment should fit the crime in an economy with different trading opportunities open to all, note that the above comparison is across equilibria of *different* economies.

We have also shown that asymmetric contract mights appear in such an environment, "legalising" robbing on behalf of one agent and prescribing his partner to give up her good without anything in return. There is a positive demand for such contracts as well (on behalf of the proposer), and it increases in the gains from trade, however, such contracts will always be broken by the "accepting" party, who co-signs this unfair deal in a hope of succeeding in stealing herself.

Let us briefly discuss our key assumptions.

Anonymity. Classical results from the repeated games are commonly used in the literature studying emergence of "social norms" in societies where there is a way to credibly transmit information about past actions of all players and a way to coordinate a collective response (punishment), say, as in Kandori (1995), where the agents play a repeated prisoners' dilemma. Calvert (1998) explicitly introduces a centralizing institution (clearinghouse of information) to keep track of individual reports about partner's actions. In one of the equilibria, sufficiently patient players use the clearinghouse to sustain cooperation, see also Dixit (2003a) for an overview. There, a low discount factor restricts the severity of punishment that a community can impose upon deviators and, consequently, the level of cooperation that will be achieved. In our environment, it *enhances* the value added by an enforcement agency, as the interaction is annulled if a violation is detected, hence punishing the infractor by delaying his consumption.

Co-existence of a third party enforcement and social norms brings new insights, and it has been analysed in different contexts in the literature. Dhillon and Rigolini (2010) provide a connection between reputation of the firms, formal enforcement (which is a "public good/bad" aimed at alleviating the opportunistic behaviour of firms who are better informed and, in addition, can use bribes to reduce its effectiveness) and frequency of productivity shocks. The norms supporting cooperation emerge in Dixit (2003b) and in De Mesquita and Stephenson (2006) along with a third-party enforcement, while the traders remain in the marketplace independently of the outcome of an interaction. Bidner and Francois (2009) study possible paths of development for both institutions supporting trade (trade contracts are endorsed by default there) and norms (understood as fixed actions taken by some agents with a stochastic connection between the payoff and the fraction of people following the norm).

Fafchamps (2002) studies the factors that affect the emergence of "relational contracting" where future interactions might depend on past histories and identity of the agents, hence certain "norms of behaviour" might emerge without any pressure from an external agency. Remarkably, the case study of Fafchamps and Minten (2001) and the survey of Fafchamps (2002) suggest that such norms might be weak or ineffective even in markets in which the participants are *not* frequently renewed.

Enforcement provision (supply) as a black box. Taking the technology (parametrised by C, ω) as given is not as restrictive as it might first appear. It is clear from theorem 1.2, that if agents can endorse a trading contract for free, having a choice of punishment, they would have chosen the maximal available one. In such a world, C can be interpreted as a limited liability constraint,[25] or an external limitation on punishment (say, not to be "cruel and unusual"); and the same is true for the choice of ω, which can be restricted due to costs of verification. Besides, an additional possible reason for a punishment to be bounded is that, just as criminals may sometimes escape unscathed, it is also possible that innocents are punished as though they were criminals. This possibility is not explored in the paper. But even here the

[25]We would like to thank the anonymous referee for suggesting this interpretation.

agents are not always interested in maximising punishment, as the gain from endorsing the contract might decrease in C (as in case 1 or theorem 1).

Our approach has another advantage: since we derive demand for *all* possible combinations of C, ω, one could, using information about "production costs" determine an optimal pricing schedule, or a menu that a monopolist could offer to the agents, potentially requesting higher price for higher C and ω. Whatever the internal organisation of the enforcement agency is, as long as the traders believe that ω is the likelihood of "being caught" and C is the decrease in payoff, the demand should be unchanged. How this belief is induced is a separate and potentially interesting question.

It is true that an enforcer might be "corrupt" in a sense that he might be interested in accepting a bribe from the observed violator. But, if the bribe is C (the most that can be extracted from the agent), and the goods are still re-instated, the results are unchanged.

The contract negotiation protocol. Recall, we chose a simple protocol with one of the agents (randomly selected) proposes and the other can either sign or leave. This eliminates some unpleasant (and possibly not very realistic) equilibria of the simultaneous offer game, however it is possible that more elaborate protocols would eliminate some equilibria that emerge here.

Contract annulment in case of detected violation. Quite surprisingly, the possibility to continue trading upon the observed contract violation, (possibly, along with the take-it-or-leave-it nature of negotiation) is responsible for the emergence of the asymmetric contracts, since, recall, it is the "bonus" that the proposer gets ($\delta V - C$), if positive, generates the willingness to pay. The annulment also serves as an additional punishment, and in case of trading contracts, is a substitute for C.

Though we view our choice as historically relevant, see Greif (2005), analysing demand for agencies that play a different role might be no less appealing, e.g., as in the model of Boukouras (2011), where the enforcer can also impose transfers among agents.

Stationarity. We conjecture that the value (V) constistent with a non-stationary equilibrium should lie in convex cone spanned by zero and the values of the stationary equilibria that we consider. So, since our aim was to calculate the highest willingness to pay (and the game under anarchy has a unique equilibrium), the assumption is not restrictive provided the conjecture.

A Appendix

A.1 Supporting equilibria with asymmetric contracts

Proof of lemma 2. Assume the robbing contract is accepted. Then for both agents rob is a strictly dominant strategy, hence the expected payoff is $u_{RR} = \frac{1-\omega}{2}(G+\delta V)+\frac{\omega}{2}(2\delta V - C)$. Robbing contract is accepted if $u_{RR} \geq \delta V$ or equivalently, when $V \leq \frac{(1-\omega)G-C\omega}{\delta(1-\omega)}$.

Null contract dominates robbing contract if $G+\delta V \geq (1-\omega)(G+\delta V)+\omega(2\delta V - C)$ or $V \leq \frac{G+C}{\delta}$, which, if $C \geq 0$, is above the upper bound $(\frac{(1-\omega)G-C\omega}{\delta(1-\omega)})$ necessary for acceptance of the robbing contracts. ■

Lemma 4 *1. Assume $C \geq 0$. If an asymmetric contract is offered, then the proposer is prescribed to rob; upon acceptance, the two agents rob. Acceptance requires $V \leq \frac{(1-\omega)G-C\omega}{\delta}$. If only asymmetric contracts are offered in an equilibrium then $V = V_{RT} \equiv \frac{(2-\omega)G-2C\omega}{4-\delta(2+\omega)}$.*

2. Assume both rob-trade and null contracts are accepted, then the rob-trade contract dominates the null contract iff $V \geq \frac{C}{\delta}$.

Proof. (1): Assume an asymmetric contract is accepted. Call the agent who is prescribed to rob, "aggressive" and the agent who is prescribed to trade, "tame". Since *rob* is the strictly dominant strategy for the aggressive, the tame should follow the prescription of the contract, if and only if his payoff from robbing is negative. But then he gets zero by accepting the contract, which can be consistent with an equilibrium only if $V = 0$. Assume it is, so since $V < \frac{G}{\delta}$, a null contract will be accepted. But then there is a profitable deviation: the tame can refuse to interact with the aggressive, instead, waiting till he becomes a proposer (which will happen with a strictly positive probability), offer a null contract, which yields a strictly positive value, contradiction.

Hence, if the asymmetric contract is ever endorsed, both should rob. It follows that the proposer will never be tame: he can do better by offering null, since $w(G, V) > r(C, V)$, as $C \geq 0$, unless null is rejected, i.e., $\delta V > G$. But in latter case the payoff of the tame proposer, $\frac{1}{2}(\delta V + (1-\omega)G - \omega C)$ is always smaller than the payoff from waiting one period, $\delta V > G > (1-\omega)G - \omega C$.

For the tame to accept, his payoff, $\frac{1}{2}((1-\omega)G + \delta V - C\omega)$ should be at least as high as the value of waiting, δV. Hence, V can not exceed $\frac{(1-\omega)G-C\omega}{\delta}$.

If only rob-trade contracts are offered, since a given agent becomes a proposer with probability half, the value $V = V_{RT}$ of the tradeable in such an equilibrium should be half the sum of the payoffs of the proposer ($\frac{1}{2}(G + \delta V(1+\omega) - C\omega)$), and the tame ($\frac{1}{2}(\delta V + (1-\omega)G - \omega C)$), i.e., $V_{RT} = \frac{(2-\omega)G - 2C\omega}{4-\delta(2+\omega)}$.

(2): Should a proposer offer a null contract or a rob-trade one if both will be accepted? Rob-trade dominates null iff $G + \delta V \leq G + \delta V(1+\omega) - C\omega$ or, equivalently, $\delta V \geq C$. ■

Proof of proposition 1. By lemma 2, we can ignore the robbing contracts and it is sufficient to check that when $V = V_{RT}$ asymmetric contracts are accepted and dominate both the null and the trading contracts.

By lemma 4, if only asymmetric contracts appear in equilibrium, then $V = V_{RT} = \frac{N}{D}$, with $N \equiv (2-\omega)G - 2C\omega$ and $D \equiv 4 - \delta(2+\omega)$. Since $D > 0$ and by lemma 4.2 $V_{RT} \geq \frac{C}{\delta}$, which is non-negative, and so is N. The latter inequality, which is necessary for rob-trade contract to dominate null is equivalent to $\delta \geq \frac{4C}{N+(2+\omega)C}$ and by lemma 4.1 the acceptance criterion is $\delta V_{RT} \leq L \equiv (1-\omega)G - C\omega$, which is equivalent to $\delta \leq \frac{4L}{N+(2+\omega)L}$.

If $N = 0$, so is V_{RT}, hence rob-trade can dominate null only if $C = 0$, but $N = 0$ also implies (by construction) $\frac{C}{G} = \frac{2-\omega}{\omega} > 0$, hence the asymmetric contracts can not be accepted in this case.

If $N > 0$, the implied interval for δ is non-empty if $C \leq L$, or equivalently, $c = \frac{C}{G} \leq \frac{1-\omega}{1+\omega}$ (which also implies $N > 0$, i.e., $c < \frac{2-\omega}{\omega}$).

If the proposer expects his partner to choose *trade* with probability $\gamma = 0$ upon signing the trading contract, then offering trading contract is equivalent to offering the robbing contract, which is dominated by null, by lemma 2.

Hence it is sufficient to consider equilibria in which a partner is expected to trade with probability $\gamma > 0$ upon endorsing a trading contract.

Let $\rho(c) = \rho_1(c)$, if $c \leq \frac{4-4\delta+2\delta\omega-\delta\omega^2}{\delta\,\omega^2+(4-4\,\delta)\,\omega}$ and $\rho(c) = \rho_2(c)$ otherwise, where $\rho_1(c) = \frac{(3\,c+1)\,\delta\,\omega^2+(-4\,\delta-4\,c)\,\omega+4}{(4\,c+2)\,\delta\,\omega^2-6\,\delta\,\omega-4\,\delta+8}$, and $\rho_2(c) = \frac{(2\,c+1)\,\delta\,\omega^2+(-2\,c-3)\,\delta\,\omega+2\,\delta}{(2\,c+1)\,\delta\,\omega^2-3\,\delta\,\omega-2\,\delta+4}$.

If $\gamma > 0$, the trading contract will be accepted iff $\gamma G + (1-\gamma)\delta\omega V \geq \delta V$, or when $\gamma \geq \frac{\delta V(1-\omega)}{G-\delta\omega V}$, substituting $V = V_{RT}$, we get $\gamma \geq \rho_2$.

The proposer is better off with the asymmetric contract iff $\frac{1}{2}(G + \delta V(1+\omega) - C\omega) \geq \gamma G + (1-\gamma)\delta\omega V$, which is equivalent to $\gamma \leq \frac{G+\delta(1-\omega)V - C\omega}{2(1-\delta\omega V)}$, and substituting $V = V_{RT}$, we get $\gamma \leq \rho_1$.

So, for to support the equilibrium we need $\gamma \leq \max\{\rho_1, \rho_2\}$. The difference, $\rho_1 - \rho_2$ decreases with c, since its derivative is $\frac{\omega\,(\delta\,\omega+2\,\delta-4)\,\left(\delta\,\omega^2+\delta\,\omega-4\,\delta+4\right)}{2\,(2\,c\,\delta\,\omega^2+\delta\,\omega^2-3\,\delta\,\omega-2\,\delta+4)^2} <$

0, and $\rho_1(c) \geq \rho_2(c)$ if $c \in [0, \frac{4-4\delta+2\delta\omega-\delta\omega^2}{\delta\,\omega^2+(4-4\,\delta)\,\omega}]$.

ρ is decreasing in c, since both ρ_1 and ρ_2 are with the respective derivatives $\frac{\omega\,(\delta\,\omega+2\,\delta-4)\,\left(\delta\,\omega^2-3\,\delta\,\omega+4\right)}{2\,(2\,c\,\delta\,\omega^2+\delta\,\omega^2-3\,\delta\,\omega-2\,\delta+4)^2} < 0$, and $\frac{\omega\,(\delta\,\omega+2\,\delta-4)\,\left(\delta\,\omega^2-3\,\delta\,\omega+4\right)}{2\,(2\,c\,\delta\,\omega^2+\delta\,\omega^2-3\,\delta\,\omega-2\,\delta+4)^2} < 0$. ■

A.2 Supporting equilibria with trading contracts

A.2.1 Trading only

Lemma 5 *Assume only trading contracts are endorsed, then, the set of equilibrium values of γ consists of*

1. *roots in $[0,1]$ of polynomial F, defined in proposition 2:*

$$\begin{aligned}
F(\gamma, c) &\equiv \gamma^2 a(c) + \gamma b(c) + k(c), \text{ with} \\
k(c) &= (c\omega - (1-\omega))(1-\delta\omega); k \geq 0 \textit{ iff } c \geq \bar{c} \geq 0 \\
b(c) &= (1+\omega)(1-\delta) + c\omega; b \geq 0 \textit{ iff } c \geq \tilde{c} = \frac{(1+\omega)(1-\delta)}{-\omega} \\
a(c) &= \delta(c\omega^2 - (1-\omega^2)); a \geq 0 \textit{ iff } c \geq \frac{1-\omega^2}{\omega^2} \geq \bar{c},
\end{aligned} \tag{1}$$

2. *unity, if $F(1,c) > 0$, and*
3. *zero, if $F(0,c) < 0$.*

Proof. Let V_t be the value of the good, held by an agent who always trades, and V_r be that of a perpetual robber. From the specification of payoffs (section 3.2),

$$V_t(\gamma, G) = \frac{\gamma G}{1-\delta\omega(1-\gamma)}, \tag{2}$$

$$V_r(\gamma, C, G) = \frac{(\gamma+1)((1-\omega)G - C\omega)}{\delta(1-\omega)(1-\gamma) + 2(1-\delta)} \tag{3}$$

If $V_t(1, G) > V_r(1, C, G)$, then there is an equilibrium where everyone always trades; if $V_t(0, G) < V_r(0, C, G)$, then there is one with everyone always robbing; finally if there is $\gamma \in [0,1]$ such that $V_t(\gamma, G) = V_r(\gamma, C, G)$ then a fraction γ of agents chooses to trade at any t. Finally, since

$$\begin{aligned}
F(\gamma, c) &= \kappa(\gamma) G[V_t(\gamma, 1) - V_r(\gamma, c, 1)], \text{ where} \\
\kappa(\gamma) &\equiv (\delta(1-\omega)(1-\gamma) + 2(1-\delta))(1-\delta\omega(1-\gamma))
\end{aligned} \tag{4}$$

and $\kappa(\gamma) > 0$ for any $\gamma \in [0,1]$, the sign of $F(\gamma, c)$ coincides with the sign of the difference $V_t(\gamma, G) - V_r(\gamma, C, G)$, and their sets of zeros (for any given C, G) coincide as well, hence the conclusion. ■

Corollary 1 *If $F(x,c) > 0$ for all $x \in [0,1]$ then in any equilibrium $\gamma = 1$, and if $F(x,c) < 0$ for all $x \in [0,1]$ then in any equilibrium $\gamma = 0$.*

Remark 4 *F is increasing in c. $F(1,c) \leq 0$ iff $c \leq \underline{c} = \frac{\delta - \omega}{\omega}$.*

Lemma 6 *Assume $c \neq \frac{1-\omega^2}{\omega^2}$. The extremum of $F(\cdot, c)$ is $\gamma^e(c) \equiv \frac{1}{2}\frac{(1+\omega)(1-\delta)+c\omega}{(1-\omega^2-c\omega^2)\delta}$, which is positive iff $c \in]\tilde{c}, \frac{1-\omega^2}{\omega^2}[$, and is zero only at $\tilde{c}$. $\gamma^e(c) > 1$ iff $c \in]c^e, \frac{1-\omega^2}{\omega^2}[$ and is unity only at c^e where $c^e \equiv \frac{2\delta(1-\omega)-(1-\delta)}{2\delta\omega+1}\frac{(\omega+1)}{\omega}$.*

Proof. Since $b(c) < 0$ implies $a(c) < 0$, $\gamma^e = -\frac{b(c)}{2a(c)}$ is strictly positive iff $b(c) > 0$ and $a(c) < 0$, or equivalently, $\tilde{c} < c < \frac{1-\omega^2}{\omega^2}$. $\gamma^e = 0$ iff $b(c) = 0$ iff $c = \frac{(1+\omega)(1-\delta)}{-\omega}$. The formula for the extremum follows by a direct computation, hence the first claim. So $\gamma^e(c)$ is strictly increasing in c and $\gamma^e(c) \geq 1 \Rightarrow c \geq c^e$; $c^e < \frac{1-\omega^2}{\omega^2}$, hence the second claim. ■

Remark 5 *$\delta > \frac{1}{2} \Leftrightarrow \underline{c} < c^e$, $\delta = \frac{1}{2} \Leftrightarrow \underline{c} = c^e$.*

Lemma 7 *If $\delta > 1/2$ there exists a unique $\underline{\underline{c}} \in]\tilde{c}, \underline{c}[$, such that the discriminant $H(c) = b^2(c) - 4a(c)k(c)$ of F is zero at $\underline{\underline{c}}$. $\gamma_L(\underline{\underline{c}}) = \gamma_H(\underline{\underline{c}}) \in]0,1[$. H is strictly positive on $]\underline{\underline{c}}, \underline{c}[$ and for any c in that interval there are two roots of F, $0 < \gamma_L(c) < \gamma_H(c) < 1$ with $\gamma_H'(c) > 0$ on $]\underline{\underline{c}}, \underline{c}]$, $\lim_{c \searrow \underline{\underline{c}}} \gamma_H'(c) = +\infty$. Also, $F(x,c) < 0$ for $c < \underline{\underline{c}}$ for any $x \in [0,1]$.*

Proof. $\tilde{c} < \underline{c}$ by definition, and by remark 5, $c^e > \underline{c}$, so by lemma 6, $0 < \gamma^e(\underline{c}) < 1$. By remark 4, unity a root of F at $\underline{c}$, and by the last inequality, it is the upper root. By the same inequality, the lower root, $\gamma_L(\underline{c})$, being below the extremum, is strictly lower than the upper one, and is strictly positive, since $k(\underline{c}) < 0$. It implies that $H(\underline{c}) > 0$, and since $a(\underline{c}) < 0$, $F(x, \underline{c}) > 0$ for $x \in]\gamma_L(\underline{c}), 1[$. For every γ in this interval there is a unique $c < \underline{c}$ that turns it into a root of the quadratic F, as F is strictly monotone and continuous in c and $F(\gamma, \tilde{c}) < 0$ for $\gamma \in [0,1]$. Hence $H(\tilde{c}) < 0$, and since H is continuous in c, there should be $\tilde{c} < \underline{\underline{c}} < \underline{c}$ such that $H(\underline{\underline{c}}) = 0$. Then also, using the classical formulae, $\gamma_L(\underline{\underline{c}}) = \gamma^e(\underline{\underline{c}}) = \gamma_H(\underline{\underline{c}})$, and this value is strictly inside the unit interval by lemma 6, since $\tilde{c} < \underline{\underline{c}} < \underline{c} < c^e$. Clearly, also, at $\underline{\underline{c}}$, F is zero at its extremum. Since F is strictly increasing in c, it is negative for $c < \underline{\underline{c}}$, and is strictly positive at $\gamma^e(\underline{\underline{c}})$ for any $c > \underline{\underline{c}}$, thus $H(c)$ changes sign only once in $]\tilde{c}, \underline{c}[$, hence the uniqueness of $\underline{\underline{c}}$.

The upper root is increasing in c for $\underline{c} \geq c > \underline{\underline{c}}$. Indeed, using the classical formula, $\gamma_H(c) = \frac{-b(c)-\sqrt{H(c)}}{2a(c)}$,

$$\gamma_H'(c) = \frac{-b'(c) - \frac{H'(c)}{2\sqrt{H(c)}}}{2a(c)} - \frac{\gamma(c)a'(c)}{a(c)} = \frac{-b' - 2\gamma(c)a'}{2a(c)} - \frac{H'(c)}{4a(c)\sqrt{H(c)}} \tag{5}$$

where $a' = \omega^2\delta, b' = \omega, k' = \omega(1-\delta\omega) > 0$. Now $H'(c) = 2b(c)b' - 4a'k(c) - 4k'a(c) > 0$ since $k(c) < 0, a(c) < 0, b(c) > 0$. This implies $\gamma'(c) > 0$, and also, since by definition, $H(\underline{\underline{c}}) = 0$, the statement about the limit. ■

Lemma 8 *$F(\gamma, c) < 0$ for any $\gamma \in [0,1]$ iff*

1. *if $\delta \leq 1/2$ and $c < \underline{c}$;*
2. *if $\delta > 1/2$ and $c < \underline{\underline{c}}$, with $\underline{\underline{c}}$ defined in lemma 7.*

Remark 6 *If $\delta \leq \frac{1}{2}$ then $\gamma_L(\underline{c}) = 1$.*

Proof. Let $\tau = \sup\{t : F(x,t) \leq 0 \quad \forall x \in [0,1]\}$. Since F is strictly monotone in c, the inequality $F(\cdot, c) \leq 0$ on $[0,1]$ is satisfied for all $c < \tau$. Since by remark 4, $F(1,c) > 0$ for $c > \underline{c}$, the threshold τ should be below $\underline{c}$. Then claim 2 follows directly from lemma 7.

So remains the case where $\delta \leq \frac{1}{2}$. By remark 5, $\underline{c} \geq c^e$ and hence by lemma 6, $\gamma^e(\underline{c}) \geq \gamma^e(c^e) = 1$, but unity is a root of F at $c = \underline{c}$ by remark 4, and so it is the lower root (as stated in remark 6). Hence for any $\gamma < 1 = \gamma_L(\underline{c})$, F does not change sign, and, because $a(\underline{c}) < 0$, so F is concave, it is negative below the lower root. So, we have $F(x, \underline{c}) \leq 0$ for all $x \in [0,1]$, so $\tau \geq \underline{c}$, but by the argument above, $\tau \leq \underline{c}$, hence $\tau = \underline{c}$, as claimed in 1. ■

Proof of proposition 2. By corollary 1 and lemma 8 the only equilibrium value of γ is zero if either $\delta \leq \frac{1}{2}$ and $c < \underline{c}$ or if $\delta > \frac{1}{2}$ and $c < \underline{\underline{c}}$, where $\underline{\underline{c}}$ is as defined in lemma 7. This proves the first case in both paragraphs (1 and 2) of the statement.

By corollary 1, unity is the only equilibrium value of γ when $c \geq \frac{1-\omega^2}{\omega^2}$, since then all the coefficients of F are non-negative. For $c \in]\overline{c}, \frac{1-\omega^2}{\omega^2}[$, $a(c) < 0$, but $b(c), k(c) > 0$, hence the lower root is negative and (the strictly concave)

F is strictly positive between the roots. But by remark 4, for any c in that interval $F(1,c) > 0$ and hence $F(x,c) > 0$ for any $x \in [0,1]$, so $\gamma = 1$ for all such c and, combining with the above, it is unity for any $c > \bar{c}$.

For the rest, using lemma 5, it is sufficient to identify all roots of F inside the unit interval and check its sign at the endpoints.

If $\delta \leq 1/2$, by remark 6, at $c = \underline{c}$ the lower root of F is unity, so γ can equal to 1. Since $a(\underline{c}) < 0$, $F(x,\underline{c}) < 0$ for $x \in [0,1[$, the only additional equilibrium value for γ besides unity is zero. Next, at $k(\bar{c}) = 0$, $\gamma_L(\bar{c}) = 0$, and so this is one of the equilibrium values. Also, as above, by concavity, and remark 4, $F(x,\bar{c}) > 0$ for all $x \in]0,1]$, implying γ can only be unity at $\bar{c}$, in addition. By continuity and strict monotonicity of F, for every $\gamma \in]0,1[$ there is $c \in]\underline{c}, \bar{c}[$ that turns it into a (lower) root of F. Again, since $a(c) < 0$, F is strictly positive between the two roots, and by remark 4, $F(1,c) > 0$ for all $c \in]\underline{c}, \bar{c}[$, so unity is an equilibrium value of γ as well. Finally, since $k(c)$ is negative on this interval, zero is the third value that γ can attain there.

If $\delta > 1/2$ then by lemma 7 $\underline{\underline{c}}$ is unique and at $c = \underline{\underline{c}}$ the two roots of F coincide inside the unit interval, and, since $k(\underline{\underline{c}}) < 0$, the two values that γ can attain, are as stated. Further, by lemma 7, for any $c \in]\underline{\underline{c}}, \underline{c}[$, the two roots of F are in the unit interval and hence are the values for γ; in addition, since $k(c) < 0$, there is also the third value, zero. Next, at $c = \underline{c}$, the upper root is unity, $F(1,\underline{c}) = 0$, and the lower one is still strictly inside the unit interval, as $k(\underline{c}) < 0$, hence the three values for γ are as stated. Next, by monotonicity of F in c, for $c > \underline{c}$ the upper equilibrium value remains unity, so for $c \in]\underline{c}, \bar{c}[$ there are three equilibrium values: zero, unity and $\gamma_L(c)$. At $c = \bar{c}$, there are only two values for γ: $0, 1$, as the lower root of F is zero, $\gamma_L(\bar{c}) = k(\bar{c}) = 0$. ■

A.2.2 Characterisation of existence

Lemma 9 *Assume $C \geq 0$. An equilibrium where only trading contracts are endorsed exists iff either $c \geq \frac{1-\omega}{\omega}$ or if $c < \frac{1-\omega}{\omega}$ and $\gamma^*(c) \geq \zeta(c) = \max\{\gamma_{T0}, \min\{\eta(c), \gamma_{TTR}(c)\}\}$, where $\gamma_{T0} \equiv \frac{1-\delta\omega}{2-\delta\omega-\delta}$, $\eta : c \mapsto \frac{(1-c\omega-\omega)(1-\delta\omega)}{\delta(1-\omega(1-c\omega-\omega))}$ and $\gamma_{TTR} : c \mapsto \frac{(1-\omega\delta)(1-c\omega)}{2-\delta-\delta\omega+\delta\omega^2 c}$. Both γ_{TTR} and η are strictly decreasing for $c \geq 0$.*

Proof. If $\gamma^*(c) = 0$, in case trading contract is endorsed, it is a strictly dominant strategy for both agents to rob, hence the payoff to each is the same as under the robbing contract. So, by lemma 2 trading is dominated in this case.

Hence, $\gamma^*(c)$ has to be strictly positive for the trading contract to be offered. Acceptance of this contract requires $\delta V_{TT} \leq \gamma^*(c)G+(1-\gamma^*(c))\omega\delta V_{TT}$, or, $\delta V_{TT} \leq \frac{\gamma^*(c)G}{1-(1-\gamma^*(c))\omega}$, which always holds, since $V_{TT} = \frac{\gamma^*(c)G}{1-(1-\gamma^*(c))\omega\delta}$.

Null contracts are accepted whenever trading are, since, $V_{TT} < G < \frac{G}{\delta}$. Trading dominates null iff $V_{TT} > \frac{1}{2}(G + \delta V_{TT})$, or, equivalently, $V_{TT} > \frac{G}{2-\delta}$, which is equivalent to $\gamma^*(c) \geq \gamma_{T0} \equiv \frac{1-\delta\omega}{2-\delta\omega-\delta} \in [0, 1]$. Note, since $c \geq \frac{1-\omega}{\omega}$ implies $\gamma^*(c) = 1$, this inequality always holds.

Finally, trading has to dominate rob-trade. The latter contracts are accepted, by lemma 4, iff $\delta V_{TT} \leq (1-\omega)G - C\omega$, which, since $c \geq 0$, translates to $\gamma^*(c) \leq \eta(c) = \frac{(1-c\omega-\omega)(1-\delta\omega)}{\delta(c\omega^2+\omega^2-\omega+1)}$. It is violated for $c \geq \frac{1-\omega}{\omega}$, hence the first case is proved. Trading contracts dominate asymmetric ones iff $V_{TT} \geq \frac{G-C\omega}{2-\delta(1+\omega)}$, which is equivalent to $\gamma^*(c) \geq \gamma_{TTR}(c) = \frac{(1-c\omega)(1-\delta\omega)}{c\delta\omega^2-\delta\omega-\delta+2}$.To sum up, if $c < \frac{1-\omega}{\omega}$, γ^* has to be above either $\eta(c)$ or $\gamma_{TTR}(c)$ and above γ_{T0}, hence the second case claimed.

γ_{TTR} and η are decreasing, since the derivatives (on $\mathbb{R}_+$), are respectively, $-\frac{\omega\,(\delta\,\omega-1)\,(\delta\,\omega+\delta-2)}{(c\,\delta\,\omega^2-2\,\delta\,\omega-\delta+2)^2} < 0$ and $\frac{\omega\,(\delta\,\omega-1)}{\delta\,(c\,\omega^2+\omega^2-\omega+1)^2} < 0$. ■

Proof of proposition 3. By lemma 9, ζ is decreasing, and may be constant only when equal to $\gamma_{T0} < 1$. Since $\gamma^*(c)$ is increasing and is constant and strictly positive only when equal to unity, the difference, $\gamma^* - \zeta$ is increasing and may be constant only if strictly positive. Hence, if there is a non-negative solution to $\gamma^*(c) = \zeta(c)$, it is unique. Thus, using 9, the threshold is $\inf\{c \geq 0 | \gamma^*(c) \geq \zeta(c)\}$, which has to be below $\frac{1-\omega}{\omega}$. ■

A.3 Determinants of the Demand for Enforcement

Proof of theorem 1. (1): By definition 3, the demand in this case is the difference between the payoff to the proposer of the asymmetric contract $\frac{1}{2}(G + (1+\omega)\delta V_{RT} - \omega C)$ and that of the null $1/2(G + \delta V_{RT})$, or $\omega r(C, V_{RT})$ so the result follows by the direct computation, using the expression of V_{RT} from lemma 4.1.

(2): Similarly, by definition 3, the demand is $V_{TT} - 1/2(G+\delta V_{TT})$, where $V_{TT} = \frac{\gamma^* G}{1-\delta\omega(1-\gamma^*)}$ for $\gamma^* > 0$ by its definition (2) and the specification of payoffs. The rest follows from proposition 2.

If neither rob-trade nor trading contracts can be supported in an equilibrium, then there is no demand for contract endorsement, provided the punishment is positive, hence the last claim. ∎

Proof of theorem 2. By theorem 1, one can directly calculate the effect of G using the formula for demand in this case: $D^*(G, C; \delta, \omega) = \frac{1}{2}Gz(c)$, where

$$z(c) = \frac{\gamma_H(c)(2 - \delta - \delta\omega) + \delta\omega - 1}{1 - \delta\omega(1 - \gamma_H(c))}$$

since $\gamma^*(c) = \gamma_H(c)$. Also $\frac{d(c)}{dG} = -\frac{C}{G^2} = -\frac{c}{G}$. Hence

$$\frac{dD^*(G, C; \delta, \omega)}{dG} = \frac{1}{2}[z(c) - cz'(c)] \tag{6}$$

$$z'(c) = \gamma_H'(c)\frac{2 - \delta - \delta\omega}{1 - \delta\omega(1 - \gamma_H(c))} - \frac{z(c)\delta\omega}{1 - \delta\omega(1 - \gamma_H(c))} \tag{7}$$

To show part (1), note that at $c = \beta$, the demand for enforcement is zero, so $D^*(\frac{C}{\beta}, C, \delta, \omega) = 0 = z(\beta)$. Hence in this case

$$\frac{dD^*(\frac{C}{\beta}, C; \delta, \omega)}{dG} = \frac{-\beta}{2}\gamma_H'(\beta)\frac{2 - \delta - \delta\omega}{1 - \delta\omega(1 - \gamma_H(\beta))} \tag{8}$$

By definition of β, $\underline{c} > \beta \geq \underline{\underline{c}}$, so by lemma 7, $\gamma_H'(\beta) > 0$. Since $\beta > 0$, the derivative at this point is negative.

To show the last part of the statement (2), use $\lim_{c \searrow \underline{\underline{c}}} \gamma_H'(c) = +\infty$ (by Lemma 7), $z(\underline{\underline{c}}) = 0$ and equations (6)-(7). ∎

Lemma 10 *Assume $\delta > \frac{1}{2}$. $\underline{\underline{c}} > 0$ iff either $\omega \leq \frac{1}{2}$ or $\omega > \frac{1}{2}$ and $\delta > \delta_L(\omega)$, where $\delta_L(\omega)$ is the lower root of the quadratic $h(\delta, \omega) \equiv (1 + \omega)(1 - \delta)^2 - 4\delta(1 - \omega)^2(1 - \delta\omega)$.*

Proof. At $\underline{\underline{c}}$ the maximum of the parabola $(F(\cdot, c))$ is zero. So we have to check the sign of $F(\gamma^e(0), 0)$. If it is negative, then $\underline{\underline{c}} > 0$. $a(0) = -\delta(1-\omega^2)$; $b(0) = (1 + \omega)(1 - \delta)$; $k(0) = -(1 - \omega)(1 - \delta\omega)$; $\gamma^e(0) = \frac{1-\delta}{2\delta(1-\omega)}$. So,

$$F(\gamma^e(0), 0) = (1 + \omega)\frac{(1 - \delta)^2}{4\delta(1 - \omega)} - (1 - \omega)(1 - \delta\omega)$$

Its sign is the sign of $h(\delta, \omega)$, which is quadratic in δ. For $\omega \in [0, 1]$, it changes sign in δ: $h(1, \omega) < 0$ and $h(0, \omega) > 0$, and hence is decreasing on

$[0,1]$, it is convex, and so $h(\delta,\omega) < 0$ iff $\delta > \delta_L(\omega) = \frac{3-2(1-\omega)\omega-\omega-(2(1-\omega))^{\frac{3}{2}}}{4\omega^3-8\omega^2+5\omega+1}$ (the lower root of h). This root is an increasing function of ω, with $\delta_L(\frac{1}{2}) = \frac{1}{2}$ and $\delta_L(1) = 1$. ■

Proof of lemma 3. Let c_x be the argument (c) where γ_{T0} intersects the $\min\{\gamma_{TTR}(c), \eta(c)\}$. Since γ_{TTR} and η are strictly decreasing in the relevant range, $[0, \underline{c}[$, by lemma 9, $c_x = \min\{\frac{\delta}{2-\delta}, \frac{\omega}{1-\omega} - \frac{\delta}{(2-\delta)\omega}\}$. Hence, for $c \geq \frac{\delta}{2-\delta}$, $\zeta(c) = \gamma_{T0}$.

(1): By construction, $\gamma^e(\alpha) = \gamma_{T0}$, and by positive monotonicity of γ^e for $c \leq \frac{1-\omega^2}{\omega^2}$ (lemma 6 and remark 4, assuring $\alpha < \underline{c}$), we have $\gamma^e(\alpha) \geq \gamma^e(\beta)$, so $\gamma_{T0} \geq \gamma^e(\beta)$, but γ_{T0} is one of the roots of the quadratic F at $c = \beta$ (by construction of β), so it has to be the upper root. Hence $\gamma_H(\beta) = \gamma^*(\beta) = \gamma_{T0}$. Finally, since β then is in $[\underline{\underline{c}}, \underline{c}[$, γ^* is strictly increasing (by lemma 7) so, by definition of the threshold, $t_+ = \beta$.

(2): Define $f : c \mapsto F(\gamma^e(c), c)$, it (strictly) increases in c for $c \in [0, \frac{1-\omega^2}{\omega^2}[$ (lemma 6), and is negative at $c = \frac{\delta}{2-\delta}$ by assumption, so, since by construction of $\underline{\underline{c}}$, $f(\underline{\underline{c}}) = 0$ we have $\underline{\underline{c}} \geq \frac{\delta}{2-\delta}$. Combining $F(\gamma^e(\frac{\delta}{2-\delta}), \frac{\delta}{2-\delta}) \geq F(\gamma_{T0}, \frac{\delta}{2-\delta})$ (by construction of the extremum γ^e); with the assumption $0 \geq F(\gamma^e(\frac{\delta}{2-\delta}), \frac{\delta}{2-\delta})$ and the definition of β, $F(\gamma_{T0}, \beta) = 0$, we get $F(\gamma_{T0}, \cdot)$ is non-positive for $c \in [\frac{\delta}{2-\delta}, \beta]$. In addition, since $\beta > \alpha$, similarly to the previous point, $\gamma^e(\beta) > \gamma^e(\alpha) = \gamma_{T0}$, so γ_{T0} has to be the lower root of F at $c = \beta$, and, so γ_{T0} is below the lower root of F for $c \in [\frac{\delta}{2-\delta}, \beta]$, and, clearly, as the upper root is increasing in c, $\gamma_{T0} = \gamma_L(\beta) < \gamma_H(c)$ for $c > \beta$. So, in this case any strictly positive upper root of F is above $\zeta(c) = \gamma_{T0}$, so the threshold is $\underline{\underline{c}}$, the smallest c, at which F starts having real roots. Finally, $\underline{\underline{c}} > 0$ is by lemma 10. ■

Proof of proposition 4. Proposition 3 assures the demand is positive in this case. If $c > \overline{c}$, then the conclusion trivially follows from theorem 1. If $t_+ \leq c < \overline{c}$, then, by theorem 1

$$\frac{dD^*}{d\delta} = \frac{G}{2}\big[z_\delta + z_\gamma \frac{d\gamma^*}{d\delta}\big] \tag{9}$$

$$\frac{dD^*}{d\omega} = \frac{G}{2}\big[z_\omega + z_\gamma \frac{d\gamma^*}{d\omega}\big] \tag{10}$$

where

$$z_\gamma = \frac{(\delta\omega - 1)(\delta - 2)}{(\gamma^*\delta\omega - \delta\omega + 1)^2} \tag{11}$$

$$z_\omega = \frac{(\delta - 2)(\gamma^* - 1)\gamma^*\delta}{(\gamma^*\delta\omega - \delta\omega + 1)^2} \tag{12}$$

$$z_\delta = \frac{(2\gamma^*\omega - 2\omega + 1)\gamma^*}{-(\gamma^*\delta\omega - \delta\omega + 1)^2} \tag{13}$$

Note that $z_\gamma > 0$.

To prove that the demand is increasing in ω, therefore, it is left to show that the last summand in (10) is positive. First, $z_\omega > 0$ because $\gamma^* > 0$ which follows by lemma 7, given $c \geq t_+ \geq 0 > \tilde{c}$. Hence it is sufficient to show that $\frac{d\gamma^*}{d\omega} > 0$. Recall, γ^* is the zero of the quadratic polynomial F defined in proposition 2 and for the relevant range of parameters the first coefficient (a) is negative, hence F decreases at γ^*. So, by implicit function theorem, the sign of $\frac{d\gamma^*}{d\omega}$ is the sign of $\frac{\partial F}{\partial \omega}$, which is equal to $(\gamma^*)^2 a_\omega + \gamma^* b_\omega + k_\omega > 0$ since $a_\omega = 2\omega\delta(c+1) > 0$, $b_\omega = 1 - \delta + c > 0$, and $k_\omega = (c+1)(1-\delta\omega) + \delta(1 - c\omega - \omega) > 0$.

Similarly, demand is decreasing in δ. First, $z_\delta < 0$ since $\gamma^* \geq \gamma_{T0}$ by lemma 9 and $\gamma_{T0} > \frac{2\omega-1}{2\omega}$. Second, the sign of $\frac{d\gamma^*}{d\delta}$ is the sign of $\frac{\partial F}{\partial \delta}$, which is equal to $(\gamma^*)^2 a_\delta + \gamma^* b_\delta + k_\delta < 0$ since $a_\delta = c\omega^2 - (1-\omega^2) < 0$, and $\gamma^* b_\delta + k_\delta = -[\gamma^*(1+\omega) + \omega^2 c - \omega(1-\omega)] < 0$, since $\gamma^* \geq \gamma_{T0} > \frac{\omega}{1+\omega}$. $\blacksquare$

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