The Role of Consumption-Labor Complementarity as a Source of Macroeconomic Instability

Baruch Gliksberg*

University of Haifa

June, 2010

The equilibrium ramification of a balanced budget rule are scrutinized in a one sector growth model augmented with investment frictions and a non-separable utility function in consumption and leisure. Edgeworth-complementarity between consumption and labor is formulated so as to generate a positive co-movement of consumption, output, and hours worked, as found in the data. Calibration of the model to the U.S. economy provides evidence that a balanced budget rule with a Taylor type monetary policy induce determinate equilibria.

JEL Classification: C62, C63, *E4, E52, E61, E62, E63.*

Keywords: Fiscal-Monetary policy; Non-Separable Utility; Consumption-Labor Complementarity; Endogenous Labor; Stabilization; Determinacy; Investment;

* Department of Economics, University of Haifa, , Haifa Israel. Phone: (972) 4 824 0116, Email: baruchg@econ.haifa.ac.il

1. Introduction

It seems plausible that versions of business cycle models that exhibit a positive co-movement between consumption and labor may also exhibit indeterminacies. A noteable attempt to link between indeterminacies and the positive co-movement between consumption and labor is found in Benhabib and Wen (2004). They show that under indeterminacy, aggregate demand shocks are able to explain not only aspects of actual fluctuations that standard RBC models predict fairly well, but also aspects of actual fluctuations that standard RBC models cannot explain, such as the hump-shaped, trend reverting impulse responses to transitory shocks found in US output [Cogley and Nason (1995)] the large forecastable movements and comovements of output, consumption and hours [Rotemberg and Woodford (1996)] and the fact that consumption appears to lead output and investment over the business cycle. Indeterminacy arises in their model due to capacity utilization and mild increasing returns to scale. Subsequent literature, such as Linnemann (2008) and Meng and Yip (2008), contributes to this literature by improving our intuition as to the causes of indeterminacy.

In Linnemann (2008), equilibrium indeterminacy can arise in a neoclassical growth model when the government continuously balances its budget through adjustments of the income tax rate[1]. Linnemann (2008) puts emphasis on a steady state in which leisure is constant although consumption may grow. In this case, complementarity between consumption and employment emerges as a stabilizing mechanism. Meng and Yip (2008) relax the restrictions commonly imposed on the magnitude of capital externalities in one-sector models with Cobb-Douglas technology. They find that indeterminacy can arise either if utility is separable in consumption and leisure and

[1] The reasons for indeterminacy under such policy are discussed extensively in Schmitt-Grohé and Uribe (1997).

there are negative capital externalities; or: utility is non-separable and the social elasticity of production with respect to capital is greater than one. In addition, with Cobb-Douglas technology they show that leisure must be a normal good for indeterminacy to occur [that is, the presence of income effects on the demand for leisure is a necessary condition for indeterminacy]. There is no restriction in meng and Yip (2008) analysis on whether consumption and labor are complements or substitutes.

At this time the literature provide no clue as to the direction of causality where equilibrium is indeterminate and consumption co-moves with labor, so it remains a matter of faith. Either, as in Benhabib and Wen (2004) under indeterminacy, aggregate demand shocks explain the large forecastable movements and comovements of output, consumption and hours, or , complementarities between consumption and labor are responsible both to indeterminacies a-la Benhabib and Wen (2004) and to the positive co-movement found in the data. It should be noted however, that the assumption of additive separability of utilities with respect to consumption and leisure has been typically made more for convenience than from conviction. Basu and Kimball (2002) and Kimball and Shapiro (2008) reject additive separability and arrive at the view that the utility function exhibits complementarity between consumption and labor by considering facts about lung-run labor supply. Hall (2008) and Raj (2006) also share this view. Their view also points to predictions in the context of Benhabib and Wen (2004). For example, complementarity between consumption and labor provide a straightforward channel for monetary policy to cause an increase in output. When labor and consumption are complementary, the increase in consumption is enough to cause output to increase as long as interest and wealth effects are not too large.

Note that in most RBC models it is implicit that the government seeks to stabilize the conomy. Among the goals of the policy stands firmly the intent to eliminate fluctoations due to self fulfillinf expectations. In such a case, consumption-labor complementarity can overturn policy outcomes.

This paper puts emphasis on characterizing policy rules that potentially subdue the self fulfilling fluctutions that come to pass from the complementarity between consumption and hours worked. It maintains, however, the assumption that the consolidated government operates a balanced budget. In simple settings the conditions under which monetary policy can lead to indeterminacy under a balanced budget requirement are well understood: active interest rules within a balanced budget requirement generate determinacy and passive rules generate indeterminacy. Prominent papers in this literature include Benhabib et. al. (2001) [who discuss monetary policy rules where money and consumption are Edgeworth complements and the policy affects households allocation at the private level between financial assets and capital assets via an arbitrage channel] and Huang and Meng (2007) [who consider an effect of policy on households via the arbitrage channel and an effect on firms via a pricing channel in a model with monopolistic competition and nominal price rigidity]. This paper contributes to existing literature by considering an additional channel: a labor channel, where consumption and labor are assumed Edgeworth complements. Accordingly, it is assumed throughout that the utility function is non-separable in consumption and leisure and, in line with Hall (2008), that consumption and labor are complements. The upshot of this paper is that controlling the real rate of interest is a necessary condition for the determination of expectations. The model calibrated to the U.S economy provides evidence that: a) operating an interest rate rule such that the expected real rate of interest is above its steady state level during booms and below its steady state level during recessions is sufficient to induce equilibrium determinacy; b) operating an interest rate rule such that the expected real rate of interest is constant at all times is sufficient to induce equilibrium determinacy; c) a non-monetary version of the model is expected to exhibit indeterminacies as demonstrated in Meng and Yip (2008)[2].

The rest of the paper is organized as follows: section 2 illustrates a model with endogenous labor-leisure choice where consumption and labor are Edgeworth complements. This model is extended to include a concave investment technology and a cash-in-advance constraint. The cash-in-advance constraint is introduced so as to mimic the role of complementarity between consumption and money. Section 3 contains local stability analysis with the least amount of restrictions over the functional forms of utilities and production technology. In section 4 long run elasticities and deep structural parameters are calibrated to the U.S. economy. Results show that for a plausible range of parameters, under a requirement that the consolidated budget is balanced throughout, local-real-determinacy is ensured where monetary policy is neutral or active. Section 5 concludes.

[2] This version of the article does not formally prove this conjecture.

2. A Model with Endogenous Labor-Leisure Choice and Frictions in Investment

It is assumed throughout the paper that the consolidated government runs a balanced budget within which the central bank operates an interest rate feedback rule. Dupor (2001) and Benhabib et al. (2001) discuss the issue of local real determinacy in a continuous time model where the monetary authority sets a nominal interest rate as a function of the instantaneous rate of inflation. The policy considered here follows this line and is also in one line with the forward-looking policy considered by Schmitt-Grohé and Uribe (2007) and Carlstrom and Fuerst (2005) in their discrete-time models[3].

Money enters the economy via a cash-in-advance constraint on consumption. Following Benhabib et. al (2002), and to avoid steady state multiplicity, attention is restricted to equilibria with a strictly positive nominal interest rate. This approach receives substantial support in Schmitt-Grohé and Uribe (2007)[4]. Finally, to keep the focus on labor channel effects and to abstract from cost-channel effects it is assumed throughout that nominal prices are flexible and that markets are perfectly competitive.

2.1. The Economic Environment

Households – The model is a continuous time, flexible price version of Schmitt-Grohé and Uribe (2007) with endogenous labor-leisure choice and endogenous investment. The economy is populated by a continuum of identical infinitely long-lived

[3] As we know, the instantaneous rate of inflation in a continuous-time setting is the right-derivative of the logged price level and thus, the discret-time counterpart of a countinuous-time policy rule that sets the interest rate in response to the instanteneous rate of inflation is characterized by forward-looking policy that responds to expected future inflation.

[4] Schmitt-Grohé and Uribe (2007) show that welfare is maximized where inflation target is close to zero. Consequently, the nominal rate of interest at the efficient steady state is positive.

households, with measure one. The representative household's lifetime utility is given by

(1)

$$U = \int_0^\infty e^{-\rho t} u(c, 1 - L) dt$$

where $\rho > 0$ denotes the rate of time preference, c denotes consumption. It is assumed that households are endowed with one unit of leisure. L and $l = 1 - L$ denote labor and leisure, respectively. $u(c,l)$ is twice differentiable, strictly increasing in both arguments and concave where consumption and leisure are Edgeworth substitutes $(u_{cl} < 0)$. We also assume that $u_{cc} - u_{cl} u_c / u_l < 0$ and $u_{ll} - u_{cl} u_l / u_c < 0$ which implies that consumption and leisure are normal goods. Apart from being plausible, this assumption is imperative in order to give rise to indeterminacies a-la Meng and Yip (2008).

It is also assumed that consumption and money balances are Edgeworth complements. To keep the analysis simple and tractable while maintaining the consumption-money complementarity assumption, money enters the economy via a cash-in-advance constraint on consumption[5]. In addition to money, households can store wealth in government-issued non-indexed bonds and physical capital. Bonds pay a net nominal interest of $R > 0$. Capital is either utilized for production or consumed paying an adjustment cost. The household's budget constraint is therefore described as:

$$c + I + \dot{b} + \dot{m} = (R - \pi) b - \pi m + f(k, L) - \tau \qquad (2)$$

where I is the flow of investment, b is the real value of government bonds, m is the real value of money balances, k is the stock of capital, τ is a real lump-sum tax and π is the rate of inflation. Note that all variables are time-dependent (the time argument is

[5] Feenstra (1986) demonstrates a functional equivalence between using real money as an argument of the utility function and entering money into a liquidity constraint. Specifically, he argues that cash-in-advance constraints can be viewed as a special case of a utility function that includes real balances with zero elasticity of substitution between goods and money.

omitted to keep notation simple). Finally it is assumed that the function, *f(k,L),* is twice differentiable, strictly increasing and displays a constant returns to scale production technology. We further assume that factor markets are competitive, thus, production factors are paid their marginal product. By defining $a \equiv b + m$ as the real value of non-capital wealth the household's budget constraint becomes:

$$\dot{a} = (R - \pi)a - Rm + rk + wL - c - I - \tau \tag{3}$$

Where r and w denote capital rent and labor compensation, respectively. The household's consumption is then subject to the cash-in-advance constraint $F(T) = \int_t^{t+T} c(s)ds \le m$. Normalizing T to 1 the CIA constraint can be approximated as:[6]

$$c \le m \tag{4}$$

Finally, assuming that the stock of capital depreciates at a rate δ, the household's lifetime maximization problem becomes

$$\begin{aligned}
&Max \int_0^\infty e^{-\rho t} u(c, 1-L)dt \\
&s.t. \\
&\dot{a} = (R - \pi)a - Rm + rk + wL - c - I - \tau \\
&\dot{k} = \varphi(I) - \delta k \\
&c \le m
\end{aligned}$$

With the following no-Ponzi-game condition $Lim_{t\to\infty} e^{-\int_0^t [R(s) - \pi(s)]ds} [a(t) + k(t)] = 0$

and where $\varphi(I)$ is increasing and concave with $\varphi(0)=0$. This specification suggests that

[6]This version of CIA is similar to Rebelo and Xie (1999). Formally, the cash-in-advance constraint is $F(T) = \int_t^{t+T} c(s)ds \le m$. A Taylor series expansion gives $F(T) = \mathrm{T}c(t) + \frac{1}{2}\mathrm{T}^2\dot{c}(t) + \cdots$ and so (4) can be interpreted as a first-order approximation. Finally, and without loss of generality to subsequent analysis, T is normalized to 1.

adjustments to the stock of capital are costly and that adjustment costs are increasing with investment and convex[7].

The household chooses sequences of *{c,I,L,m}* so as to maximize its lifetime utility, taking as given the initial stock of capital *k(0),* and the time paths of $\tau, R,$ and π.

An optimal program must choose *c,m,L* and *I* so as to maximize the current-value Hamiltonian $H_1 \equiv u(c,1-L)+\lambda\left[(R-\pi)a-Rm+rk+wL-c-I-\tau\right]+\mu\left[\varphi(I)-\delta k\right]+\xi(m-c)$ Thus, the necessary conditions for an interior maximum of the household's problem are

$$u_c(c,1-L)=\lambda+\xi \tag{5}$$

$$\frac{\mu}{\lambda}=\frac{1}{\varphi'(I)} \tag{6}$$

$$w=\frac{u_l(c,1-L)}{\lambda} \tag{7}$$

$$\xi=\lambda R \tag{8}$$

$$\xi \geq 0; \xi(m-c)=0 \tag{9}$$

Where λ_t and μ_t (in the current-value Hamiltonian the time subscripts are omitted to simplify notation) are co-state variables interpreted as the marginal valuation of a unit of financial assets and installed capital, respectively, and ξ_t is a Lagrange multiplier associated with the CIA constraint.

Second, the co-state variables must evolve according to the law

[7] In general, macroeconomic continuous time modeling could be misleading in the sense that it does not correctly approximate the behavior of the discrete time model of arbitrarily small periods. Carlstrom and Fuesrt (2005) point out that modeling policy issues in continuous time could end up with conclusions that are opposite to the conclusions drawn from a discrete-time counterpart of the model. They attribute the opposite conclusions to the difference in timing in the no-arbitrage condition of investing in bonds and capital between the two settings: while the continuous-time setting entails a contemporaneous no-arbitrage condition, a similar no-arbitrage condition in the discrete-time setting involves only future variables which bring a zero eigenvalue into the linearized dynamic system. One way to overcome contemporaneous features of no-arbitrage in continuous time macroeconomic models that enter at the "back door" as the period length gets shorter is to introduce adjustment costs to capital. For proof see Gliksberg (2009).

$$\dot{\lambda} = \lambda[\rho + \pi - R] \tag{10}$$

$$\dot{\mu} = -\lambda r + (\rho + \delta)\mu \tag{11}$$

Since attention is restricted to steady states with non negative inflation targets, the nominal rate of interest is positive near the steady state. As a result, equation (8) implies that ξ, the Lagrange multiplier associated with the cash-in-advance constraint in non zero. It then follows from (9) that $m=c$ and from (8) and (5) that $u_c(c,1-L) = \lambda(1+R)$. The interpretation is simple: near a steady state with positive nominal interest rate holding money entails opportunity costs. Thus, equilibrium amount of (real) money that minimizes the opportunity cost associated with money holding while still satisfying condition (4) is $m=c$. Accordingly, the law of motion for the real value of financial assets becomes

$$\dot{a} = (R - \pi)a + rk + wL - c(1 + R) - I - \tau \tag{12}$$

and the law of motion for capital is

$$\dot{k} = \varphi(I) - \delta k \tag{13}$$

We study equilibria close to the steady state and we assume that fiscal policy is passive [according to Leeper (1991)], therefore, the transversality condition holds.

Let η denote the ratio between the two co-state variables, $\eta \equiv \frac{\mu}{\lambda}$. Accordingly, $\dot{\eta} = \frac{\dot{\mu}}{\lambda} - \eta\frac{\dot{\lambda}}{\lambda}$, and thus, substituting in equations (10) - (11) and rearranging yields the evolution of η:

$$\dot{\eta} = -f_k(k,L) + \eta(R - \pi + \delta) \tag{14}$$

Equations (10), (13) – (14) fully describe the optimal instantaneous decision of the representative household as it takes the time paths of τ, R, and π as given.

The Government

Following Benhabib et. al. (2001) it is assumed that monetary policy takes the form of an interest-rate feedback rule whereby the nominal interest rate is set as an increasing function of instantaneous inflation. Specifically, it is assumed that

$$R = R(\pi) \tag{15}$$

Where *R(π)* is continuous, non-decreasing and strictly positive, and there exists a steady-state, (π^*), where $\pi^* > -\rho$ such that $R(\pi^*) = \rho + \pi^*$. It is further assumed that the monetary authority reacts to an increase in the rate of inflation by increasing the nominal rate of interest. That is, $\frac{\partial}{\partial \pi} R(\pi^*) > 0$. Following the terminology of Leeper (1991), the government operates under a passive fiscal policy and levy lump-sum taxs. It is assumed that government disbursements include purchases and interest payments over the outstanding debt. Thus, the consolidated government's nominal budget constraint is

$$Pg + RB = \dot{M} + \dot{B} + P\tau \tag{16}$$

where *B* and *M* denote nominal stocks of bonds and money, respectively, *P* is the level of nominal prices, and *g* denotes government purchases. The central bank imposes the desired interest rate by controlling the price of riskless nominal bonds and exchanging money for bonds at any quantities demanded at that price. In that sense, the nominal rate of interest is exogenous and the M and B are endogenous. Simple algebraic manipulations of equation (16) yield that $a \equiv m + b$ evolves according to:

$$\dot{a} = (R - \pi)a - Rm - (\tau - g) \tag{17}$$

Equilibrium – In equilibrium, the goods market, the factors market and the assets market clear. Equations (10), (13) – (14) imply that the evolution of λ, η and k fully describe a rational expectations equilibrium given the time paths of τ, R, and π as (exogenously) given. Thus, our discussion will focus on whether the representative agent can construct an isolated equilibrium trajectory in the (λ,η,k) space given a predetermined level of k and a common knowledge with respect to the stance of monetary policy.

Conjecture that an isolated equilibrium exists. Then, equations (18), (19) and (20) describe instantaneous equilibrium in the goods market, labor market and the assets market respectively:

$$f(k, L(\lambda,\eta,k)) = c(\lambda,\eta,k) + I(\lambda,\eta,k) + g \tag{18}$$

$$f_L(k, L(\lambda,\eta,k)) = \frac{u_l(c(\lambda,\eta,k), 1 - L(\lambda,\eta,k))}{\lambda} \tag{19}$$

$$\lambda = \frac{u_c(c(\lambda,\eta,k), 1 - L(\lambda,\eta,k))}{1 + R(\pi(\lambda,\eta,k))} \tag{20}$$

also, Equation (6) demonstrates that the rate of investment depends only on η. Thus,

$$\eta = \frac{1}{\varphi'(I)} \Rightarrow I = I(\eta) \tag{21}$$

2.2. **Equilibrium Dynamics**

Partially deriving equations (18)-(20) with respect to λ,η and k [appendix A.1] yields 10 equations with 10 unknowns. Solving this set yield:

(22)

$$I_\eta = -\frac{1}{\eta^2\varphi''(I)}, \quad c_\lambda = wL_\lambda, \quad c_\eta = wL_\eta - \frac{1}{\eta^2\varphi''(I)}, \quad c_k = f_k(k,L) + wL_k;$$

$$\pi_\lambda = -\frac{1 + R + L_\lambda(-wu_{cc}(c,1-L) + u_{cl}(c,1-L))}{\lambda R_\pi(\pi)}$$

$$\pi_\eta = \frac{\frac{1}{\eta^2\varphi''(I)}u_{cc}(c,1-L) - L_\eta(-wu_{cc}(c,1-L) + u_{cl}(c,1-L))}{\lambda R_\pi(\pi)}$$

$$\pi_k = \frac{f_k(k,L)u_{cc}(c,1-L) - L_k(-wu_{cc}(c,1-L) + u_{cl}(c,1-L))}{\lambda R_\pi(\pi)}$$

$$L_\lambda = -\frac{w}{\lambda f_{LL}(k,L) - wu_{cl}(c,1-L) + u_{ll}(c,1-L)}$$

$$L_\eta = \frac{u_{cl}(c,1-L)}{\eta^2\varphi''(I)\left(\lambda f_{LL}(k,L) - wu_{cl}(c,1-L) + u_{ll}(c,1-L)\right)}$$

$$L_k = \frac{-\lambda f_{kL}(k,L) + f_k(k,L)u_{cl}(c,1-L)}{\lambda f_{LL}(k,L) - wu_{cl}(c,1-L) + u_{ll}(c,1-L)}$$

Thus, the dynamics of all the variables in the economy can be described by (λ,η,k) which evolve according to:

$$\dot{\lambda} = F(\lambda,\eta,k)$$
$$\dot{\eta} = G(\lambda,\eta,k)$$
$$\dot{k} = H(\lambda,\eta,k)$$
$$\dot{a} = [R(\pi(\lambda,\eta,k)) - \pi(\lambda,\eta,k)]a(\lambda,\eta,k) - R(\pi(\lambda,\eta,k))m(\lambda,\eta,k) - \tau$$

where

$$F(\lambda,\eta,k) \equiv \lambda[\rho + \pi(\lambda,\eta,k) - R(\pi(\lambda,\eta,k))] \tag{23}$$

$$G(\lambda,\eta,k) \equiv -f_k(k, L(\lambda,\eta,k)) + \eta[R(\pi(\lambda,\eta,k)) - \pi(\lambda,\eta,k) + \delta] \tag{24}$$

$$H(\lambda,\eta,k) \equiv \varphi(I(\eta)) - \delta k \tag{25}$$

and where the transversality condition is $Lim_{t\to\infty} e^{-\int_0^t [R(\pi(\lambda,\eta,k)) - \pi(\lambda,\eta,k)]ds}[a(\lambda,\eta,k) + k] = 0$

3. Equilibrium and Local Real Determinacy

Linear models with infinite horizon generally admit infinitely many rational expectations solutions. Consequently, some selection devices are needed to narrow the set of applicable solutions. This paper emphasizes a selection device according to Evans and Guesnerie (2005) who show that the saddle-path solution is bound to be selected from a multiple of applicable solutions.

Definition - *Equilibrium displays Local-Real-Determinacy (LRD) if there exists a Saddle-Path stable solution in the* (λ, η, k) *space.*[8] *Otherwise equilibrium displays Non-LRD.*

Local Real Determinacy

Equations (5), (7), (23)–(25) imply that in the steady state $R(\pi^*) = \rho + \pi^*$, $\lambda^* = \dfrac{u_c(c^*, 1-L^*)}{1+\rho+\pi^*}, \eta^* = \dfrac{f_k(k^*, L^*)}{\rho+\delta}$, $\varphi(I(\eta^*)) = \delta k^*$, $w^* = \dfrac{u_l(c^*, 1-L^*)}{\lambda^*}$ [asterisk throughout denote steady state levels]. Also, a linear approximation of the dynamic system near the steady state (λ^*, η^*, k^*) is obtained through the system

$$\begin{bmatrix} \dot{\lambda} \\ \dot{\eta} \\ \dot{k} \end{bmatrix} = \underbrace{\begin{bmatrix} -(R^*_\pi - 1)\lambda^* \pi^*_\lambda & -(R^*_\pi - 1)\lambda^* \pi^*_\eta & -(R^*_\pi - 1)\lambda^* \pi^*_k \\ -f^*_{kL} L^*_\lambda + (R^*_\pi - 1)\eta^* \pi^*_\lambda & \rho + \delta - f^*_{kL} L^*_\eta + (R^*_\pi - 1)\eta^* \pi^*_\eta & -f^*_{kk} - f^*_{kL} L^*_k + (R^*_\pi - 1)\eta^* \pi^*_k \\ 0 & -\dfrac{\varphi'(I^*)}{\varphi''(I^*)\eta^{*2}} & -\delta \end{bmatrix}}_{A} X \begin{bmatrix} \lambda - \lambda^* \\ \eta - \eta^* \\ k - k^* \end{bmatrix}$$

where A is the Jacobian of equations (23)-(25) and R^*_π denotes $\dfrac{\partial R(\pi)}{\partial \pi}$ at π^*.

[8]Isolated equilibria that are not saddle-path stable are ignored. Evans and Guesnerie (2005) show that rational agents coordinate on saddle-path solutions. LRD also excludes non-isolated equilibria that either converge to the steady state or cycle (both periodically and non-periodically) near the steady state.

The solution of the characteristic equation of A depends on R_π^* that exhibits the stance of monetary policy near the steady state. Note that

$$\theta_1\theta_2\theta_3 = Det(A) = -(R_\pi^* - 1)\lambda^* Det\underbrace{\begin{bmatrix} \pi_\lambda^* & \pi_\eta^* & \pi_k^* \\ -f_{kL}^* L_\lambda^* & \rho + \delta - f_{kL}^* L_\eta^* & -f_{kk}^* - f_{kL}^* L_k^* \\ 0 & -\dfrac{\varphi'(I^*)}{\varphi''(I^*)\eta^{*2}} & -\delta \end{bmatrix}}_{B}$$

$$= -\frac{R_\pi^* - 1}{R_\pi^*} Det\underbrace{\begin{bmatrix} -1 + R - \Omega^* L_\lambda^* & \dfrac{1}{\varphi''(I^*)\eta^{*2}} u_{cc}^* - \Omega^* L_\eta^* & f_k^* u_{cc}^* - \Omega^* L_k^* \\ -f_{kL}^* L_\lambda^* & \rho + \delta - f_{kL}^* L_\eta^* & -f_{kk}^* - f_{kL}^* L_k^* \\ 0 & -\dfrac{\varphi'(I^*)}{\varphi''(I^*)\eta^{*2}} & -\delta \end{bmatrix}}_{\hat{B}}$$

where

$$\Omega^* \equiv -w^* u_{cc}(c^*, 1 - L^*) + u_{cl}(c^*, 1 - L^*)$$

And that

$$\theta_1 + \theta_2 + \theta_3 = Trace(A) = (R_\pi^* - 1)(\eta^* \pi_\eta^* - \lambda^* \pi_\lambda^*) + \rho - f_{kL}^* L_\eta^*$$

Where $\theta_i, i = 1,2,3$ denote the eigenvalues of A. Thus, with one predetermined variable, a necessary condition for LRD is that $\theta_1\theta_2\theta_3 = -\dfrac{R_\pi^* - 1}{R_\pi^*} Det(\hat{B}) < 0$.

4. Solution, Calibration, and Computation

According to equations (5) and (7) the shadow real wage as seen by the household is

$$w(c,L)=\frac{u_l(c,1-L)}{u_c(c,1-L)}(1+R).$$

Define the elasticity of the wage with respect to consumption when labor is held constant as $\zeta(c,\overline{L})\equiv\frac{\partial\ln(w(c,\overline{L}))}{\partial\ln(c)}=c\frac{u_{cl}}{u_l}-c\frac{u_{cc}}{u_c}$ and the elasticity of intertemporal substitution for consumption $s(c,\overline{L})$ and the consumption-constant elasticity of labor supply $h(\overline{c},L)$ by

$$\frac{1}{s(c,\overline{L})}\equiv -c\frac{u_{cc}}{u_c}\quad\text{and}\quad\frac{1}{h(\overline{c},L)}\equiv\frac{\partial\ln(w(\overline{c},L))}{\partial\ln(L)}=-L\frac{u_{ll}}{u_l}+L\frac{u_{cl}}{u_c}.$$

Finally define $\omega(c,L)\equiv\frac{w(c,L)L}{c}=\frac{L}{c}\frac{u_l(c,1-L)}{u_c(c,1-L)}(1+R)$ and note that $\lambda=\frac{u_c(c,1-L)}{1+R}$

Since the Hessian $\begin{bmatrix}u_{cc} & u_{cl}\\ u_{cl} & u_{ll}\end{bmatrix}$ is symmetric, the three elasticities, $\zeta(c,\overline{L})$ $s(c,\overline{L})$ and $h(\overline{c},L)$, plus $\omega(c,L)$ and λ determine all local first and second derivatives of the function $u(c,1-L)$ apart from overall scale of felicity which has no economic meaning. Accordingly, simple algebra obtains

$$u_c{}^*=\lambda^*(1+R^*);\quad u_l{}^*=\lambda^* w^*;\quad u_{cc}{}^*=-\frac{\lambda^*(1+R^*)}{c^*s^*};\quad u_{cl}{}^*=\frac{w^*\left(s^*\zeta^*-1\right)\lambda^*}{c^*s^*};$$
$$u_{ll}{}^*=\frac{w^*}{L^*}\left[-\frac{\lambda^*}{h^*}+\frac{\omega^*\left(s^*\zeta^*-1\right)\lambda^*}{(1+R^*)s^*}\right]$$

where asterisk denote steady state levels[9]. A similar exercise obtains that

$$I^*=\frac{c^*(\alpha+\omega-1)}{1-\alpha};\ f_k{}^*=\frac{c^*\alpha\omega}{k^*(1-\alpha)};\ f_{kk}{}^*=-\frac{c^*\alpha\omega}{k^{*2}};\ f_{kL}{}^*=\frac{c^*\alpha\omega}{k^*L^*};\ f_{LL}{}^*=-\frac{c^*\alpha\omega}{L^{*2}}$$

[9] The utility is separable in its arguments iff $s^*\zeta^*=1$ whereas $s^*\zeta^*<1$ imply that consumption and labor are complements.

(proof in appendix A.2)

and

$$\varphi(I^*) = \delta k^*; \quad \varphi'(I^*) = \frac{k^*(1-\alpha)(\rho+\delta)}{c^*\alpha\omega}; \quad \varphi''(I^*) = -\frac{2k^*(1-\alpha)^2\left(\rho(1-\alpha-\omega)+\delta(\alpha-1)(\omega-1)\right)}{c^{*2}\alpha\omega(\alpha+\omega-1)^2}$$

(proof in appendix A.3)

Where $\alpha,\delta,\rho,\omega,\zeta,h,s$ are obtained from estimates of stationary time series.[10],

At this point, finding the restrictions on monetary policy so as to induce LRD is straight forward: First, steady state levels $u_c^*, u_l^*, u_{cc}^*, u_{cl}^*, u_{ll}^*, I^*, f_k^*, f_{kk}^*, f_{kL}^*, f_{LL}^*$,

$\varphi(I^*), \varphi'(I^*), \varphi''(I^*)$ are substituted into equation (22) to obtain

$I_\eta^*, c_\lambda^* c_\eta^* c_k^*, L_\lambda^*, L_\eta^*, L_k^*, \pi_\lambda^*, \pi_\eta^*, \pi_k^*$ that depend only on the long run estimate of

$\alpha,\delta,\rho,\omega,\zeta,h,s$. Then, steady state levels

$I_\eta^*, c_\lambda^* c_\eta^* c_k^*, L_\lambda^*, L_\eta^*, L_k^*, \pi_\lambda^*, \pi_\eta^*, \pi_k^*, \varphi''(I^*)$ are substituted into the Jacobian matrix A.

Table 1 – Structural Parameters and Elasticities

Parameter	Range	Description
s	[0.2,0.5]	Elasticity of intertemporal substitution
h	[1,2]	Consumption-constant elasticity of labor supply with respect to wages
ζ	[0.8,1.2]	Labor-constant elasticity of wages with respect to consumption
ω	[0.8,0.9]	Long-run ratio of wage income to consumption
α	0.3	Capital share of output
ρ	0.03	Rate of subjective time preference
δ	0.1	Rate of capital depreciation

[10] $\varphi''(I)<0$. Thus, the values of $\alpha,\delta,\rho,\omega$ should be calibrated so as to ensure that $\varphi''(I)$ is negative at the steady-state investment. Specifically, $\varphi''(I^*) < 0 \Leftrightarrow \omega < \dfrac{(1-\alpha)(\rho+\delta)}{\rho+\delta-\alpha\delta}$

.

According to the theory, ω equals labor income divided by consumption expenditure. Taking nominal wages and salaries from the U.S National Income Accounts and dividing by nominal spending on non-durable consumption and services gives an average ratio of 0.9. Basu and Kimball (2002) use the prices as perceived by consumer so they define ω using after-tax wage. Thus they use ω=0.8 as their preferred value. It should be noted however, that in the present context, results are robust to ω as long as $\omega < \frac{(1-\alpha)(\rho+\delta)}{\rho+\delta-\alpha\delta}$ which is essential to ensure that the investment technology is concave. For that matter, and given the assumed values of α,ρ,δ, ω must be less than 0.91.

For all values specified in Table 1 $Det(\hat{B}) > 0$ which imply that the Taylor principle is necessary to ensure local real determinacy. Furthermore, the upshot of explicitly solving the characteristic equation of A for values of Table 1 is that an active policy stance, $R_{\pi}^{*} > 1$, induces two unstable eigenvalues and one stable eigenvalue and hence local-real-determinacy. A neutral policy stance, $R_{\pi}^{*} = 1$, brings about one stable eigenvalue, one zero eigenvalue and one unstable eigenvalue and hence local-real-determinacy. A passive policy stance, $R_{\pi}^{*} < 1$, brings about two stable eigenvalues and one unstable eigenvalue and hence equilibrium is Non-LRD[11].

[11] Methematica© code used to calculate the results is available at request from the author.

5. **Conclusion**

This paper considers local real determinacy (LRD) as a prerequisite for macroeconomic stability. LRD was examined under a simple forward-looking interest rate feedback rule and a passive fiscal-policy with non distorting taxation where both labor supply and investment are endogenous. The focus is on labor market dynamics that derive from complementarities between consumption and hours worked. The model is specified so as to apply for a wide range of utilities through transformation from ethereal expressions to long run elasticities.

The model gives rise to a labor market channel. This channel of monetary policy, in an economy calibrated to the US economy, was found to act together with the arbitrage channel of monetary policy. The Taylor principle is shown to induce LRD. Furthermore, a neutral policy stance, such that the expected real rate of interest is constant in and off the steady state, achieves a similar upshot.

Appendix A

A.1

$$I_\eta = -\frac{1}{\eta^2 \varphi''(I)}$$

$$w L_\lambda = c_\lambda$$

$$w L_\eta = c_\eta + I_\eta$$

$$f_k(k,L) + w L_k = c_k$$

$$1 + R = u_{cc}(c,1-L)\, c_\lambda - u_{cl}(c,1-L)\, L_\lambda - \lambda R_\pi(\pi)\, \pi_\lambda$$

$$u_{cc}(c,1-L)\, c_\eta - u_{cl}(c,1-L)\, L_\eta = \lambda R_\pi(\pi)\, \pi_\eta$$

$$u_{cc}(c,1-L)\, c_k - u_{cl}(c,1-L)\, L_k = \lambda R_\pi(\pi)\, \pi_k$$

$$\lambda f_{LL}(k,L)\, L_\lambda = -w + u_{cl}(c,1-L)\, c_\lambda - u_{ll}(c,1-L)\, L_\lambda$$

$$\lambda f_{LL}(k,L)\, L_\eta = u_{cl}(c,1-L)\, c_\eta - u_{ll}(c,1-L)\, L_\eta$$

$$\lambda(f_{kk}(k,L) + f_{LL}(k,L)\, L_k) = u_{cl}(c,1-L)\, c_k - u_{ll}(c,1-L)\, L_k$$

A.2

It is assumed throughout that a large number of firms, each of which produces a homogenous commodity using a constant returns-to-scale production-technology, operate in a competitive setup of markets for goods and factors. Accordingly, $f(k,L) = f_k(k,L)k + f_L(k,L)L$. And, the capital share of output is $\frac{f_k(k,L)k}{f_k(k,L)k + w(c,L)L} = \alpha$.

Taking partial derivatives of both sides of $f(k,L) = f_k(k,L)k + f_L(k,L)L$ and assuming the market clearing conditions for goods and factor markets, we obtain a set of equations

$$\begin{aligned}
f(k,L) &= f_k(k,L)k + w(c,L)L \\
\frac{f_k(k,L)k}{f(k,L)} &= \alpha \\
f_{kk}(k,L) &= -\frac{(1-\alpha)f_k(k,L)}{k} \\
f_{LL}(k,L) &= -\frac{\alpha w(c,L)}{L} \\
f_{kL}(k,L) &= -f_{LL}(k,L)\frac{L}{k} \\
f(k,L) &= c + I
\end{aligned}$$

Of which the solutions with respect to I and the partial derivatives of f are

$I^* = \frac{c^*(\alpha+\omega-1)}{1-\alpha}$; $f_k{}^* = \frac{c^*\alpha\omega}{k^*(1-\alpha)}$; $f_{kk}{}^* = -\frac{c^*\alpha\omega}{k^{*2}}$; $f_{kL}{}^* = \frac{c^*\alpha\omega}{k^*L^*}$; $f_{LL}{}^* = -\frac{c^*\alpha\omega}{L^{*2}}$ where

$\omega \equiv \frac{w^*L^*}{c^*}$ and asterisk denote steady state levels

A.3

A second order Taylor expansion of the investment technology $\varphi(I)$:

$\varphi(I) = \varphi(I^*) + \varphi'(I^*)(I-I^*) + \frac{1}{2}\varphi''(I^*)(I-I^*)^2$ Specifically it is used to approximate $\varphi(0)=0$. Rearranging the Taylor expansion for $\varphi(0)=0$ implies that $\varphi''(I^*) = 2\frac{\varphi'(I^*)I^* - \varphi(I^*)}{I^{*2}}$. Substituting in $\varphi(I^*) = \delta k^*$; $\varphi'(I^*) = \frac{k^*(1-\alpha)(\rho+\delta)}{c^*\alpha\omega}$

yields $\varphi''(I^*) = -\frac{2k^*(1-\alpha)^2\left(\rho(1-\alpha-\omega) + \delta(\alpha-1)(\omega-1)\right)}{c^{*2}\alpha\omega(\alpha+\omega-1)^2}$

References

Basu, S. and M. Kimball, 2002, Long-Run Labor Supply Elasticity and the Elasticity of Substitution for Consumption, University of Michigan working paper.

Benhabib J., Schmitt-Grohé S., and Uribe M., 2001, Monetary Policy and Multiple Equilibria, American Economic Review, 91,167-186.

Benhabib J., Schmitt-Grohé S., and Uribe M., 2002, Avoiding Liquidity Traps, Journal of Political Economy, 110, 535-563.

Benhabib J. and Y. Wen, 2004, Indeterminacy, Aggregate Demand, and the Real Business Cycle, Journal of Monetary Economics, 51, 503-530

Bennett R. L., R. Farmer, 2000, Indeterminacy with Non-Separable Utility, Journal of Economic Theory 93, 118-143

Carlstrom , C.T. and Fuerst T.S., 2005, Investment and Interest Rate Policy: A Discrete Time Analysis, Journal of Economic Theory, 123(1), 4-20.

Cogley T. and Nason, J., 1995. Output dynamics in real-business-cycle models. American Economic Review, 85, 492–511.

Dupor, B., 2001, Investment and Interest Rate Policy, Journal of Economic Theory, 98, 81-113.

Evans, G.W. and R. Guesnerie, 2005, Coordination on saddle-path solutions: the eductive viewpoint- linear multivariate models, Journal of Economic Theory, 124, 202-229.

Feenstra, R.C., 1986, Functional Equivalence between Liquidity Costs and the Utility of Money, Journal of Monetary Economics, 17 , 271-291

Gliksberg, B., 2009, Monetary Policy and Multiple Equilibria with Constrained Investment and Externalities, Economic Theory, 41, 443-463

Hall, R., 2008,Aggregate Implications of Indivisible Labor, Incomplete Markets, and Labor-Market Frictions, Journal of Monetary Economics, 55, 980-82

Huang, K. X. D. and Q. Meng, 2007, Capital and Macroeconomic Instability in a Discrete-Time Model with Forward-Looking Interest Rate Rules, Journal of Economic Dynamics and Control, 31(8), 2802-26.

Kimball, M. and M.D. Shapiro, 2008, Labor Supply: Are the Income and Substitution Effects both Large or Both Small? NBER working paper series w14208.

Leeper, E.M., 1991, Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies, Journal of Monetary Economics, 27, 129-147

Linnemann L., 2008, Balanced Budget Rules and Macroeconomic Stability with Non-separable Utility, Journal of Macroeconomics, 30, 199-215

Meng, Q. and C.K. Yip, 2004, Investment, Interest Rate Rules, and Equilibrium Determinacy, Economic Theory, 23, 863-878.

Meng, Q. and C.K. Yip, 2008, On Indeterminacy in One-Sector Models of the Business Cycle with Factor-Generated Externalities, Journal of Macroeconomics, 30, 97-110

Raj, C., 2006, A New Method of Estimating Risk Aversion, American Economic Review, 96(5),1821-34.

Rebelo, S. and D. Xie, 1999, On the Optimality of Interest Rate Smoothing, Journal of Monetary Economics, 43, 263-82

Schmitt-Grohé S., and Uribe M., 1997, Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability, Journal of Political Economy, 105, 976-1000

Schmitt-Grohé, S. and M. Uribe, 2007, Optimal Simple and Implementable Monetary and Fiscal Rules, Journal of Monetary Economics, 54, 1702-1725

Rotemberg, J. and Woodford, M., 1996. Real-business-cycle models and the forecastable movements in output, hours, and consumption. American Economic Review 86,71–89.