Abstract

We are interested in why minorities are so often at a disadvantage relative to the majority. Efforts made to assimilate, and time, are two elements that work to bring minorities into line with the majority. A third element, the degree to which the majority welcomes the minority, also plays a role. We examine the consequences for assimilation and harassment of growth in the minority population, time, and the role of a political entity. Over time, the assimilated and the non-assimilated members of the minority group exhibit different interests in assimilating and in maintaining their cultural identity, and we discuss how this affects the minority’s position over time.

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Minorities, Assimilation and Harassment

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Introduction
Minority groups participation in labor markets is quite complex and in many ways different from that of majority citizens. Studies of minorities around the world show, with few exceptions, that they tend to earn wages substantially below those of comparable majority workers. Partly, this reflects a failure on the part of the minority group to undertake the effort to assimilate with the majority. The “lack of effort” can arise from the desire to maintain a cultural heritage or separate identity which would be lost or reduced if the group assimilated. The failure to take active steps to assimilate can also arise in the face of high adjustment costs, such as inadequate language skills, intergenerational familial conflicts, and, in the case of immigrants, lack of knowledge about the host country labor market.

Various indicators are used to measure the degree to which minority groups have assimilated. The most common measures in the economics literature are wages and earnings, and there is an extremely large literature that examines the rate and degree of decline in wage and earnings differences among groups. Recently employed other indicators include labor force participation, poverty, and education, more frequently now being looked at over several generations. Moreover, for immigrants and their descendants, as length of time in the host country increases, assimilation occurs and immigrant earnings tend to approach those of comparable majority workers. On occasion, minority workers outperform majority workers.

Efforts made to assimilate, and time, are two elements that work to bring minorities into line with the majority. A third element, the degree to which the majority welcomes the minority, also plays a role. Often, the majority is less than welcoming, blaming the minority for depressing wages and displacing majority
workers – i.e., causing majority unemployment. This presumption has very strong policy implications and is implicit in the calls for increased regulation of immigration heard worldwide. Yet, there is mixed evidence on the impact of minorities on majority wages and employment – it depends on whether they are substitutes or complements (Gang and Rivera-Batiz 1994). Whether minorities actually lower wages and increase employment, or not, the perception exists. Because of this perception the majority may take active steps to discourage minority assimilation – discrimination, isolation, and so on. We refer to these majority activities as harassment.

We are interested in why minorities are so often at a disadvantage relative to the majority, and the circumstances under which their status changes or stagnates over time. Assimilation efforts, harassment and time are the three elements that determine how well the minority is doing compared to the majority. We proceed by constructing a model in which there are four actors: the members of the majority and the organization that represents them, and members of the minority and the organization that represents them. These organizations (or political entities) are institutions that are able to overcome the free-rider problem individual members of each group have in moving from the actions they desire to take, to actually taking the actions. The organizations could be, for example, political parties, trade organizations, unions, or thugs. While the organizations may represent the members of the group, the interests’ of the organization and that of its members do no always coincide.

We examine the consequences for assimilation and harassment of increases in the numbers of members of the minority, time, and the role of the political entity. Over time, the political entity representing the minority and the members of the minority exhibit different interests in assimilating and in maintaining their cultural identity. We discuss how this affects the minority’s position over time. Our work adds to the blossoming literature on majority – minority conflict and resolution, assimilation, and the reestablishment of cultural identity (see, for example, Alesina and La Ferrara, 2000, Anas, 2002, Bisin and Verdier, 2000, Dustmann, Fabbri and Preston, 2004, Kahanec, 2004, and Lazear, 1999).
The model
Consider a firm that has two factors of production: workers who are part of the majority, \( L_m \), and workers who are members of a minority, \( L_m \). For simplicity, we assume that there is only one minority group.\(^1\) We use the term minority and majority in terms of power. The whites in South Africa under apartheid are to be seen as the majority while the blacks are the minority. We normalize the efficiency level of majority workers to unity; the minority’s productive/efficiency level equals \( g(\cdot)Pr(\cdot) \). The two functions, \( g(\cdot) \) and \( Pr(\cdot) \), play important roles in the determination of production and wages; aside from these, labor is homogeneous.\(^2\)

\( Pr(\cdot) \), where \( 0 \leq Pr(\cdot) \leq 1 \), is a function of two elements: (1) The effort invested by the majority in order to prevent the minority from assimilating into the majority. These activities include harassing members of the minority, not cooperating with them, discriminating against them, and so on (hereafter, "harassment"). Such activities decrease the minority’s productivity and thus their efficiency.\(^3\) Denote the harassment level by \( h \). (2) The effort invested by the minority in assimilating. These activities affect the minority's efficiency level positively. The more the minority assimilates, its productivity increases, as cooperation increases between the majority and the minority. Denote assimilation activity by \( a \).\(^4\)

We further assume that over time, in a natural way, the minority assimilates or the majority gets used to them and views them more as equals (Chiswick, 1978, Duleep and Regets, 2002). We therefore introduce an element of time into the minority’s productivity. Productivity increases with time, though it cannot be higher than the majority’s. Thus \( g(\cdot) \) is such that for \( t \geq 1, \ 0 < g(t) \leq 1 \).\(^5\) An example of such

\(^1\) We use the terms minority and majority to refer to both the groups as a whole and individual members of each group.
\(^2\) Introducing productivity/efficiency as increased by diversity would enhance our results.
\(^3\) This is similar to the cooperation and harassment activities described in insider-outsider theory (Lindbeck and Snower, 1998).
\(^4\) Assimilation is not always beneficial for the minority; see Epstein (2003) for a discussion of migrant assimilation. For now, we ignore such possibilities; we will return to discuss them later in this paper.
\(^5\) If we allow the minority to be more productive/efficient than the majority, corresponding to the popular opinion about Asians and other groups in America, our results below will be more extreme.
a function is \( g(t) = e^{-\frac{t}{\tau}}, \) with \( \tau > 0. \) In this example, over a long period of interaction between the minority and the majority, \( t = \infty, \) the time weight equals one unit, \( g(t) = 1. \) For all other levels of \( t, \) \( t < \infty, \) the weight is less than one, \( g(t) < 1. \)

The productivity weight that the minorities receive equals \( g(\cdot) \Pr(\cdot). \) This proportion reflects the productivity and efficiency of the minority relative to the majority group, with the majority group investing in harassment activities and the minority investing in assimilation activities (\( a \) and \( h \) are positive). We focus on the unique interior Nash equilibria. The function \( \Pr(\cdot) \) has the following properties:

\[
\frac{\partial \Pr(h, a)}{\partial h} < 0 \quad \text{and} \quad \frac{\partial \Pr(h, a)}{\partial a} > 0.
\]

The representative firm's production function at time \( t \) \((t \geq 1)\) is given by

\[
Q_t(L) = f(L_n + g(t)\Pr(h, a)L_m),
\]

such that

\[
\frac{\partial f(L)}{\partial L} > 0, \quad \frac{\partial^2 f(L)}{\partial L^2} < 0, \quad \frac{\partial^3 f(L)}{\partial L^3} = 0, \quad \frac{\partial g(t)}{\partial t} > 0 \quad \text{and} \quad \frac{\partial^3 g(t)}{\partial t^3} < 0.
\]

We assume decreasing returns to scale for labor. Moreover, we assume that the third derivative of the production function with respect to labor equals zero, \( \frac{\partial^3 f(L)}{\partial L^3} = 0. \)

This assumption simplifies our calculations. Below we show where this assumption is used and that it is not critical for our results.

Let \( W_n \) be the majority worker's wage, and \( W_m \) be the minority's wage. We assume that the wages the majority and minority receive equals their marginal product values. We could assume that the majority workers have market power over their employers; this would not change our results.

Normalizing the price of the product to unity, the profits of the firm are given by
The first order conditions for maximization are

\[ \frac{\partial \pi}{\partial L_n} = f' - W_n = 0 \Rightarrow f' = W_n, \]

and,

\[ \frac{\partial \pi}{\partial L_m} = g(t) \Pr(h, a) f' - W_m = 0 \Rightarrow g(t) \Pr(h, a) f' = W_m. \]

Equation (4) represents the wage condition for majority workers and (5) represents the wage condition for minority workers.

**Harassment and majority utility**

We specify the majority’s utility quite simply as

\[ u_n(\cdot) = C_n, \]

where \( C_n \) is the majority’s consumption level. We assume that each worker consumes his entire income in each period; this means that the consumption level equals the majority's wage level (the price level is normalized to unity). Therefore,

\[ u_n(\cdot) = C_n = f'(L_n + g(t) \Pr(h, a) L_m). \]

Utility is a positive function of harassment activities by the majority and a negative function of the assimilation activities by the minority. Each of the majority individuals would want to invest in harassment activities, however, as a result of free riding it is not rational for each individual separately to do so.

Now assume that there exists a political entity that represents the majority. The utility of this entity is a positive function of the worker’s utility, \( u_n(\cdot) \). The total quantity of harassment activity set by this political entity equals \( h \). Denote the utility of the political entity representing the majority by \( U_N(\cdot) \). The political entity is able to overcome the free rider problem.
The political entity representing the majority, hereafter the “majority”, determines the level of harassment so as to maximize utility. From (4) it is clear that increasing the level of harassment decreases the productivity level of the minority. This is similar to decreasing the labor force available to the firm, thus increasing the wage level of the majority.6

The majority can benefit from harassing the minority, as these activities will increase their own wages. On the other hand, there is a cost to harassment that decreases the utility of the majority.

One could also think of majority utility as increasing in the harassment level, as the majority may have positive utility just from harassment. This may be the case for some in the majority group; however, overall we assume that it costs majority group to participate in harassment activities. These activities take time and effort and thus decrease the utility of the majority. The utility of the political entity representing the majority is thus a function of the majority representative agent and the level of harassment activities: \( U_N(u_n, h) \). To simplify we assume, using (4), (6) and (7), that the utility of the political entity representing the majority’s utility can be written as7

\[
U_N(.) = f'(L_n + \Pr(h,a)g(t)L_m) - h,
\]

where \( f'(L_n + \Pr(h,a)g(t)L_m) = \frac{\partial f(L_n + \Pr(h,a)g(t)L_m)}{\partial L} \).

The majority’s objective is to maximize its utility by determining its optimal harassment level. The first order condition determining the optimal harassment level is given by \( \frac{\partial U_N(.)}{\partial h} = 0 \), thus

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6 We could also model the behavior of a second political entity, one that represents the interests of capital owners (the firm) in the majority. The capital owners would react like the minority (since the minority decreases the firm’s marginal costs), so the results would not change.

7 Assuming a general function such as \( U_N \left( f'(L_n + \Pr(h,a)g(t)L_m) - h \right) \) would not change the results.
\( \Delta_N = \frac{\partial \Pr(h, a)}{\partial h} g(t) f'''(L_n + g(t) \Pr(h, a) L_m) L_n L_m - 1 = 0 , \)

where \( f'''(L_n + \Pr(h, a) g(t) L_m) = \frac{\partial^2 f(L_n + g(t) \Pr(h, a) L_m)}{\partial L^2} . \)

The first order condition therefore satisfies,

\[ \frac{\partial \Pr(h, a)}{\partial h} = \frac{1}{f'''(L_n + g(t) \Pr(h, a) L_m) g(t) L_n L_m} . \]

Remember that both \( \frac{\partial \Pr(h, a)}{\partial h} \) and \( f'''(L_n + \Pr(h, a) g(t) L_m) \) are negative.

In order for the harassment level determined in (9) to maximize the majority's utility, the second order condition must hold. The second order condition for maximization is given by \( \frac{\partial^2 U_m (L)}{\partial h^2} < 0 . \) Thus,

\[ \frac{\partial^2 \Pr(h, a)}{\partial h^2} f'''(L_n + g(t) \Pr(h, a) L_m) g(t) L_n L_m + \]

\[ \left( \frac{\partial \Pr(h, a)}{\partial h} \right)^2 f''''(L_n + g(t) \Pr(h, a) L_m) g(t)^2 L_n^2 L_m^2 < 0 . \]

Under the assumptions made above, see (2), the third derivative of the production function with respect to the labor equals zero, \( \frac{\partial^3 f(L)}{\partial L^3} = 0 . \) This assumption simplifies matters; alternatively, it can be assumed that \( \left( \frac{\partial \Pr(h, a)}{\partial h} \right)^2 f''''(L_n + g(t) \Pr(h, a) L_m) g(t)^2 L_n^2 L_m^2 \) is very small or equals zero.

The second order condition can thus be written as

\[ \frac{\partial^2 \Pr(h, a)}{\partial h^2} f''''(L_n + g(t) \Pr(h, a) L_m) g(t) L_n L_m < 0 . \]
From (2) we know that $f''(L_m + g(t)Pr(h,a)L_m) < 0$, so in order for the second order conditions to hold it must be that

$$\frac{\partial^2 \Pr(h,a)}{\partial h^2} > 0 \quad \text{or} \quad \frac{\partial^2 (1-\Pr(h,a))}{\partial h^2} < 0.$$  

Namely, the proportion of the minority that does not assimilate $(1-\Pr(h,a))$ faces decreasing returns to harassment, $h$.\(^8\)

**Assimilation and minority utility**

The utility of the minority is of the same form of that of the majority. We specify the minority’s utility function simply as

$$u_m() = C_m,$$

where $C_m$ is the minority’s consumption level. We assume that each worker consumes his entire income in each period. Therefore,

$$u_m() = W_m = g(t)Pr(h,a)f''(L_m + g(t)Pr(h,a)L_m).$$

Utility is a negative function of harassment and a positive function of assimilation activities. Each minority individual would want to invest in assimilation; however, while it is rational for all together to invest, as a result of free riding it is not rational for each individual separately to do so.\(^9\)

Now assume that there exists a political entity that represents the minority and is able to overcome the free rider problem. The political entity might be a group representing minority rights, interests, and so on. The utility of this entity is a positive

\(^8\) The proportion of the minority that has assimilated affects both the wages of the minority and majority.

\(^9\) It has been frequently noted that while people may hold prejudices about a group, often they do not hold these views about individuals from the group whom they know. The costly assimilation we refer to here is in terms of how the local population thinks of the minority as a group. Individuals in the minority may benefit from, for example, learning the language and will be willing to invest.
function of the number of assimilated minority workers. Assimilation has many
benefits for the minority group: first, it increases their wages; second, if part of the
minority assimilates, the benefits of their assimilation will provide the rest of the
minority with benefits – this arises because the minority group can act as a network
(Rauch, 2001); and third, for international traders such connections help in importing
and exporting products at lower costs. In this last example, if the minority consists of
a group of immigrants, as they assimilate into the host country, network externalities
increase, enabling increased profits and increased imports, thus increasing the utility
of the international importer (exporter) minority (in other words, there is an extra
externality, see Epstein and Gang, 2005). In general, the utility of the political entity
representing the minority will be a function of the degree to which the minority
population has assimilated into the majority, \(g(t)\Pr(h, a) L_m\).

Denote the utility of the political entity representing the minority by
\(U_M (g(t)\Pr(h, a) L_m, a)\) such that,

\[ U(M)(\cdot) = R(g(t)\Pr(h, a) L_m) - a. \]

\(R(.\) is the rent associated with the assimilation of the minority. This level of
assimilation is represented by the term \(g(t)\Pr(h, a) L_m\). We assume that as the level
of assimilation increases, \(g(t)\Pr(h, a) L_m\) increases, that is, the rent also increases.

Some of the minority participates in assimilation activities and thus have a cost of \(a\)
for each \(a\) unit of effort for the purpose of assimilating. To simplify, we assume for
now that the rent equals \(R(g(t)\Pr(h, a) L_m) = r g(t)\Pr(h, a) L_m\). Therefore, the utility
of the political entity representing the minority group becomes

\[ U(M) = r \Pr(h, a) g(t) L_m - a. \]

As assumed above, as assimilation activities increase \(\Pr(h, a)\) increases. The first
order condition for maximization of minority utility is given by \(\frac{\partial U(M)}{\partial a} = 0\),
namely,
The first order condition is satisfied if

\[
\Delta_M = \frac{\partial \Pr(h, a)}{\partial a} \frac{1}{g(t)L_m r} = 0.
\]

Remember we assumed that \(\frac{\partial \Pr(h, a)}{\partial a} > 0\), see (1). In order to insure that the solution is the level that maximizes minority utility it must hold that \(\frac{\partial^2 U_M}{\partial a^2} < 0\).

Therefore, it must also hold that

\[
\frac{\partial^2 \Pr(h, a)}{\partial a^2} < 0.
\]

In other words, (20) assumes that there are decreasing returns to investing in assimilation. From (19) we can conclude that,

*Increasing the size of the minority, or the rent associated with assimilation, \(r\), will, given the level of harassment, increase the level of assimilation. Moreover, over time, the minority will invest more effort in assimilating.*

The political entity representing the minority seeks aggregate rents as described above, not rents per capita. Thus for this political entity, growth of the minority population wanting to assimilate is the same as increasing rents. Rising rents increase the benefits for the minority political entity and thus increase the entity’s returns to investing in assimilation. With time, the minority naturally integrates into the majority; increasing assimilation efforts speeds up the process.

*Equilibrium*
The majority group invests in harassment and the minority invests in assimilation \((a\ and\ h\ are\ positive)\). We focus on the unique interior Nash equilibria. We now wish to consider the effects changes in the size of the minority (rents to the minority’s political entity) have on the equilibrium levels of harassment and assimilation efforts.

By differentiation of the first order conditions (see (8) and (18)), the Nash equilibrium effort levels satisfy the following conditions for \(L=L_F\),

\[
\frac{\partial h^*}{\partial L} = \frac{\partial \Delta_N}{\partial a} \frac{\partial \Delta_M}{\partial L} - \frac{\partial \Delta_M}{\partial a} \frac{\partial \Delta_N}{\partial L} ,
\]

and,

\[
\frac{\partial a^*}{\partial L} = \frac{\partial \Delta_M}{\partial h} \frac{\partial \Delta_N}{\partial L} - \frac{\partial \Delta_N}{\partial h} \frac{\partial \Delta_M}{\partial L} .
\]

From (8) and (18) we obtain

\[
\frac{\partial \Delta_N}{\partial a} = \frac{\partial^2 \Pr}{\partial h \partial a} g(t) f^{'''} L_n L_m ; \quad \frac{\partial \Delta_N}{\partial h} = \frac{\partial^2 \Pr}{\partial h^2} g(t) f^{''''} L_n L_m ^2 ; \quad \frac{\partial \Delta_N}{\partial L} = \frac{\partial \Pr}{\partial h} g(t) f^{'''} L_n ;
\]

(23) and,

\[
\frac{\partial \Delta_M}{\partial a} = \frac{\partial^2 \Pr}{\partial a^2} g(t) L_m r ; \quad \frac{\partial \Delta_M}{\partial h} = \frac{\partial^2 \Pr}{\partial a h} g(t) L_m L_n r ; \quad \frac{\partial \Delta_M}{\partial L_m} = \frac{\partial \Pr}{\partial a} g(t) r .
\]

Substituting (23) into (21) and (22) and using the first order conditions (9) and (19) we obtain
\[
\frac{\partial h^*}{\partial L_m} = \frac{g(t)^2}{H} \left( \frac{\partial^2 \text{Pr}}{\partial h \partial a} f'' L_n - \frac{\partial^2 \text{Pr}}{\partial a^2} r \right),
\]

(24) and,

\[
\frac{\partial a^*}{\partial L_m} = \frac{L_n}{g(t)H} \left( \frac{\partial^2 \text{Pr}}{\partial a \partial h} r - \frac{\partial^2 \text{Pr}}{\partial h^2} f'' L_n \right),
\]

where, \( H = g(t)^2 L_n^2 L_m^2 r f'' \left( \frac{\partial^2 \text{Pr}}{\partial a^2} - \left( \frac{\partial^2 \text{Pr}}{\partial h \partial a} \right)^2 \right). \)

Since \( f'' < 0; \left( \frac{\partial^2 \text{Pr}}{\partial h \partial a} \right)^2 > 0 \) and \( \text{Sign} \left( \frac{\partial^2 \text{Pr}}{\partial a^2} \right) = -\text{Sign} \left( \frac{\partial^2 \text{Pr}}{\partial h^2} \right), H > 0. \)

The ability of the minority to convert its assimilation efforts into productivity and efficiency can be represented by the marginal effect of a change in the minority’s assimilation effort on the marginal productivity effect, \( \frac{\partial \text{Pr}(h)}{\partial a} \) (where \( 0 \leq \text{Pr}(h) \leq 1 \)).

By assumption, this marginal productivity effect is declining with the minority’s own assimilation efforts. Changes in assimilation efforts also affect, however, the majority’s marginal productivity level. The minority has an advantage in terms of ability if a change in majority’s effort positively affects the minority’s marginal productivity level. In other words, a positive (negative) sign of the cross second-order partial derivative of \( \text{Pr}(h,a), \frac{\partial^2 \text{Pr}}{\partial a \partial h} \), implies that the minority has an advantage (disadvantage) when majority group efforts to harass the minority change. For a given combination of efforts \((h,a)\), the ratio between the effect of a change in the minority’s effort on the marginal productivity level and the effect of a change in majority group effort, \( \frac{\partial^2 \text{Pr}}{\partial a \partial h} \left( \frac{\partial^2 \text{Pr}}{\partial a^2} \right) \), is a measure of the asymmetry between the abilities of both groups to affect the minority’s productivity level.  

\(^{10}\) In the same way one can calculate the measure of asymmetry in terms of the other group.

\(^{10}\) For a general discussion on how effort activities are transferred from effort to performance see Epstein and Nitzan (2005).
From (24) we obtain

**Lemma 1**

(a) For \( \frac{\partial^2 Pr}{\partial h \partial a} = 0 \), \( \frac{\partial h^*}{\partial L_m} > 0 \) and \( \frac{\partial a^*}{\partial L_m} > 0 \).

(b) For \( \frac{\partial^2 Pr}{\partial h \partial a} < 0 \), \( \frac{\partial h^*}{\partial L_m} > 0 \) and \( \frac{\partial a^*}{\partial L_m} = 0 \) if \( \frac{\partial^2 Pr}{\partial h \partial a} < \frac{\partial^2 Pr}{\partial h^2} < \frac{f''L_m}{r} \).

(c) For \( \frac{\partial^2 Pr}{\partial h \partial a} > 0 \), \( \frac{\partial a^*}{\partial L_m} > 0 \) and \( \frac{\partial h^*}{\partial L_m} = 0 \) if \( \frac{\partial^2 Pr}{\partial h \partial a} > \frac{\partial^2 Pr}{\partial a^2} > \frac{r}{f''L_m} \).

By Lemma 1 (a), if the contestants are symmetric in equilibrium in terms of their abilities, then growth in the minority population will increase both harassment and assimilation efforts.

By Lemma 1 (b), if the majority group has an advantage, as defined above, over the minority population, then growth in the minority population will increase harassment; however it is not clear what will happen to the level of assimilation activities. This ambiguity depends on the measure of asymmetry between the two groups, \( \frac{\partial^2 Pr}{\partial h \partial a} \), growth in the U.S., and the ratio between the effects of the marginal efficiency of their investment, \( \frac{f''L_m}{r} \). The main idea here is that even though harassment activities increase, the majority is so strong that it may not be worthwhile for the minority to try to fight the majority. Therefore, the minority may well decrease its efforts to assimilate. This will depend on what each group can gain from such activities.

By Lemma 1 (c), if the minority has an advantage over the majority, then an increase in the minority population will increase assimilation activities, however it is not clear what will happen to the level of harassment activities. This ambiguity
depends on the measure of asymmetry between the two groups, \( \frac{\partial^2 \Pr}{\partial a \partial h} \), and the ratio between the effects of the marginal efficiency of their investment, \( \frac{r}{f'' L_n} \).

Recall our assumption \( f''' = 0 \), so when increasing \( L_n \), \( f'' \) doesn’t change and thus the ratio decreases. Thus the relative advantage of the minority decreases, while it increases for the majority.

As we described earlier, we define the majority to be the strong group. It does not have to be the biggest group. Therefore, we assume that the majority is the “majority” because it has the advantage over the minority and thus it holds that \( \frac{\partial^2 \Pr}{\partial h \partial a} < 0 \). We conclude that,

**Proposition 1:** \( \frac{\partial h^*}{\partial L_m} > 0 \) and \( \frac{\partial a^*}{\partial L_m} = 0 \) if \( \frac{\partial^2 \Pr}{\partial a \partial h} < \frac{r}{f'' L_n} \).

Growth in the minority population, given that the majority group is relatively stronger, increases the majority’s harassment of the minority. As the minority’s population grows, efforts to assimilate on the part of the minority will increase if the measure of the asymmetry between the abilities of both groups to affect the minority’s productivity exceeds the ratio between the effects of the marginal efficiency of their investments on their rents. Here it “pays” for the minority not to give up in the face of greater harassment, but to fight instead and further their assimilation. Similarly, if the measure of the asymmetry between the abilities of both groups to affect the minority’s productivity is less than the ratio between the effects of the marginal efficiency of their investments on their rents, the minority will not find it worthwhile to fight the majority population and will reduce its assimilation efforts as its population increases. As the minority’s population grows, assimilation efforts by the minority will decrease if the majority population is so strong that it is worthwhile for the minority to try to fight the majority. Therefore, the minority may well decrease its
efforts to assimilate. This will depend on what each group can gain from such activities.

Let us now consider how a change in the rent received by the political entity representing the minority population, \( r \), affects the level of harassment against the minority and its assimilation efforts. As presented above in (21) and (22), differentiating of the first order conditions (see (8) and (18)), the Nash equilibrium effort levels satisfy the following conditions,

\[
\frac{\partial h^*}{\partial r} = \frac{\partial \Delta_N \partial \Delta_M - \partial \Delta_M \partial \Delta_N}{\partial \Delta_N \partial \Delta_M - \partial \Delta_M \partial \Delta_N} \quad \text{and} \quad \frac{\partial a^*}{\partial r} = \frac{\partial \Delta_F \partial \Delta_N - \partial \Delta_N \partial \Delta_F}{\partial \Delta_F \partial \Delta_N - \partial \Delta_N \partial \Delta_F}.
\]

The calculations described in (23) hold. We, however, calculate two additional components:

\[
\frac{\partial \Delta_N}{\partial r} = 0 \quad \text{and} \quad \frac{\partial \Delta_M}{\partial r} = \frac{\partial \Pr}{\partial a} g(t) L_F.
\]

Using (26) together with (23), (25) and (26) we obtain

\[
\frac{\partial h^*_i}{\partial r} = \frac{g(t)^2 \partial^2 \Pr}{H \partial h^* \partial a} f'' \frac{\partial \Pr}{\partial a} L_m^2 L_n^2
\]

(27) and,

\[
\frac{\partial a^*}{\partial r} = \frac{g(t)^2}{H} \left( - \frac{\partial^2 \Pr}{\partial h^2} f'' L_m^2 L_n^2 \frac{\partial \Pr}{\partial a} \right),
\]

where \( H \) is defined in equation (23).

**Lemma 2:** \( \text{Sign} \left( \frac{\partial h^*_i}{\partial r} \right) = - \text{Sign} \left( \frac{\partial^3 \Pr}{\partial h^2 \partial a} \right) \) and \( \frac{\partial a^*}{\partial r} > 0. \)
Increasing the rent associated with assimilation efforts increases assimilation activities by the minority. However, it is not clear what will happen to harassment activities. If the minority has an advantage over the majority in turning effort into ability, \( \frac{\partial^2 \text{Pr}}{\partial h a} > 0 \), then increasing rents associated with assimilation decreases harassment.\(^{11}\) The idea here is that the minority has a lot more to gain from its assimilation activities relative to what the majority can obtain and, at the same time, it can have an advantage in turning effort into efficiency. These two elements will cause the minority to increase its assimilation activities, while they will decrease the majority's harassment efforts.

With the majority as the strong population we obtain

**Proposition 2:** Increasing the rent associated with assimilation activities will increase both harassment and assimilation efforts by both groups (\( \frac{\partial h^*}{\partial r} > 0 \) and \( \frac{\partial a^*}{\partial r} > 0 \)).

Let us now consider how time affects assimilation activities. Will we see greater or fewer assimilation and harassment efforts over time? As \( g(t) \) is increasing in \( t \), we look at the effect of changes in \( g(t) \) on the levels of harassment and assimilation activities. Use (21) and (22) for \( L=g(t) \) and the fact that (recall \( f'''=0 \)),

\[
(28) \quad \frac{\partial \Delta_N}{\partial g(t)} = \frac{\partial \text{Pr}}{\partial h} f^{''} L_n^a L_n \quad \text{and} \quad \frac{\partial \Delta_H}{\partial g(t)} = \frac{\partial \text{Pr}}{\partial a} L_n^r.
\]

By substituting (28) and (2) into (21) and (22) and using the first order conditions (9) and (19) we obtain

\(^{11}\) Effort refers to activities such as assimilation and harassment, while ability tells us how effort translates into outcomes – does one unit of effort give us one percent or ten percent of the desired outcome
\[
\frac{\partial h^*}{\partial g(t)} = \frac{L_n}{H} \left( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} f'' L_N - \frac{\partial^2 \Pr}{\partial \alpha^2} r \right)
\]

(29) and

\[
\frac{\partial a^*}{\partial g(t)} = \frac{L_n L_m}{H} \left( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} r - \frac{\partial^2 \Pr}{\partial h^2} f'' L_n \right).
\]

From (29) we have,

**Lemma 3**

(a) For \( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} = 0 \), \( \frac{\partial h^*}{\partial t} > 0 \) and \( \frac{\partial a^*}{\partial t} > 0 \).

(b) For \( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} < 0 \), \( \frac{\partial h^*}{\partial t} > 0 \) and \( \frac{\partial a^*}{\partial t} > 0 \) if \( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} = 0 \) if

\[
\frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} \left( \frac{\partial^2 \Pr}{\partial h^2} \right) < \frac{f'' L_n}{r}.
\]

(c) For \( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} > 0 \), \( \frac{\partial a^*}{\partial t} > 0 \) and \( \frac{\partial h^*}{\partial t} > 0 \) if \( \frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} = 0 \) if

\[
\frac{\partial^2 \Pr}{\partial \alpha \partial \alpha} \left( \frac{\partial^2 \Pr}{\partial h^2} \right) > \frac{r}{f'' L_n}.
\]

By Lemma 3(a), if the contestants are symmetric in equilibrium in terms of their abilities then over time the majority group increases its harassment activities and the minority increases its assimilation activities. By Lemma 3(b) if the majority group has an advantage over the minority, then over time harassment activities increase, however, it is not clear what happens to assimilation efforts. By Lemma 3(c) if the minority has an advantage (in turning effort into performance) over the majority, then over time assimilation activities increase, however, it is not clear what happens to the level harassment efforts. The reason for these results are the same type of reasons presented after Lemma 1.

Given that the majority population is the stronger group we obtain:
Proposition 3: Harassment will increase over time while assimilation efforts may increase or decrease.

\[
\frac{\partial a^*}{\partial t} = 0 \quad \text{if} \quad \frac{\partial^2 \Pr}{\partial a \partial h} > \frac{f''(L_u)}{r}.
\]

A general model of assimilation

Now let us return to the political entity representing the minority population. Assume that assimilation has two different types of effects on the utility of an individual: (1) it increases the wages of the individual and, (2) it decreases the utility of the individual as he is losing his identity and uniqueness. Therefore, we rewrite the utility of the entity in the following way:

\[
U(t)_{M} = R(\Pr(h, a) g(t)L_m, (1 - \Pr(h, a)) g(t)L_m) - a.
\]

The first component is the number of assimilated minority members, while the second component in the number of non-assimilated minority members. Increasing any of these components increases the utility of the political entity:

\[
\frac{\partial R(\Pr(h, a) g(t)L_m, (1 - \Pr(h, a)) g(t)L_m)}{\partial (\Pr(h, a) g(t)L_m)} > 0 \quad \text{and} \quad \frac{\partial R(\Pr(h, a) g(t)L_m, (1 - \Pr(h, a)) g(t)L_m)}{\partial ((1 - \Pr(h, a)) g(t)L_m)} > 0.
\]

The objective of the political entity is to maximize its utility by determining the level of assimilation activities. The first order condition is:

\[
\frac{dU(t)_{M}}{da} = \left( \frac{\partial R(\Pr(h, a) g(t)L_m)}{\partial (\Pr(h, a) g(t)L_m)} - \frac{\partial R(\Pr(h, a) g(t)L_m)}{\partial ((1 - \Pr(h, a)) g(t)L_m)} \right) \frac{\partial ((1 - \Pr(h, a)) g(t)L_m)}{\partial a}.
\]

12 Another way of thinking about this is from the political entity's view: this group’s existence is a function of harassment and the minority’s failure to undertake effective assimilation activities. In order for the political entity to survive it needs to help those it represents and at the same time ensure that they still need its services. If all members of the minority fully assimilate then there will be no place for this political entity to exist.
In the case we described above it is clear that we were only talking about the first component of (32).

Under this more generalized case, it is clear that if $\Pr(h,a) = 1$ then the benefit of an individual will be low (and for the political entity it will be zero and it will not have any reason for existence). Therefore, if the level of assimilation is sufficiently high then the assimilated individuals will want to invest in resisting assimilation so as to differ from the local population, thus holding on to their heritage and traditions.

Assume for $t=t^*$ the assimilation activity that maximizes (32) equals $a_t^*$. Over time, as the assumed level of assimilation increases, if the level of harassment in time $t=t^*+1$ equals the level of harassment activities at time $t=t^*$, then it is clear that the level of assimilation activities of those who have already assimilated will decrease from time period $t=t^*$ to period $t=t^*+1$. Namely, $a_t^* > a_{t+1}^*$. In other words it may well be the case that after a certain period of time the level of assimilation activities will decrease. Let us now return to the minority worker. Given (5), minority workers who have a low level of assimilation will always want to invest effort in assimilation activities while it is not clear that the political entity that represents all the minority will always want to do so. Moreover, if the level of assimilation is sufficiently high, it may well be the case that the minority worker will continue investing in assimilation activities while the political entity will invest in anti-assimilation activities, for example, preserving the heritage of the minority group, etc. These activities are aimed at preserving home country traditions and emphasizing the differences between the majority population and the minority.

We conclude therefore that,

**Proposition 4**

There exists an inverse U-shaped relationship between the rent obtained from assimilation activities and time for the political entity representing the minority. With time the majority will continue to invest in harassment activities against the minority, whereas the members of the minority who have low levels of assimilation will invest in assimilation activities while those with a high level assimilation will invest in anti-assimilation activities. The political entity representing the minority will increase its assimilation activities until a certain point in time, $\tilde{t}$, and beyond this time will
decrease assimilation activities and may even invest in anti-assimilating activities \((a_1<0)\).

This proposition states that the members of the minority benefit from assimilation and will invest (as much as they can given free rider problems, etc.) in assimilation activities. Denote the minority’s members assimilation activities by \(a_2 > 0\). At the beginning, the political entity representing the minority will benefit from assimilation and thus will increase their investment in assimilation activities. Over time, after \(t\), the political entity benefits less from assimilation of the minority group as its members are becoming more and more assimilated. As a result, after period \(\tilde{t}\) assimilation activities by political entity decrease and may even become negative. Negative assimilation activities can be thought of as anti-assimilation activities (these activities of the political entity are denoted by \(a_1<0\)), and include activities aimed at preserving home country traditions and emphasizing the differences between the majority population and the minorities. At the same time the majority population will continue harassing the minorities, while individual minorities continue to invest in assimilation activities.

It may well be the case that as the members of the minority continue in their assimilation activities after a level of \(a_2^*\) (see figure) and the political entity will invest effort in anti-assimilation activities, so that \(a_1\) becomes negative (see figure). Thus we will see that the political entity is fighting to prevent assimilation or at least full assimilation while the members of the minority that have low levels of assimilation fight to increase assimilation. Both the majority population and the political entity will be fighting assimilation and the members of the minority with low levels of assimilation will be fighting to increase assimilation. Over time, therefore, we may well see that the political entity raises assimilation efforts and fights harassment, but after a certain point they go against their fellow country-men and decrease assimilation activities, even engaging in anti-assimilation activities to hold on to their rent and not let the minority fully assimilate into the host country.

**Concluding remarks**

We posit that the degree of assimilation is a matter of majority attitudes and minority desires. There is a conflict, or at least a potential conflict, between the majority and
the minority over the position of the minority in the economy and society. This potential conflict is acute between both the majority and the minority, and within the minority community (see Gradstein and Schiff, 2005 and Gradstein and Justman, 2005). The majority’s attitudes towards minorities (is the majority group welcoming, do they attempt to integrate minorities), the minorities desire to integrate and the will of minority representatives determine the degree of integration, as represented in our model by the relative productivity of the two groups.

We explicitly model the actors involved in assimilation, in developing desires to maintain ethnic identity, and in trying to isolate and distinguish minorities. We examine how these actors interact, affecting the assimilation path.

Our picture of assimilation is highly simplified – with time and effort minorities assimilate into majority culture until they cannot be differentiated in terms of consumption preferences or wages. The majority group harasses minorities to forestall and prevent this, or at least to keep the gains from the process out of the hands of minorities. In the latter part of the paper, we examine the case where the representatives of the minority, facing a loss of their own positions, reject assimilation. Over time it is assumed that, in a natural way, the minority assimilate or the majority gets used to them and sees them more as equals. While highly simplified, the model allows us to obtain insights that are useful for understanding richer assimilation stories (Bun and Kiong, 1993, Gang and Zimmermann, 2000, Gradstein and Schiff, 2005, Gradstein and Justman, 2005, Rapoport and Weiss, 2001).

The intensity of assimilation activities by minorities and harassment activities by the majority group generally depends on how symmetric the minority and majority are in terms of their abilities, and their relative marginal efficiency of investment in these activities. Over time, the minority’s representatives and some of the members of the minority exhibit different interests in assimilation and in maintaining their cultural identity, and the interplay of their conflict with the actions of the majority over time provides further insights.
References

Alesina, Alberto and Eliana La Ferrara, 2000, Participation in Heterogeneous Communities, Quarterly Journal of Economics, (August), 847-904.


Figure
Assimilation activities of political entity and employed minorities

\[ a_2 \]
Level of assimilation

\[ a_2^* \]

Assimilation activities \( a_1 \)