Affective Decision Making in Insurance Markets

[Job Market Paper]

Anat Bracha
Yale University, Department of Economics.
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Abstract

This paper suggests incorporating affective considerations into models of decision making and insurance decisions in particular. The model is based on, and explores the implication of, a distinction between two internal thinking processes. This distinction is supported by numerous studies in psychology, and is a formalization of what is often referred to as the dual processes theory. More specifically, the decision maker is modeled as having two internal accounts – the rational account and the mental account. In the context of insurance choice, the rational account decides on the insurance level that maximizes expected (perceived) utility, while the mental account chooses the risk perception that maximizes expected utility net of mental costs. In order to reach a decision, the two accounts interact. This interaction is modeled as an intrapersonal simultaneous move game. Choice is characterized by the pure strategy Nash equilibria of the game. Hence, choice is composed of both risk perception and insurance level and reflects consistency between the two accounts. This framework naturally separates between report and choice tasks. In addition, it captures framing (attentional) effects, and gives rise to both optimism and pessimism. In the insurance markets, the presence of affective considerations allows for negative correlation between objective risk and insurance level, and implies that the absolute risk aversion property of the utility function cannot be extracted from the data. Moreover, the mental account suggests a linkage between models of risk and uncertainty.

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1 Introduction

The recent development of economic decision making models under risk and uncertainty is a result of a mutually enriching discourse between formal axiomatic models and empirical evidence. The axiomatization of expected utility with objective probabilities by von Neumann and Morgenstern (1944) served as the basis of analysis. However, ample empirical and experimental studies question its descriptive adequacy (e.g., Allais, 1953; Grether and Plott, 1979). That, in turn, has engendered other decision theories, such as prospect theory (Kahneman and Tversky, 1979), expected utility with rank dependent probability weight, and mixed fanning\(^1\) (for review see Camerer, 1995; Camerer, Loewenstein and Rabin, 2004). In order to accommodate the observable behavior, the new theories depart from the expected utility paradigm by introducing a probability weighting function\(^2\) or a different shape of the utility function. Probability weighting function captures a well recorded phenomenon where probability weight, as judged by the agent, is different from the objective probability (Kahneman and Tversky, 1979; Camerer, Loewenstein and Rabin, 2004). In spite of the above differences, the new theories share the expected utility-type representation, i.e., a unitary process in which the agent maximizes some weighted average of future utility. In calculating the weighted average of future utility, probability weights are taken as given and independent of the size of outcomes.\(^3\)

Although the separability of weights and outcomes has an intuitive appeal, it is often violated. Ample empirical evidence suggests that in the mind of the agent probability judgment and the size of outcomes are not independent, e.g., Edwards (1955, 1961) and Irwin (1953). In fact, the numerous records of optimism bias are an evidence of such dependency. Optimism bias is the tendency of people to think about good outcomes as more likely, and bad outcomes as less likely, than they actually are (Weinstein, 1980; Armor and Taylor, 2002). This is a widespread phenomenon; optimism bias was shown to be correlated with normal mental health (Taylor and Brown, 1988), and to be present in various domains, one of which is risk perception (e.g., Slovic, 2000).

Dependency of probability judgment and outcomes leads to another questionable feature of decision making models: a fixed weighting function. By simple introspection it is safe to conclude that we do not have a fixed weighting function, i.e., a fixed probability judgment. This is related to
the previous point that probability weight and the size of outcomes are not independent; if weights assigned to future events are not independent of the size of outcomes then probability weights are endogenous to the problem and therefore not fixed.

Affective decision making (ADM) suggests a static model of decision making where the process of forming probability weight (thereafter perceived probability) is endogenous and explicitly modeled. The model extends on expected utility theory in a new way, and is illustrated by focusing on the insurance markets.

For the insurance markets, the relevant probability distribution is personal risk. Risk is defined as the agent’s probability distribution over future states of the world. As mentioned, the agent works with a perceived, rather than objective risk, and she is optimistically biased. According to the psychology literature, one of the mechanisms underlying optimism bias is motivated reasoning, where the agent’s beliefs are motivated by emotional goals (e.g., Kunda, 1990). Motivated reasoning suggests that the agent has preference over beliefs, which is a source of many psychological biases, including optimism bias. Preference over beliefs, and the biases resulting from motivated reasoning were incorporated into economic models. It has been shown that psychological biases have an interesting and important impact on information acquisition and optimal actions (e.g., Akerlof and Dickens, 1982; Rabin and Schrag, 1999; Koszegi, 2000; Bénabou and Tirole, 2001; Bodner and Prelec, 2001; Yariv, 2002; Eyster, 2002; Brunnermeier and Parker, 2002; Caplin and Leahy, 2004). Similarly, the current model explores the impact of optimism bias on insurance decision, however takes a different modeling approach.

More specifically, ADM models the agent as having two inner accounts— the rational account and the mental account. The rational account coincides with the standard economic model. That is, for a given risk perception (perceived probability distribution), it maximizes expected utility with respect to insurance level. The mental account is where risk perception is formed. In accordance with psychology research of motivated reasoning, the agent acts as if she chooses her risk perception. In particular, the agent selects an optimal risk perception to balance two contradicting forces: (1) affective motivation and (2) a taste for accuracy. Affective motivation is the desire to hold favorable personal risk perception (optimism) for the given insurance level and is captured
using an expected utility term. That is, the mental account, like the rational account, wishes to maximize expected utility albeit doing so with respect to risk perception. Although the mental account desires favorable risk beliefs, it does come at a cost. In particular, the agent bears a mental cost for holding beliefs other than her base rate. Base rate is the belief that minimizes the mental cost function and can be thought of as the agent’s probability reference point, or best assessment. That is, a taste for accuracy. Indeed, this mental process is an abstraction of a much more complex process. However, I believe it captures the essence of it.

Note that the mental process maps payoffs, which are determined by the rational account’s insurance choice, into optimal risk perception. This, in turn, changes the optimal insurance level for the rational account. In order to reach a decision, the two accounts interact and aim for consistency. This interaction is modeled as a simultaneous move intrapersonal game. Consistency of the two accounts is characterized by the pure strategy Nash equilibria of the game and these are the candidates for choice.

The distinction between the two processes is based on research in psychology and neuroscience (Epstein, 1994; Wilson and Dunn, 2004; Camerer, Loewenstein and Prelec, 2004; Damasio, 1994). In psychology the distinction is between two systems, while neuroscience views decision making as a result of an interaction between various modules of the brain. These modules are often coarsely classified into two systems (upstairs, downstairs; old and new). In contrast to a common view that the two systems are competing, this model views them as complements. This is a result of the nature of the problem at hand and is consistent with recent developments in neuroscience (Damasio, 1994).

The main conclusions arising from this model are that (1) affective considerations generally lead to multiple equilibria. This implies that people with the same information can have different beliefs about their risk and take different actions, (2) affective motivation can lead to both records of optimism and pessimism; the key is that it is always optimism in the mind of the agent, (3) the Nash equilibria set consists of two types of equilibria, one of which allows for negative correlation between objective risk and insurance. The other equilibrium type captures an enhancement effect. That is, the presence of the mental account enhances the choice of the rational account. The possibility of
negative correlation between insurance and objective risk is consistent with stylized facts in the life insurance market and suggests that educating the public about its higher-than-perceived-risk can lead to a counter intuitive result of less insurance. Moreover, for the insurance markets the model suggests that (4) in the presence of affective considerations, the absolute risk aversion property of the utility function cannot be concluded from the data, and (5) the insurance premium affects risk perceptions. Also, we show that this model makes a natural distinction between choice and report tasks, and its framework, once applied to other contexts, is consistent with stylized facts outside the insurance markets. Note that multiplicity of equilibria means that there exists a set of possible perceived probabilities, which reflects uncertainty. However, the rational account is a model of risk. Therefore, ADM can be viewed as a bridge between risk and uncertainty.

The remainder of this paper is organized as follows: section 2 summarizes the psychology and economic literature and argues that the process underlying risk perception choice is as described above. Section 3 gives the setting of the rational and mental accounts. Section 4 proves the existence of a pure strategy Nash equilibrium for the intrapersonal game and provides comparative statics to examine the impact of changes in income, insurance premium and base rate on the individual’s insurance decision. Section 5 discusses related economic studies and implications of the model, in and outside the context of insurance. For example, it discusses the relationship between absolute risk aversion, income and insurance choice, as well as the cautious optimism phenomenon. Section 6 concludes.

2 Related Literature

2.1 Risk Perception

In order to characterize the thinking process of forming risk perception, one has to turn to the psychology literature. Since this paper is concerned with the choice behavior of a single decision maker, it is important to understand the forces underlying perceived personal risk. Casual observations show that most people exhibit the “this is not going to happen to me” phenomenon. More scientifically put, most people, according to psychological experiments, believe that they are less likely than the average to be injured in a car accident or be involved in other bad experiences,
but more likely than others to experience a positive event such as living longer, having a healthy life, and being successfully employed (Weinstein, 1980; Armor and Taylor, 2002). In other words, on average, people tend to be unrealistically optimistic, and this is correlated with mental health (Taylor and Brown, 1988).

Psychologists explain optimistic predictions as a way to achieve emotional and motivational goals such as good mood and self-confidence. The literature on motivated cognition (e.g., Kunda, 1990) suggests that confirmatory bias, optimism, cognitive dissonance, self-esteem and many other well-recorded psychological phenomena are due to individuals extracting utility from beliefs per se. That is, people have preferences over beliefs: people want to believe they are better than average, less likely to be ill, unemployed or unhappy. Differently put, the psychology literature suggests that people act as if they choose their beliefs. It is interesting to note that optimism bias is not exclusive to cases where an outcome is correlated with personal abilities; experiments show that subjects’ prediction of what will occur is highly correlated with what they would like to see happen, rather than with what is objectively likely to happen (Irwin, 1953). This is true even in events that are totally random (Taylor and Brown, 1988 and references within).

In the context of risk perception, motivated reasoning means that individuals are choosing their personal risk perception and the desire for holding favorable beliefs guides them in that process, i.e., in choosing risk perceptions individuals are optimistically biased. Indeed research on risk perception finds optimism bias to be one of the sources in forming risk perception (Slovic et al, 1982). However, if motivation were the only driving force in the process of forming beliefs in general, and risk perception in particular, then people would generally hold arbitrary beliefs, which is not the case. Kunda (1990) argues that motivated cognition is restricted by personal experience, prior belief, knowledge or, in general, reality. In other words, individuals hold the most favorable risk perceptions that they can justify, i.e., that are reasonable.

In experiments regarding probability judgment, people are shown to use heuristics such as anchoring and adjustment (Tversky and Kahneman, 1974; Shiller, 2000), representativeness, and availability (Gilovich, Griffin and Kahneman, 2002) to form their beliefs. These heuristics are related to motivated reasoning where agents are argued to balance motivation and accuracy. For
example, consider the anchoring an adjustment heuristic. This heuristic was demonstrated in experiments using an exogenous anchor, to emphasize the thinking process the decision maker use. However, in reality anchors are not provided and are instead endogenous. Therefore, it is not obvious what are the anchors decision makers use in reality. Motivated reasoning gives a possible answer – agents are anchored to motivated beliefs and adjust to reality as they perceive it (base rate). In fact, Kunda (1990) acknowledges that anchoring and adjustment might not be distinguishable from motivated reasoning. Representativeness and availability can be seen as the cognitive strategy that the agent employs in order to justify her motivated beliefs. That is, the mind conveniently makes a representative evidence, that is in support of the motivated beliefs, available.

2.2 Dual Processes

After reviewing the psychology literature with its implications for personal perception and, in particular, perceived personal risk, one needs to think about how the latter leads to choosing insurance. In other words, one should link together probability judgment and choice behavior. This brings us to the distinction often made in psychology, and supported by neuroscience, between two systems of reasoning.

The modern, most influential, mapping of the human mind into multiple processes was made by Freud. Freud suggests a distinction between two processes: the unconscious (primary process), and the conscious (secondary process). His hypothesis is that the primary process is symbolic and associative while the secondary process is a rational-thinking process. Throughout the years, many more scholars in various fields of psychology made similar distinctions between two processes, albeit distinguishing the two by different traits: a verbal and nonverbal process, logical and prototypical systems, explicit and implicit, analytical and intuitive information processing and more (Wilson and Dunn, 2004; Epstein, 1994 and references within). Generally speaking, then, the psychology literature agrees on the distinction Freud proposes between two processes and agrees that one is a more deliberate, rational reasoning process and the other one is more intuitive and emotionally based. In contrast to Freud, psychologists now believe that the intuitive mode is not the source that undermines people’s attempt at rational thinking, but merely a different type of reasoning.
system. Epstein (1994), recognizing the similarities between the various dual process theories, developed a general theory of personality, the Cognitive-Experimental Self Theory (CEST), to encompass all such dual process models by making a distinction between the rational system – which is a deliberate, effortful, abstract system – and the experimental system – which is intuitive and emotionally driven. Recently, Kahneman (2003) introduced this distinction to economists, and argued that it should be incorporated in economic decision making. Moreover, advances in neuroscience research support the hypothesis of a modular brain (e.g., Damasio, 1994; Camerer, Loewenstein and Prelec, 2004 and reference within). The hypothesis of modular brain has already been incorporated into economic decision making models. Thaler and Shefrin (1981) study the problem of self-control in the principal-many agents framework. More recent examples, drawing on new insights from neuroscience, are Bernheim and Rangel’s (2004) model of addiction using two modes of decision processes, and Benhabib and Bisin’s (2004) model of consumption and self-control.

Forming risk perception, as discussed above, involves emotional motives such as feeling good about oneself or one’s future. Moreover, by introspection, forming risk perception is generally an unconscious process, i.e., agents do not have access to it. Therefore, it seems natural that the task of forming risk perception is generally an intuitive one performed by the experimental system in CEST and which we label the mental account, defined in section 3.2. Indeed, Wilson and Dunn (2004) argue that a source of self-knowledge failure is the inaccessibility of the mind to mental processes that involve perception, self esteem and alike. Thus, they agree that many self-perceptions are formed in implicit mental processes, which they argue, are generally unconscious. In contrast, choosing optimal action, such as insurance, is a deliberate task which demands logical effort, and it is therefore labeled the rational account. We follow the distinction psychologists often make between the rational and mental accounts and hypothesize that each is partially in charge of the insurance decision. The insurance decision is modeled as an outcome of the interaction of the two accounts, in accordance with the psychology literature. However, in contrast to the psychology literature where the two systems are often taken as rivals, this study views the two processes as complements. The reason for this difference is that one can decompose the decision making into
two main components; one is dominated by the rational account and the other is dominated by the mental account. In order to reach a decision, each process uses all available information including that supplied by the other process.

2.3 Preferences Over Beliefs in Economics

Some of the biases that are said to be explained by motivated cognition have been recognized in economic studies and were shown to play an important role in information acquisition, strategic interactions and consumption choices. To mention a few, Akerlof and Dickens (1982) examine the consequences of cognitive dissonance for workers in a hazardous profession, and show that cognitive dissonance might prevent workers from purchasing available safety equipment. Koszégi (2000) show that also ego utility (capturing an agent’s utility from positive beliefs about herself) can lead to suboptimal actions. More specifically, ego utility leads the agent to draw information strategically, generating distorted beliefs and suboptimal actions from the standard point of view. Bénabou and Tirole (2001) study the value agents put on self-confidence, modeled as a probability distribution over possible individual abilities. They show that above a certain threshold of self-confidence, agents will not acquire information. That will lead to non informative, self-handicapping activities. Similarly, Bodner and Prelec (2001) propose a model in which the agent’s utility is composed of instrumental and dispositional elements. Since the agent’s ability is not fully known, she extract information on her own ability by interpreting her actions. However, they assume that motivation is not self transparent which gives rise to self-signaling. Two interesting levels of agent’s sophistication are considered – “face value” and “true” interpretation. Face value is when, in the process of interpreting her action, the agent fails to realize that her action are a result of motivational goals. True interpretation is when the agent does take motivational goals into account in interpreting her actions. In the latter case, either the instrumental or the dispositional elements in her utility prevails and actions reveal the disposition for which they are optimal. In a strategic setup, Caplin and Leahy (2004) examine a principal agent model where the agent’s utility is influenced by her probability beliefs; in particular, the agent is potentially anxious. That is, the agent belief utility is influenced by both the probability belief and its distance from one-half (no information). The principal (expert) would like to maximize the agent’s private utility and thus needs to decide
whether to supply information about a medical operation which the agent (patient) is about to undergo. They find that information supply will depend on the agent’s anxiety: if it were only for anxiety, the expert would reveal information only for the patient whose anxiety is reduced with early resolution of uncertainty, but not otherwise. When the agent’s utility is determined by both the type of operation and anxiety, then information will be fully revealed. The axiomatic foundation for belief dependent utility is provided by Caplin and Leahy (2001) and Yariv (2001).

Brunnermeier and Parker (2002) (BP) consider an intertemporal decision making model where the agent chooses probability beliefs to maximize expected total well-being. That is, at the beginning of time the agent is choosing probability beliefs for all subsequent periods in which she will choose actions, based on her belief. This agent’s portfolio decision is shown to depart from the standard economic model: if the agent, under the standard model, invests in an asset, then BP’s agent will invest even more or alternatively will like to sell it short.

Another source for anomalous behavior is consistency bias. Eyster (2002) captures a taste for consistency by introducing a regret function that is a function of past and present actions. He shows that consistency bias affects optimal actions, and in particular explains why agents do not ignore sunk cost. That is, provided that past and present investments are strategic complements, an agent who invested too much in the past invests too much in the present. Yariv (2002) shows that a taste for consistency between beliefs in different stages can lead to either overconfidence or underconfidence, and presents conditions under which actions are persistent. Moreover, Yariv characterizes cases where a taste for consistency leads agents to prefer less information. Rabin and Schrag (1999) show that confirmatory bias can lead to overconfidence, and then examine the impact of this bias on agents’ beliefs.

To sum, psychology research suggests that people have preference over, and extract utility from, beliefs. This motivates agents to choose favorable beliefs, which are balanced against accuracy goals. This insight is indeed the link between all of the above papers, i.e., agents choose beliefs in a strategic manner, resolving a trade-off between a standard instrumental payoff and some psychologically based one. The current paper captures this trade-off by introducing an internal game between two accounts. The mental account chooses optimal beliefs (for a given action) to
maximize some _mental profit_ from beliefs. The rational account chooses optimal action (for a given belief) to maximize expected utility. Choice is a consistency point between the two accounts.

In addition, the current model embodies some existing concepts in the literature. To see that, note that the process of reaching a decision in our model can be viewed as a dynamic process, in which the two accounts are myopic and play in turns. In this process the two account constantly disagree, i.e., the agent is subject to cognitive dissonance. Choice, therefore, is a _resolution_ of cognitive dissonance. That is, in what constitutes choice, actions and beliefs are consistent. In particular, given actions beliefs are optimal, which is similar in spirit to the face-value interpretation in Bonder and Prelec. In fact, by using the notion of intrapersonal equilibrium, the ADM model encompasses both sophistication levels; choice of action reveals the optimal probability belief for which it is optimal, however, in the process of reaching a choice the agent in our model fails to realize the motivational goal embodied in her beliefs.

### 3 The Model

Consider an agent who is facing two possible future states of the world, \( s \in \{s_1, s_2\} \) with an associated wealth level of \( w \in \{w_1, w_2\} \). We make the following assumptions:

**Assumption 1** _Without loss of generality assume_ \( w_1 < w_2 \)

and denote the income shock as \( z \equiv w_2 - w_1 \).

**Assumption 2** _The agent has a strictly increasing, strictly concave, smooth utility function of wealth, \( U(w) \), with_ \( \lim_{w \to -\infty} U'(w) = \infty, \lim_{w \to \infty} U'(w) = 0 \)._

Unlike the standard insurance models, the agent is assumed _not_ to know her risk level, rather she forms a risk perception and works with that. Risk perception is defined as the perceived probability \( \beta \in [0, 1] \) of being in state \( s_1 \). Note that since we have only two states of the world, \( \beta \) captures the perceived probability distribution over future states of the world.

As argued above, the agent has a preference over beliefs, which in this context is risk perception. Acknowledging that the agent has a preference over risk perception, leads one to believe that the agent _chooses_ her perception of risk. The forces underlying this choice and the selection process
itself will be discussed in section 3.2. In general, we distinguish between two processes: the rational account is presented in section 3.1 and the mental account presented in 3.2.

In order to avoid her (perceived) risk the agent can purchase insurance $I \in (-\infty, \infty)$ to smooth her wealth across states of the world. The insurance premium rate is $\gamma \in [0,1]$ and it is fixed for all levels of insurance purchase.

### 3.1 Rational Account

The rational account chooses insurance for a given perceived risk $\beta$ to maximize expected utility. Thus, the rational account is the standard expected utility model using perceived rather than objective probabilities. More specifically, the rational account maximizes the following objective function:

$$
\max_I \left\{ \beta U(w_1 + (1 - \gamma)I) + (1 - \beta)U(w_2 - \gamma I) \right\}
$$

Where $U(\cdot)$ is the agent’s utility of wealth and $\gamma$ is the insurance premium. The optimal insurance level, $I^*$, satisfies the first order condition of this problem:

(RA) \[ \beta U'(w_1 + (1 - \gamma)I^*)(1 - \gamma) - (1 - \beta)U'(w_2 - \gamma I^*)\gamma = 0 \]

Given a fixed wealth, shock size and insurance premium, $I^*$ is a function of risk perception $\beta$ and will be denoted as $I^*(\beta)$. Since perceived risk is determined by the mental account, as discussed in section 3.2 below, one can think of $I^*(\beta)$ as the best response of the rational account for any given strategy of the mental account.

A straight-forward conclusion from the first order condition is that the best response $I^*(\beta)$ is a strictly increasing function of $\beta$ and can be more than, less than, or exactly equal to full insurance. For stating the condition on the model’s parameters which determine that, recall the definition of $z$ as the income shock size ($z \equiv w_2 - w_1$). The lemma below summarizes the result.

**Lemma 1** $I^*(\beta)$ is a continuous, strictly increasing function of $\beta$, and as $\beta \to 0$, $I^*(\beta) \to -\infty$ while as $\beta \to 1$, $I^*(\beta) \to \infty$. Moreover, if $\beta \geq \gamma$ then the optimal insurance level is $I^*(\beta) \geq z$. 

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A possible illustration of $I^*(\beta)$ is presented in Figure 1 below:

(Figure 1)

3.2 Mental Account

The psychology literature dealing with motivated cognition (e.g., Kunda, 1990) suggests that individuals extract utility from beliefs per se. Acknowledging the effect of beliefs per se on well-being leads one to think that people choose their beliefs. Thus, we allow the agent to choose her beliefs and derive the impact of that choice on perception and insurance choice.

Bénabou and Tirole (2001) argue that people get a positive utility from favorable beliefs for consumption value and signaling value among other reasons. Consumptions value simply implies that people like to hold favorable beliefs about the self. Signaling value means that if one perceives herself as better than she really is, other people will tend to believe it as well. We capture this by using an expected utility term. Note that the mental account takes the insurance level, and therefore the utility level, as given. Therefore, changing $\beta$, which is the weight assigned to the utility in state $s_1$, will change perceived expected utility. This also captures the dependency of perceived risk on payoffs. That is, the mental gain of changing $\beta$ depends upon the utilities in both states of the world.

Notice that if expected utility were the only component in deciding on optimal perception of risk, then the agent would hold arbitrary values of $\beta$, $(\beta \to 0)$ for values of $I < z$, and $(\beta \to 1)$ for values of $I > z$. However, as argued in section 2, people’s beliefs are not arbitrary and are a
result of balancing motivation with a taste for accuracy. As discussed, balancing motivation and accuracy can be viewed also as the phenomenon of anchoring and adjustment. We capture this by introducing a mental cost function $f(\beta)$ that reaches a minimum at some point $\beta = \beta_0$. Therefore the cost function will be denoted as $f(\beta; \beta_0)$. Note that the cost at $\beta_0$ can be normalized to zero, and therefore $\beta_0$ is labeled the belief reference point, or base rate; this can be thought of as the agent’s best assessment.

There are two possible sources for the cost function. First, the cognitive strategies that the mental account employs to justify her beliefs, such as availability, become more costly as one is further away from her base rate. Second, the cost function can be viewed as capturing the emotion of fear of being overly optimistic; the further away perception is from the agent’s base rate the higher is her fear. Emotions such as fear then guide her in adjusting perception closer to her base rate. This is in accordance with the psychology literature where emotions are believed to influence decisions in general (Loewenstein et al, 2001) and in particular negative emotions influence perceived risk (Johnson and Tversky, 1983; Shafir and LeBoeuf, 2002), with fear increasing it (Lerner and Keltner, 2000; Lerner et al, 2003). Negative emotions such as regret and fear affect decisions because people will attempt to avoid or minimize those emotions in the future (Bell, 1982; Mellers et al, 1998; Shafir and LeBoeuf, 2002).

According to the discussion above, the agent balances the desire of holding beliefs $\beta \neq \beta_0$ with taste for accuracy. Hence, the optimal risk perception $\beta^*$ solves the following

$$max_\beta \{\beta U(w_1 + (1 − \gamma)I) + (1 − \beta)U(w_2 − \gamma I) − f(\beta; \beta_0)\}$$

The first order condition is:

$$(M) \quad U(w_1 + (1 − \gamma)I) − U(w_2 − \gamma I) − \frac{\partial f(\beta^*; \beta_0)}{\partial \beta} = 0$$

Given a fixed income, and loss size $\beta^*$ is a function of insurance level $I$ and is indexed by $\beta_0$. Notice that the marginal mental gain$^{10}$ of infinitesimally changing beliefs, $U(w_1 + (1 − \gamma)I) − U(w_2 − \gamma I)$$^{11}$, is influenced by the insurance level while the marginal cost of this, $\frac{\partial f(\beta^*; \beta_0)}{\partial \beta}$, is determined by $\beta$ and $\beta_0$. $\beta^*(I; \beta_0)$ is the probability judgment, or belief, that balances these two forces. In other words, $\beta^*(I; \beta_0)$ is the perception that maximizes the mental well-being, for a given insurance level $I$, i.e.,
\( \beta^*(I; \beta_0) \) is the best mental response for a given strategy of the rational account. It is interesting to note that the mental best response presents the risk perception one would expect the agent to report, for a given insurance level.

To analyze the behavior of \( \beta^*(I; \beta_0) \) we assume the following:

**Assumption 3** \( f(\beta; \beta_0) \) is a continuous, smooth function of \( \beta \) and \( \beta_0 \), it is strictly convex in \( \beta \) and reaches a minimum at \( \beta = \beta_0 \).

In words, the further away \( \beta \) is from \( \beta_0 \), the greater is the psychological cost associated with it. This is, by definition, taste for accuracy. We further assume that the cost function \( f(\cdot) \) is submodular:

**Assumption 4** The mental cost function \( f(\beta; \beta_0) \) is submodular, i.e., \( \frac{\partial^2 f(\beta; \beta_0)}{\partial \beta \partial \beta_0} \leq 0 \).

This assumption implies that the marginal cost of holding a risk perception \( \beta \) is nonincreasing in \( \beta_0 \). This assumption is used in later sections.

Lastly, experiments show that people attribute a special quality to certain situations, which correspond to extreme beliefs \( \beta \in \{0, 1\} \). Thus, people would not generally choose to believe \( \beta \in \{0, 1\} \) under conditions of risk and uncertainty. That can be captured by a mental cost function that is finite between two values \( \{\beta, \bar{\beta}\} \) with \( 0 < \beta < \bar{\beta} < 1 \) and is infinite at the limits. Note that \( \{\beta, \bar{\beta}\} \) can be arbitrarily close to \( \{0, 1\} \). These arguments are summarized in Assumption 5 below:

**Assumption 5** Mental costs are positive and infinite at the boundaries, \( \lim_{\beta \to \beta} f(\beta; \beta_0) = \lim_{\beta \to \bar{\beta}} f(\beta; \beta_0) = +\infty \).

Assumption 5 implies that the marginal mental cost at the limit are \( \lim_{\beta \to \beta} \frac{\partial f(\beta; \beta_0)}{\partial \beta} = -\infty \), \( \lim_{\beta \to \bar{\beta}} \frac{\partial f(\beta; \beta_0)}{\partial \beta} = +\infty \).

Using Assumption 3 and the first order condition, the following is a straightforward conclusion

**Lemma 2** \( \beta^*(I; \beta_0) \) is a continuous and strictly increasing function of \( I \). At full insurance, \( \beta^*(I = z; \beta_0) = \beta_0 \).
Figure 2 is a possible illustration of $\beta^*(I; \beta_0)$:

![Diagram](image)

(Figure 2)

The illustration of the best response locus above assumes it follows a particular shape: $\beta^*$ is initially convex and then becomes concave in insurance $I$. This is not necessarily the case; in fact, since $\beta^*(I; \beta_0)$ balances marginal mental gain and cost, its shape with respect to $I$ depends on the effect of a change in $I$ on the marginal gain, $\frac{\partial}{\partial I} [U'(w_1 + (1 - \gamma)I) - U'(w_2 - \gamma I)]$, and the rate at which this happens, $\frac{\partial f'(\beta^*; \beta_0)}{\partial \beta}$, relative to the change in marginal cost as one changes belief $\beta$, $\frac{\partial f'(\beta^*; \beta_0)}{\partial \beta^2}$, and its speed of change $\frac{\partial^2 f'(\beta^*; \beta_0)}{\partial \beta^2}$. More specifically the shape of $\beta^*(I; \beta_0)$ depends on the following condition:

**Lemma 3** $\beta^*(I; \beta_0)$ is concave in $I$ iff

$$\frac{\partial}{\partial I} [U'(w_1 + (1 - \gamma)I)(1 - \gamma) + U'(w_2 - \gamma I)\gamma] \leq \frac{\partial^2 f'(\beta^*; \beta_0)}{\partial \beta^2}$$

and it is convex otherwise.

### 3.3 Both Accounts

Although the two accounts are discussed separately, they are thought, and assumed, to interact in the process of reaching a decision. A decision in this context is a pair of risk perception and insurance level. As argued, the rational account produces an insurance best response to a certain
risk perception $\beta$, while the mental account produces a belief best response to any strategy of the rational account, i.e., insurance level $I$. The two accounts aim for consistency; otherwise the agent suffers a cost of cognitive dissonance. This situation is summarized in an intrapersonal static game of two accounts, defined below.

**Definition 1** An intrapersonal game is a simultaneous move game of two players – the rational and the mental account. The strategy of the rational account is an insurance level, $I \in (-\infty, \infty)$, and the strategy of the mental account is a risk perception, $\beta \in (\underline{\beta}, \overline{\beta})$. The rational account’s payoff function is $g(\beta, I) \equiv \beta U(w_1 + (1 - \gamma)I) + (1 - \beta)U(w_2 - \gamma I)$, $g : (\underline{\beta}, \overline{\beta}) \times (-\infty, \infty) \to R$ and the mental account’s payoff function is $\psi(\beta, I) \equiv g(\beta, I) - f(\beta; \beta_0)$, $\psi : (\underline{\beta}, \overline{\beta}) \times (-\infty, \infty) \to R$, where $f(\cdot)$ is the mental cost function of holding beliefs $\beta$ and it reaches a minimum at $\beta_0$.

Modeling the interaction of the two accounts as a simultaneous move game captures a recent view in neuroscience research. Namely, both accounts are concerned with the task at hand. This is in the spirit of Damasio’s (1994) words:

“...The mechanisms for behavior beyond drives and instincts use, I believe, both the upstairs and the downstairs: the neocortex becomes engaged along with the older brain core, and rationality results from their concerned activity..”

The pure strategy Nash equilibria of this game, if they exist, represent consistency between the two accounts. Thus, the set of Nash equilibria are the natural choice candidates for the agent. Given the best responses of the two accounts one can illustrate graphically the set of pure strategy
Nash equilibria for this game. One possible illustration is presented in Figure 3 below.

(Figure 3)

Notice that the implied risk perception at any Nash equilibrium is generally different from the self-reported risk perception one might record. To see this, note that if an agent is asked, in a report task, to indicate her perceived probabilities, she will answer by activating only her mental account. That is, the beliefs that makes her feel good about her situation. However, when engaged in a choice task, the agent will activate both accounts and therefore her risk perception will be a part of a pure strategy Nash equilibrium of the intrapersonal game. This implies that the insurance choice need not be consistent with reported risk perception, a phenomenon recorded by Costa-Gomes and Weizsäcker (2003), albeit in a different context. This distinction is quite important, as will become clear in section 5.

Given the information on the best responses, one can argue that there always exists a pure strategy Nash equilibrium and, moreover, provide more details for one of the possible equilibria. This is discussed in the next section.

4 Results

This section is divided into two parts. First it proves the existence of pure strategy Nash equilibria for the intrapersonal game, presents sufficient conditions for a unique equilibrium, and analyzes the
effect of affective considerations on insurance outcome. Second, it provides comparative statics to show the change in choice as one changes the parameters of the model.

4.1 Nash Equilibria

To study the pure strategy Nash equilibria of the intrapersonal game, it is useful to define two types of equilibria, as follows.

**Definition 2** Let $I^* : (\beta, \overline{\beta}) \rightarrow R$ be the best response of the rational account. Let $\beta^* : R \rightarrow (\beta, \overline{\beta})$ be the best response of the mental account. Define a Prospective adjustment process $(P)$ as a sequential play where the rational and the mental accounts play in turns, $h = I^* \circ \beta^*$, where $h : R \rightarrow R$.

**Definition 3** A Nash equilibrium is of type $P$ if an adjustment process $P$ converges to it for all initial points in its neighborhood.

That is, fix an insurance level close enough to $I^{NE}$, $\dot{h}(I) > 0$ for $I < I^{NE}$ and $\dot{h}(I) < 0$ for $I > I^{NE}$. Graphically P type equilibrium is an equilibrium point where the mental account’s best response $\beta^*$ crosses the rational account’s best response $I^*$ from below. For example:

(Figure 4)
**Definition 4** Let $I^{*-1} : R \to (\beta, \beta)$ be the belief that makes $I^*$ the rational account’s best response. Let $\beta^{*-1} : (\beta, \beta) \to R$ be the insurance level that makes $\beta^*$ the mental account’s best response. Define a Retrospective adjustment process $(R)$ as a sequential play where the rational and the mental accounts play in turns, $h^{-1} = I^{*-1} \circ \beta^{*-1}$, where $h^{-1} : (\beta, \beta) \to (\beta, \beta)$.

**Definition 5** A Nash equilibrium is of type $R$ if an adjustment process $R$ converges to it for all initial points in its neighborhood.

That is, fix a belief close enough to $\beta^{NE}$, $\dot{h}^{-1}(\beta) > 0$ for $\beta < \beta^{NE}$ and $\dot{h}^{-1}(\beta) < 0$ for $\beta > \beta^{NE}$. Graphically $R$ type equilibrium is an equilibrium point where the mental account’s best response $\beta^*$ crosses the rational account’s best response $I^*$ from above. For Example:

(Figure 5)

Using the above definitions and excluding the case of tangency between the two accounts’ best responses, the Proposition below follows.

**Proposition 1** There exists a pure-strategy Nash equilibrium for the intrapersonal game. The Nash equilibria set contains an odd number of equilibria with a lowest and a highest arguments. If these equilibria are ordered with respect to the natural partial order on $R^2$, then they form a chain. Moreover, the Nash equilibria points alternate between being of type $P$ to $R$. Under Assumption 5, the extreme equilibria are of type $P$. 

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Proposition 1, using Milgrom and Roberts (1994), assures us that an agent can achieve internal consistency between her rational and mental accounts. Moreover, there is an odd number of such equilibria and generally there are multiple equilibria. Having possibly multiple equilibria, a chain guarantees a unique order of equilibria from (low risk perception, low insurance) to (high risk perception, high insurance) and, excluding the case of a unique equilibrium, there is at least one equilibrium of type R. Since all equilibria are candidates for choice, the case of a unique equilibrium is of a special interest. The following is a sufficient condition for uniqueness:

**Proposition 2** The following is a sufficient condition for a unique pure strategy Nash equilibrium of the intrapersonal game:

\[
\frac{\partial^2 f(\beta; \beta_0)}{\partial \beta^2} > -\frac{\left[U'(w_1 + (1 - \gamma)I)(1 - \gamma) + U'(w_2 - \gamma I)\gamma\right]^2}{[\beta U''(w_1 + (1 - \gamma)I)(1 - \gamma)^2 + (1 - \beta)U''(w_2 - \gamma I)\gamma^2]},
\]

\(\forall (I, \beta) \in \left[I^*(\beta'), I^*(\beta)\right] \times \left[\beta', \beta\right],\)

where \(\beta' \equiv \beta^*(I^*(\beta))\) and similarly \(\beta' \equiv \beta^*(I^*(\bar{\beta}))\).

From Proposition 2 it is easy to see that if one considers a cost function \(c f(\beta; \beta_0)\) instead of \(f(\beta; \beta_0)\), then the above condition becomes

\[
\frac{\partial^2 f(\beta; \beta_0)}{\partial \beta^2} > -\frac{\left[U'(w_1 + (1 - \gamma)I)(1 - \gamma) + U'(w_2 - \gamma I)\gamma\right]^2}{[\beta U''(w_1 + (1 - \gamma)I)(1 - \gamma)^2 + (1 - \beta)U''(w_2 - \gamma I)\gamma^2]}.
\]

Therefore, for large enough \(c\) we will have a unique equilibrium.

To better understand the effect of the mental account on choice, we split the discussion into three cases: \(\gamma = \beta_0, \gamma > \beta_0\) and \(\gamma < \beta_0\). The results are summarized in the following three propositions. Note that the graphical illustrations which accompany these propositions are assuming a specific shape of the \(\beta^*\) and \(I^*\) loci. However, the conclusions drawn are general and apply for all cases with all possible shapes of the mental and the rational best responses.

**Proposition 3** If \(\gamma = \beta_0\), then there exists at least one Nash equilibrium with full insurance and
\[ \beta = \beta_0 = \gamma. \]

Note that in this case the equilibria set contains the standard economic outcome. To prove this note that at full insurance there is no mental gain for holding beliefs \( \beta \neq \beta_0 \) but there exists mental cost. Therefore at full insurance the mental account’s best response is \( \beta = \beta_0 \). Given that \( \gamma = \beta_0 = \beta \), the rational account’s best response is full insurance. Consequently, full insurance and \( \beta = \beta_0 \) is a Nash equilibrium of this case.

**Proposition 4** If \( \gamma > \beta_0 \), then there exists at least one Nash equilibrium, with \( \beta < \beta_0 \) and \( I < I^*(\beta_0) \).
At this Nash equilibrium the agent buys less insurance relative to the standard economic model. To prove this, note that the insurance premium is higher than $\beta_0$. As a result, $I^*(\beta = \beta_0) < z$. Also, $\beta^* = \beta_0$ only at full insurance, where $I = z$. Therefore, at the point of $\beta = \beta_0$ the mental account’s best response is above the rational account’s best response. Since this relationship is reversed at the limit $\beta \to \overline{\beta}$ and both the mental and the rational best responses are increasing, the conclusion is that there exists a Nash equilibrium with $\beta < \beta_0$ and less insurance than the standard outcome.

**Proposition 5** If $\gamma < \beta_0$, then there exists at least one Nash equilibrium, with $\beta_0 < \beta$ and $I > I^*(\beta_0)$.

At this Nash equilibrium, the agent buys more insurance relative to the standard economic model. To see this, notice that because the insurance premium is lower than $\beta_0$, then $I^*(\beta = \beta_0) > z$, while only full insurance, $I = z$ will make $\beta^* = \beta_0$. Therefore, at the point of $\beta = \beta_0$ the mental account’s best response is below the rational account’s best response. Since this relationship is reversed at the limit $\beta \to \overline{\beta}$ and both the mental and the rational best responses are increasing, one can conclude that there exists a Nash equilibrium with $\beta > \beta_0$ and more insurance than the standard outcome.
Propositions 4 and 5 above make the impact of affective considerations for type P equilibria quite clear: their presence leads the decision maker to enhance the extent of her rational account decision. For example, suppose the base rate decreases such that $\gamma > \beta_0$; the rational account prescribes buying less than full insurance. The mental account, then, leads the decision maker to convince herself that she is at a lower risk, in order to feel better about her decision (motivational reasoning); this effect causes a further reduction in insurance purchase.

There are two remarks on the Nash equilibria that are worth noting. First, note that the set of choice candidates captures the essence of the idea; the set is composed of Nash equilibria of the intrapersonal game which reflect the influence of insurance decision on both payoffs (thus utility) and probability judgment. This idea is similar to the moral hazard logic; in the moral hazard setting, agents are assumed to have two actions – insurance and, say, driving style. The argument is that agents will have incentives to change their driving style after buying insurance, leading to a change in their true risk. Thus insurance indirectly affect risk. In the current set-up, buying insurance directly affects the agent’s risk perception. Therefore, the similarity is that insurance affects risk but the differences are that in the present model (1) insurance influences perceived risk, not true risk and (2) choice is predicted according to Nash equilibria implying that, in equilibrium, the insurance purchase will be consistent with risk perception. The second interesting point is that the Nash equilibria set, representing the choice candidates, contains both equilibria of type P and R. Although usually economists focus attention on type P equilibria, the traditional result of full insurance and $\beta = \beta_0 = \gamma$ could be of type R. In fact, below are the exact conditions that determine whether or not the standard outcome is stable under the prospective process. These conditions depend on the relationship between the mental cost function and the utility function.

**Proposition 6** Consider the case $\beta_0 = \gamma$. Define $\bar{w} \equiv \beta_0 w_1 + (1 - \beta_0) w_2$ and let $r(\bar{w})$ be the measure of absolute risk aversion. The full insurance standard outcome is of type $P$ if and only if

$$\frac{U'(\bar{w})}{r(\bar{w})} < \frac{\partial^2 f(\beta_0; \beta_0)}{\partial \beta^2} \beta_0 (1 - \beta_0)$$

This condition implies that there are cases with $\beta_0 = \gamma$, admitting the standard outcome, and it is an equilibrium of type $R$.\textsuperscript{18}
4.2 Comparative Statics

In this part we conduct comparative statics for both types of equilibria. Note that analyzing
the change in a unique equilibrium is qualitatively the same as any other equilibrium of type P,
regardless of uniqueness. Therefore, the exercise below covers both the unique equilibrium case
and the multiple equilibria case.

4.2.1 Changes in $\beta_0$

This part presents the changes in Nash equilibria, composed of (risk perception, insurance) as
the agent’s base rate, $\beta_0$, changes. In order to achieve that, I use the following lemma which is a
conclusion drawn from the mental best response and Assumption 4.

**Lemma 4** $\beta^*(I; \beta_0)$ is weakly increasing in $\beta_0$. Assuming a strictly submodular mental cost func-
tion $f(\beta; \beta_0)$, i.e. $\frac{\partial^2 f(\beta; \beta_0)}{\partial \beta \partial \beta_0} < 0$, then $\beta^*(I; \beta_0)$ is strictly increasing in $\beta_0$.

Lemma 4 above suggests that as the base rate $\beta_0$ increases, indicating a higher chance of being
in state $s_1$, the optimal belief for any given insurance level is nondecreasing. To see that, note that
as the base rate $\beta_0$ increases, the marginal gain from holding any belief $\beta$ is unchanged while the
marginal cost is nonincreasing. If the marginal cost does not change, then the optimal belief for
a given insurance level stays the same. If the marginal cost does change, it declines and leads to
higher perception of risk at every given insurance level.

Using Lemma 4, one can deduce the effect of a change in the base rate $\beta_0$ on both types of
Nash equilibria. This is summarized in Proposition 7 below.

**Proposition 7** Consider a type P (R) equilibrium of the intrapersonal game. This equilibrium is
weakly increasing (decreasing) in $\beta_0$. Moreover, if $\beta^*(I; \beta_0)$ is strictly increasing in $\beta_0$, then it is
strictly increasing (decreasing) in $\beta_0$.

Proposition 7 suggests that as the base rate increases, the Nash equilibrium we consider might or
might not increase, depending if it is equilibrium of type P or R. If it is a retrospective equilibrium
then, as $\beta_0$ increases, it will consist of insurance level and risk perception which are less than or
equal to their previous level. This result is counter intuitive, as one would expect an increase in
the base rate to generate a higher Nash equilibrium with higher insurance and higher perception of risk just like the case for a type P equilibrium. However, this intuition, in the retrospective case, captures only one aspect: the change in $\beta^*$. Ceteris Paribus, the optimal belief is higher for a positive incremental change in $\beta_0$. However, the intrapersonal game is composed of both rational and mental considerations. If the rational and the mental account reinforce each other approaching the local Nash equilibrium, as is the case of type P equilibria, then one would maintain the intuitive result. By definition, in the neighborhood of the type R equilibrium, the insurance decision feeds beliefs in the opposite direction of the local Nash equilibrium. Therefore, for the retrospective equilibrium, an increase in the base rate results in a decrease in insurance choice because $\beta^*$ increases with $\beta_0$. This result could have policy implications implying that manipulating the base rate upwards can cause the agent to choose less insurance and hold more favorable risk perception!

4.2.2 Changes in income and shock size

This part studies the influence of a change in traditional insurance parameters, such as income and shock size, on the decision of an agent who is subject to both rational and mental considerations.

**Proposition 8** (i) Suppose that the income $w_2$ increases while the shock size, $z$, stays constant. Then, the change in choice is given by the following tables, distinguishing between choice due to type P or type R equilibrium as well as distinguishing between cases of a utility function with decreasing absolute risk aversion (DARA), constant absolute risk aversion (CARA) and increasing absolute risk aversion (IARA).

<table>
<thead>
<tr>
<th></th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I &lt; z$</td>
<td>?</td>
<td>$NE \uparrow$</td>
<td>$NE \uparrow$</td>
</tr>
<tr>
<td>$I = z$</td>
<td>unchanged</td>
<td>unchanged</td>
<td>unchanged</td>
</tr>
<tr>
<td>$I &gt; z$</td>
<td>?</td>
<td>$NE \downarrow$</td>
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<th></th>
<th>DARA</th>
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<td>$I &lt; z$</td>
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<td>$NE \downarrow$</td>
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<td>$I &gt; z$</td>
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<td>$NE \uparrow$</td>
<td>$NE \uparrow$</td>
</tr>
</tbody>
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where ? means that all kind of changes are possible.
(ii) Suppose that the shock size, \( z \) increases, while \( w_2 \) stays constant. Then, the impact of this change on the Nash equilibria is not clear.

Since choice is according to a Nash equilibrium and is composed of (risk perception, insurance), increase in equilibrium (or choice) means that both insurance and risk perception increases. Bearing this in mind, Proposition 8 states that if the utility function exhibits CARA, the initial choice consists of less than full insurance and follows a type P equilibrium, then as income increases choice will increase, leading to more insurance purchase and higher risk perception. However, in contrast to the expected utility model, such prediction in the current model is not constant; for instance if initially choice consists of more than full insurance, is due to a type P equilibrium and we have CARA utility function, then higher income leads to purchasing less insurance. Moreover, part (ii) of Proposition 8 implies that, ceteris paribus, an agent facing higher possible loss will not necessarily purchase more insurance. This is in contrast to the standard insurance model, which predicts more insurance purchase for such a change.

4.2.3 Changes in insurance premium

This section examines the influence of a change in insurance premium on insurance decision. Generally speaking, I find results different from the standard outcome. A summary of the results is presented below.

**Proposition 9** Suppose the insurance premium increases. Then, the impact of this change on the Nash equilibria is as follows:

| Type P | \( I > z \) | ? | ? | ? | \( I = z \) | \( NE \downarrow \) | \( NE \downarrow \) | \( NE \downarrow \) | \( 0 < I < z \) | \( NE \downarrow \) | \( NE \downarrow \) | ? | \( I < 0 \) | ? | ? |
|--------|-------------|---|---|---|-------------|-----------------|-----------------|-----------------|-------------|-----------------|-----------------|---|-------------|---|---|---|
| Type R | \( I > z \) | ? | ? | ? | \( I = z \) | \( NE \uparrow \) | \( NE \uparrow \) | \( NE \uparrow \) | \( 0 < I < z \) | \( NE \uparrow \) | \( NE \uparrow \) | ? | \( I < 0 \) | ? | ? | ? |
where ? means that all kind of changes are possible.

Proposition 9’s results suggest that there are cases where insurance companies can increase insurance premium and not suffer of lower insurance purchases. This depends on the agent’s initial insurance level, and the type of equilibrium her choice follows.

5 Discussion

This section discusses (1) the relationship of affective decision making with the existing literature, (2) the implications of the model on risk perception and optimism, (3) provides a possible explanation for various stylized facts in the insurance markets, and (4) experimental studies in psychology that this model can indirectly explain. Explaining phenomena outside the insurance context suggests that the framework of dual processes in decision making is more general and can, with some adjustment, be applied to decision making in other contexts.

5.1 Related literature

I would like to return to three particular papers that are closely connected to the current paper, and discuss in more substance the crucial differences with the current model. Of the related literature, the closest model to ADM is optimal expectations by Brunnermeier and Parker (2002) (BP). Optimal expectations is an intertemporal decision making model in which the agent chooses probability beliefs to maximize expected total well-being. In that sense, both models have the agent choosing her perceived probability to maximize a similar objective function. Indeed, this objective function can be viewed as an analogue to the mental objective function appearing in the current paper. In calculating this objective function, BP assume that the mental cost of distorting beliefs equals the expected utility with respect to the true probability distribution; in contrast, the ADM does not assume an explicit functional form. Hence, ADM allows for other cost functions that may be more realistic. In addition, the BP agent plays a dynamic game between what I label, for the sake of comparison, the “BP mental account” (BPMA) and the “BP rational account” (BPRA). The BPMA moves first by choosing perceived probabilities for all subsequent periods; the BPRA moves in each of the following periods by choosing optimal actions for the given perceived
probability. It is assumed that BPMA has a perfect foresight, perfectly anticipating how the future will unfold. Therefore, the BP construction is, in effect, a choice of one variable—probability belief. Moreover, BP framework can be viewed as a choice of an optimal prior, after which belief dynamics are Bayesian. In contrast, the ADM is a static model where the mental and rational accounts move simultaneously. Indeed, as was shown in the text, one can define a dynamic adjustment process which converges to a Nash equilibrium of the ADM model. However, in this dynamic process the two accounts are myopic; they play in turns, and belief dynamics are not Bayesian. Since the ADM model is simultaneous, ADM is, in effect, a decision of two variables: probability beliefs and action. The two cannot be reduced to a problem in one variable as probability belief and actions are mutually determined.

The affective decision making framework makes the distinction between two types of processes. A distinction between two processes has already been incorporated into economic models; examples are Bernheim and Rangel’s (2004) (BR) model of addiction, and Benhabib and Bisin’s (2004) model of self-control. These models and others, model the two systems, or decision modes, to be mutually exclusive. In contrast, the two processes in ADM are working together in order to reach a decision. Hence, ADM is essentially a model of one decision making mode, that is composed of two inner processes. This difference is a result of the questions the papers are addressing. In the model of addiction and the model of self-control, both processes decide on action, while in this paper one is choosing action and the other is forming perceptions; both are necessary for choice.

5.2 General Discussion

The ADM model shows that allowing motivational (affective) reasoning to interact with rational considerations gives rise to possibly multiple equilibria with different probability judgments and actions. Multiple equilibria are consistent with the casual observations that different people hold different beliefs despite being exposed to the same events or objective information. Hence, this model suggests that affective motivations is one reason for this.

Note that the adjustment processes, either prospective or retrospective, discussed to define equilibrium type can be viewed as a way of selection mechanism. If the agent indeed uses either one of such selection mechanism then choice will be path dependent— the observed choice depends
on whether the agent first chooses belief or insurance. This is in accordance with lab experiments where choices were demonstrated to depend upon the attribute subjects were induced to focus on. In the insurance context, a prospective process implies that manipulating subjects to report their risk perception first will, generally, lead to lower insurance purchase, relative to the case where subjects are manipulated to think about insurance first.\textsuperscript{19} However, as the mental cost of distorting one’s beliefs are getting larger, framing (attentional) effects get arbitrarily small.\textsuperscript{20}

Lastly, an insight that arises from this model is that it is possible for the experimenter to record both pessimism and optimism (relative to $\beta_0$). However, recording pessimism is possible only \textit{because} the action taken (insurance) can change a state from being a ‘bad’ state to being a ‘good’ state. Thus, if the available action can not change the bad state into a good state, we would record only optimism and people will purchase insurance which is less than optimal. This might hold also in other fields similar to insurance, such as pharmaceutical drugs consumption. This discussion leads one to think about issues of control. A negative relationship between sense of control and perceived risk is a well recorded phenomenon. This seems to be in contrast to this model; unless we distinct between the sources of control sensation – available actions versus other factors such as familiarity. Accepting such a distinction, this model suggests that the ability to take effective actions can lead an experimenter to record pessimism, while the other factors (which may or may not be familiarity) leads to record optimism.

5.3 Risk and Uncertainty

The reader might wonder at this point whether this model is a model of risk or uncertainty. Recall the distinction between risk and uncertainty; in both, the agent is faced with a future which outcome is not certain. Risk is the case where the agent knows the probability distribution over future event. Uncertainty, is the case where the agent is \textit{uncertain} about this probability distribution; the uncertainty is captured by having a \textit{set} of possible probability distributions over future events. Having these definitions in mind, it is clear that the \textit{rational account} is a model of risk; the rational account takes the mental account’s probability judgment and acts as if it is the true probability distribution. However, the agent’s choice is a model of uncertainty since she does not know a priori the perceived probability: multiple equilibria mean there is a set of perceived probabilities which
the agent can believe. The linkage between risk and uncertainty is the mental account.

5.4 Insurance Market

5.4.1 Income, insurance choice and the DARA hypothesis

Eisenhauer (1997) studies the life insurance market and argues that this market is not actuarially fair. Yet he finds that life insurance purchases increase with wealth. Controlling for household size and age$^{21}$, Eisenhauer concludes that the findings reject the DARA hypothesis. Indeed one can find other explanations for this finding, such that the data is still consistent with the DARA hypothesis. For example, wealthy individuals might have bequest motivations or they might have a different utility function altogether. The ADM model suggests another explanation; if the agent is subject to affective motivations then it is possible to have a utility function exhibiting DARA and to observe greater insurance purchase with an increase in income (see Proposition 8). In fact, ADM implies that in the presence of motivated reasoning the absolute risk aversion property can not be extracted from observing changes in insurance.

5.4.2 Risk measure and insurance choice

Cawley and Philipson (1999) find that relatively risky individuals, under both self-reported and actual risk measures, are less likely to purchase life insurance, and once purchasing insurance the high-risk individuals, using self-reported risk measures, purchase less insurance. They find these results after controlling for wealth and proxis for bequest motives such as number of grandchildren, number of children, age of youngest child, average age of children, number of siblings, age of spouse and marital status. This model is consistent with such results. To see that, note that the model captures objective risk with $\beta_0$; by Proposition 7 we know that decreasing $\beta_0$, i.e., lower objective risk, the type R equilibria increase, leading to higher insurance. As Lemma 4 shows, for a given insurance level, $\beta^*(I; \beta_0)$ moves always in the direction of $\beta_0$; Hence, for any given insurance level,
lower reported risk will be associated with higher insurance. The figure below illustrates this point:

![Figure illustrating the relationship between insurance and reported risk]

5.5 Other phenomena

5.5.1 Psychology Research: Cautious Optimism

Isen et al (1988) and Nygren et al (1996) examine the influence of positive affect on decision rule in risky situations. Both papers find a phenomenon labeled “cautious optimism”. To illustrate this, consider the experiment in Nygren et al. In that experiment, participants are asked to make both numerical evaluations of verbal probabilities in three-outcome gambles and actual betting decisions for similar gambles. The participants who were induced with positive affect overestimated the probabilities of winning relative to losing, for the same phrases (optimism). However, when asked to gamble, these participants were much less likely to gamble relative to controls (cautious). The intrapersonal game is consistent with this phenomenon. Note that overestimated probabilities of winning in a report task is analogous to a shift in the mental best response, and therefore explaining cautious optimism is similar to explaining higher insurance purchase by low risk individuals (self reported measure). The key is that intrapersonal game framework makes the distinction between report and choice tasks. When engaged in a report task, participants will respond according to the mental best response. However, when engaged in a choice task, choice is according to a Nash equilibrium. The Nash equilibrium can be of type R, leading individuals with lower perceived risk to choose more conservatively, analogous to higher insurance.
Note that Nygren, Isen et al (1996) conclude that ...“these findings suggest that positive affect can promote an overt shift from a decision rule focusing primarily on probabilities to one focusing on utilities or outcome values, especially for losses.” Translating this: positive affect promotes a shift from an adjustment process where the mental account moves first to one where the rational account moves first. Indeed, such a shift, in the presence of multiple Nash equilibria, means choosing more insurance or more cautiously. However, in this model, such a shift cannot explain a higher reported win perception among the positive affect participants relative to controls.

6 Conclusions

Empirical evidence from the life insurance market as well as ample experimental evidence leads one to reevaluate economic decision theory. One of the main points arising from the psychology literature, as well as recent economic studies, is that people have preference over, and extract utility from, beliefs. Taking this point seriously and following the dual processes theory, define a game — the intrapersonal game — in which the two accounts within the self, the rational and mental accounts, play simultaneously. The choice we observe is one of the Nash equilibria of this game.

Adding the mental account to the rational account can be viewed as adding a layer to, or enriching, the standard expected utility model. If the agent is not subject to mental considerations, then we are reduced to the classic model. However, if the agent’s decision involves some mental gain and cost then this model departs from the standard one, which suggests that the standard model will not provide a sufficient explanation. In other words, this model suggests that the failure of the traditional expected utility model to explain the data is in part due to systematic mental biases.

Taking the intrapersonal framework to the insurance context allows us to examine the insurance market and conduct comparative statics, given that people prefer to think the best outcome is more likely, or in other words, are optimistically biased. Many interesting dynamics arise from this framework; in particular it suggests a possible explanation for Cawley and Philipson’s (1999) finding in the life insurance market. The suggested model can explain this since negative correlation between risk (actual and reported) and insurance decision is possible. In fact, this has interesting
policy implications: educating the public to realize its higher-than-perceived risk can lead to an opposite reaction - lower insurance purchase!

Furthermore, this model is consistent (see Proposition 8) with empirical results showing that risk aversion, as deduced from insurance decision, increases with wealth (Eisenhauer, 1997). However, since the ADM model suggests that the observed data is due to the interaction between the mental and rational account, then one cannot conclude the utility function risk aversion characteristics from the data. For example, we can have a DARA utility function and still have insurance choice increasing with wealth. Like the effect of an increase in income, an increase in insurance premium can lead either to higher or lower insurance purchase. This will depend on the utility function, the equilibrium type of the initial choice, as well as the ratio between initial insurance level and the shock size. Thus, insurance companies might actually have an incentive to decrease their insurance premium.
7 Appendix

Proof of Lemma 1. Recall that $I^*$ satisfies condition (RA). By the Implicit Function Theorem and assumption 2, we get:

\[
\frac{\partial I^*}{\partial \beta} = -\frac{\partial \text{RA}/\partial \beta}{\partial \text{RA}/\partial I^*} = -\frac{U'(w_1 + (1-\gamma)I^*)(1-\gamma) + U'(w_2 - \gamma I^*)\gamma}{\beta U''[(w_1 + (1-\gamma)I^*)(1-\gamma)]^2 + (1-\beta)U''[(w_2 - \gamma I^*)\gamma]^2} > 0
\]

Note that by assumption 2 (1) $U''(\cdot) < 0$, therefore $\partial G/\partial I^* \neq 0$ (2) for all values of $I \in (\beta, \bar{\beta})$.\[\beta < \gamma \] then the optimal insurance level is $I^*(\beta) > z$.

Proof of Lemma 2. Recall that $\beta^*$ satisfies condition (M). By Assumptions 2, 3, and the Implicit Function Theorem:

\[
\frac{\partial \beta^*}{\partial I} = -\frac{\partial \text{M}/\partial I}{\partial \text{M}/\partial \beta^*} = -\frac{U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma}{\beta^* f'(\beta^*; \beta_0)} > 0
\]

By condition (M) if $I = z$, $f'(\cdot) = 0 \Rightarrow \beta = \beta_0$.

Proof of Lemma 3. The proof of Lemma 2 explicitly gives $\frac{\partial \beta^*}{\partial I}$. Taking its derivative we obtain:

\[
\frac{\partial^2 \beta^*}{\partial I^2} = \frac{\partial}{\partial I} \left[ \frac{U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma}{\beta^* f'(\beta^*; \beta_0)} \right] > 0
\]

Taking the derivatives, and rearranging implies that $\frac{\partial^2 \beta^*}{\partial I^2} > 0 \Leftrightarrow \frac{\partial \beta^*}{\partial I} < 0 \Leftrightarrow \frac{\partial\left[U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma\right]}{U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma} > 0 \Leftrightarrow \frac{\partial f'(\beta^*; \beta_0)}{\partial \beta^*} > 0$

Proof of Proposition 1. By Assumption 5 all Nash equilibria constitute of beliefs $\beta \in (\underline{\beta}, \bar{\beta})$, where $0 < \underline{\beta} < \bar{\beta} < 1$. Moreover, we know that all pure strategy Nash equilibria of this game will have insurance level in the interval $[I^*(\underline{\beta}), I^*(\bar{\beta})]$. That, in turns, implies that all Nash equilibria will have perceived probabilities in the interval $[\beta^*(I^*(\underline{\beta})), \beta^*(I^*(\bar{\beta}))]$ where $0 < \underline{\beta} < \bar{\beta} < I^*(\underline{\beta}) < \beta^*(I^*(\bar{\beta})) < \bar{\beta}$. Define $\beta^*(I^*(\beta)) \equiv \underline{\beta}; \beta^*(I^*(\bar{\beta})) \equiv \bar{\beta}$; since all the Nash equilibria of the
Thus, the equilibrium points are beliefs points. However, since there is only one more equilibrium point and Π is continuous then it must be that in the neighborhood of β, Π is nonnegative. Thus, Π(β) guarantees a solution Π(β) = 0. Fix some β ∈ (β, β′). Having a continuous function Π which is Π(β) is strictly increasing in β. Consequently Π(β; β0) > 0 and Π(β; β0) < 0. Since both Π(β; β0) and Π(β; β0) are continuous in β, then Π(β; β0) is continuous in β ∀β ∈ (β, β′). Fix some β0 ∈ (β, β′). Having a continuous function Π which is Π(β; β0) > 0 and Π(β; β0) < 0 guarantees a solution Π(β; β0) = 0 in the (β, β′) interval, which is a pure strategy Nash equilibrium of the intrapersonal game as it is a point of intersection of the two best responses.

**Lowest and highest argument** — By the boundary conditions Π(β, β0) > 0 and Π(β; β0) < 0, βL(β0) ≡ inf{β|Π(β; β0) ≤ 0}, βH(β0) ≡ sup{β|Π(β; β0) ≥ 0} exist. One needs to show that βL(β0), βH(β0) are solutions. Consider βL(β0). By definition of βL, the lim sup of Π(β; β0) as β ↑ βL is nonnegative. Thus, Π(βL(β0), β0) ≥ 0. If Π(βL(β0)) = β′, then Π(β; β0) < 0 — contradiction. If Π(βL(β0)) < β′, and Π(βL(β0); β0) > 0 then continuity implies that there is some ε > 0 such that Π(βL(β0) + ε; β0) > 0 ∀β ∈ [βL(β0), βL(β0) + ε] which is a contradiction to the definition of βL(β0). Therefore, the conclusion is that Π(βL(β0); β0) = 0. The case of βH(β0) can be proved to be a solution by similar arguments.

**Odd number of equilibria** — Suppose there are two equilibria points β1 < β2. By the boundaries conditions, in the neighborhood of β1(β0) Π(β1 − ε; β0) > 0 and Π(β1 + ε; β0) < 0. However, since there is only one more equilibrium point and Π is continuous then it must be that in the neighborhood of β2(β0), Π(β2 − ε; β0) < 0 and Π(β2 + ε; β0) > 0 which is a contradiction to the boundaries conditions. This argument can be repeated for any case of even number of equilibrium points.

**A chain** — Note that the set of equilibrium points is defined as $βNE ≡ \{β|Π(β; β0) = 0\}$. Thus, the equilibrium points are beliefs $β ∈ [β′, β]$ which form a chain by definition. Since $I^*(β)$
is an increasing function, then it follows that the Nash equilibria of intrapersonal game which are vectors \((I, \beta)\) form a chain.

**Equilibrium Type** — Recall the definition of the prospective adjustment process: \(h = I^* \circ \beta^*\), where \(\beta^*\) is the mental BR and \(I^*\) is the rational BR. Given an insurance point \(I, \beta^*\) second such that \(h: R \to R\). A Nash equilibrium is of type P iff the prospective adjustment process converges to it. In other words, take a NE \((I^{NE}, \beta^{NE})\). In the neighborhood of this equilibrium point it must be the case that for \(I < I^{NE}, \dot{h} > 0\) and \(I > I^{NE}, \dot{h} < 0\). This is equivalent to requiring that the slope of \(\frac{\partial I(\beta^{NE}, \beta_0)}{\partial \beta} > \frac{\partial I^*(\beta^{NE})}{\partial \beta}\) which is equivalent to \(\Pi(\beta^{NE} - \varepsilon; \beta_0) > 0\) and \(\Pi(\beta^{NE} + \varepsilon; \beta_0) < 0\). By definition, then, \(\beta_L\) and \(\beta_H\) are of type P. A Nash equilibrium is of type R iff the retrospective adjustment process converges to it. In other words, take a NE \((I^{NE}, \beta^{NE})\). In the neighborhood of this equilibrium point it must be the case that for \(\beta < \beta^{NE}, \dot{h}^{-1} > 0\) and \(\beta > \beta^{NE}, \dot{h}^{-1} < 0\). This is equivalent to requiring that the slope of \(\frac{\partial H(\beta^{NE}, \beta_0)}{\partial \beta} < \frac{\partial I^*(\beta^{NE})}{\partial \beta}\) which is equivalent to \(\Pi(\beta^{NE} - \varepsilon; \beta_0) < 0\) and \(\Pi(\beta^{NE} + \varepsilon; \beta_0) > 0\). By continuity of \(\Pi\), then, the NE alternate from being of type P to type R.

Note that the existence, lowest and highest Nash equilibria and the chain results can be proved by defining a restricted intrapersonal game where the insurance pure-strategy space is restricted between \([I^*(\beta_0), I^*(\beta)]\) and the perceived probabilities are restricted to \(\beta \in \left[\beta^*, \beta^\prime\right]\) such that the equilibria points of the intrapersonal game are not altered. The restricted game can be shown to be a supermodular game and thus these results follow from the properties of this class of games (see Topkis, 1998). ■

**Proof of Proposition 2.** Notice that the mental account’s objective function

\[
\beta U(w_1 + (1 - \gamma)I) + (1 - \beta)U(w_2 - \gamma I) - f(\beta; \beta_0)
\]

is the potential function of our game. That is, \(\frac{\partial P}{\partial I} = 0\) is condition \((RA)\), and \(\frac{\partial P}{\partial \beta} = 0\) is condition \((M)\). Therefore the maximization of \(P\) with respect to a pair \((I, \beta)\) gives rise to the pure strategy Nash equilibria of the game. Since all Nash equilibria of the intrapersonal game are within \(\beta \in \left[\beta^*, \beta^\prime\right]\) and \(I \in \left[I^*(\beta_0), I^*(\beta^\prime)\right]\) (see Proof of Proposition 1), then one can restrict attention to this restricted intrapersonal game where both players’ strategy space are compact. By Ui (2004), a potential game with strictly concave potential function, where all players have a compact strategy
space, has a unique pure strategy Nash equilibrium. That is, the Hessian is negative definite which is given by the condition below.

\[
\frac{\left[U'(w_1 + (1 - \gamma)I)(1 - \gamma) + U'(w_2 - \gamma I)\gamma\right]^2}{-\left[\beta U''(w_1 + (1 - \gamma)I)(1 - \gamma)^2 + (1 - \beta)U''(w_2 - \gamma I)\gamma^2\right]} < \frac{\partial^2 f(\beta; \beta_0)}{\partial \beta^2}
\]

\(\forall (I, \beta) \in \left[\tilde{I}^*(\beta'), \tilde{I}^*(\beta)\right] \times \left[\beta', \beta\right] \Box\)

**Proof of Proposition 6.** As in the proof of Proposition 1 a type-P equilibrium is where at \((\beta^{NE}, I^{NE})\) \(\frac{\partial \tilde{I}^*(\beta^{NE})}{\partial \beta} \neq 0\) or \(\frac{\partial I^{*}(\beta^{NE})}{\partial I} \neq 1\).

\[
\frac{\partial I^*(\beta) \partial \beta^*(I)}{\partial \beta} \frac{\partial I^{*}(\beta^{NE})}{\partial I} = \frac{U'(w_1 + (1 - \gamma)I^{NE})(1 - \gamma) + U'(w_2 - \gamma I^{NE})\gamma}{\beta^{NE}I''(w_1 + (1 - \gamma)I^{NE})(1 - \gamma)^2 + (1 - \beta^{NE})I''(w_2 - \gamma I^{NE})\gamma} - \frac{\partial^2 f(\beta_0; \beta_0)}{\partial \beta^2}
\]

At \(I^{NE} = z\) and \(\beta^{NE} = \beta_0 = 0\), define \(w \equiv \beta_0 w_1 + (1 - \beta_0) w_2\):

\[
\frac{\partial I^*(\beta)}{\partial \beta} \frac{\partial \beta^*(I)}{\partial I} = \frac{U'(w)^2}{\beta_0(1 - \beta_0)U''(w)\frac{\partial^2 f(\beta_0; \beta_0)}{\partial \beta^2}}
\]

Rearranging:

\[
\frac{U'(\pi)}{r(\pi)} < \frac{\partial^2 f(\beta_0; \beta_0)}{\partial \beta^2} \beta_0(1 - \beta_0)
\]

\(\Box\)

**Proof of Lemma 4.** Recall that the choice of risk perception satisfies equation (M). Therefore, if the mental cost function \(f(\beta; \beta_0)\) is submodular in \(\beta_0\), i.e., \(\frac{\partial^2 f(\beta; \beta_0)}{\partial \beta_0^2} \leq 0\), and strictly convex in \(\beta\), then:

\[
\frac{\partial \beta^*}{\partial \beta_0} = \frac{\partial M}{\partial \beta^*} = -\frac{\partial^2 f(\beta^*; \beta_0)}{\partial \beta_0^2} = \frac{\partial^2 f(\beta^*; \beta_0)}{\partial \beta_0^2} > 0
\]

Furthermore, if \(\frac{\partial^2 f(\beta^*; \beta_0)}{\partial \beta_0^2} < 0\) then

\[
\frac{\partial \beta^*}{\partial \beta_0} = \frac{\partial M}{\partial \beta^*} = -\frac{\partial^2 f(\beta^*; \beta_0)}{\partial \beta_0^2} = \frac{\partial^2 f(\beta^*; \beta_0)}{\partial \beta_0^2} 
\]

\(\Box\)

**Proof of Proposition 7.** From Lemma 4, \(\beta^*(I; \beta_0)\) is weakly increasing in \(\beta_0\). Therefore \(\tilde{I}(\beta; \beta_0)\), the inverse function, is weakly decreasing in \(\beta_0\), leading \(\Pi(\beta; \beta_0) = I^*(\beta) - \tilde{I}(\beta; \beta_0)\) to be
weakly increasing in $\beta_0$. Using Lemma 1 in Milgrom and Roberts [1994], provided below, one can conclude that the P type Nash equilibria are weakly increasing in $\beta_0$. To see this, for every type P equilibrium one can find a local game that admits the same boundary conditions as the entire game and the equilibrium we consider is one of the game’s extreme points. For the local game the following define the extreme equilibria $\beta_L(\beta_0) = \inf\{\beta|\Pi(\beta) \leq 0\}$ and $\beta_H(\beta_0) = \sup\{\beta|\Pi(\beta) \geq 0\}$.

By Milgrom and Roberts these are weakly increasing in $\beta_0$. Suppose $\beta^*(I; \beta_0)$ is strictly increasing in $\beta_0$, then $\tilde{I}(\beta; \beta_0)$ is strictly decreasing in $\beta_0$. Then $\Pi(\beta; \beta_0) = I^*(\beta) - \tilde{I}(\beta; \beta_0)$ is strictly increasing in $\beta_0$ meaning that there are no $\beta$ such that $\Pi(\beta; \beta_0) = \Pi(\beta; \beta_0^*) = 0$. Using Lemma 1, that means that the extreme Nash equilibria are strictly increasing in $\beta_0$. Note that for an equilibrium of type R one can define a local game where it is one of the extreme equilibria. However, such a local game have the opposite boundary conditions and therefore the results are exactly the opposite.

**Lemma 1 [Milgrom and Roberts, 1994]:** Let $X \subset R$ and let $f, g : X \to R$. Suppose that for all $x \in X$, $g(x) \leq f(x)$. Then $\inf\{x|g(x) \leq 0\} \leq \inf\{x|f(x) \leq 0\}$ and $\sup\{x|g(x) \geq 0\} \leq \sup\{x|f(x) \geq 0\}$.

**Proof of Proposition 8.** Recall that $I^*$ satisfies condition (RA). By the Implicit Function Theorem, $\frac{\partial I^*}{\partial w_2} = -\frac{\partial RA/\partial w_2}{\partial RA/\partial I^*}$

\[
\begin{align*}
\frac{\partial I^*}{\partial w_2} &\geq 0 \Leftrightarrow \frac{U''(w_2-z+(1-\gamma)I^*)}{U''(w_2-z+(1-\gamma)I^*)} > \frac{U''(w_2-\gamma I^*)}{U''(w_2-\gamma I^*)} \\
\frac{\partial I^*}{\partial w_2} &\geq 0 \Leftrightarrow r(w_2-z+(1-\gamma)I^*) > r(w_2-\gamma I^*)
\end{align*}
\]

Recall that $\beta^*$ satisfies condition (M). By the Implicit Function Theorem and assumption 3:

\[
\frac{\partial \beta^*}{\partial w_2} = \frac{\partial M/\partial w_2}{\partial M/\partial \beta^*} = -\frac{U'(w_2-z+(1-\gamma)I)-U'(w_2-\gamma I)}{-\frac{\partial^2 f(\beta^*)}{\partial I^2}} \geq 0
\]

\[
\Rightarrow \frac{\partial \beta^*}{\partial w_2} \geq 0 \Leftrightarrow w_2-z+(1-\gamma)I \leq w_2-\gamma I \Leftrightarrow I \leq z
\]

Thus:

<table>
<thead>
<tr>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I &lt; z$</td>
<td>$\frac{\partial I^<em>}{\partial w_2} &lt; 0$, $\frac{\partial \beta^</em>}{\partial w_2} &gt; 0$</td>
<td>$\frac{\partial I^<em>}{\partial w_2} = 0$, $\frac{\partial \beta^</em>}{\partial w_2} &gt; 0$</td>
</tr>
<tr>
<td>$I = z$</td>
<td>$\frac{\partial I^<em>}{\partial w_2} = 0$, $\frac{\partial \beta^</em>}{\partial w_2} = 0$</td>
<td>$\frac{\partial I^<em>}{\partial w_2} = 0$, $\frac{\partial \beta^</em>}{\partial w_2} = 0$</td>
</tr>
<tr>
<td>$I &gt; z$</td>
<td>$\frac{\partial I^<em>}{\partial w_2} &gt; 0$, $\frac{\partial \beta^</em>}{\partial w_2} &lt; 0$</td>
<td>$\frac{\partial I^<em>}{\partial w_2} &lt; 0$, $\frac{\partial \beta^</em>}{\partial w_2} &lt; 0$</td>
</tr>
</tbody>
</table>

Recall $\Pi(\beta; \beta_0) = I^*(\beta) - \tilde{I}(\beta; \beta_0)$. $\Pi(\beta; \beta_0) > 0$ and $\Pi(\beta; \beta_0) < 0$ and equilibria of this game
is where $\Pi(\beta; \beta_0) = 0$.

$$\begin{array}{ccc}
\text{DARA} & \text{CARA} & \text{IARA} \\
I < z & \Delta\Pi \geq 0 & \Delta\Pi \geq 0 \\
I = z & \Delta\Pi = 0 & \Delta\Pi = 0 \\
I > z & \Delta\Pi \geq 0 & \Delta\Pi < 0
\end{array}$$

Using Lemma 1 in Milgrom and Roberts (1994) one can conclude the following for any equilibria of type P (see proof of Proposition 7 above for Lemma 1 and an argument why this holds for any type-P equilibrium):

$$\begin{array}{ccc}
\text{DARA} & \text{CARA} & \text{IARA} \\
I < z & \text{?} & \text{NE} \uparrow \\
I = z & \text{unchanged} & \text{unchanged} \\
I > z & \text{?} & \text{NE} \downarrow
\end{array}$$

Note that if the NE is of type R then the result is exactly the opposite.

**For the second part of the Proposition:**

An increase in the shock size $z$ will increase $I^*$ and will decrease $\beta^*$ as is shown below:

By the Implicit Function Theorem, $\frac{\partial I^*}{\partial z} = -\frac{\partial RA/\partial z}{\partial RA/\partial I^*} = -\frac{U'(w_2-z+(1-\gamma)I^*)[U'(w_2-\gamma I^*)]^2}{[U'(w_2-\gamma I^*)]U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)U''(w_2-\gamma I^*)\gamma}$

$\Rightarrow \frac{\partial I^*}{\partial z} > 0$

By the Implicit Function Theorem and assumption 3, $\frac{\partial \beta^*}{\partial z} = -\frac{\partial M/\partial z}{\partial M/\partial \beta^*} = -\frac{U'(w_2-z+(1-\gamma)I)}{\frac{\partial M}{\partial \beta^*}} < 0$

Consequently, as $z$ increases both $\tilde{I}(\beta; \beta_0)$ and $I^*(\beta)$ increase and the change in $\Pi(\beta; \beta_0) = 0$. Therefore, it is not clear how NE changes with $z$. ■

**Proof of Proposition 9.** Recall that $I^*$ satisfies condition (RA), and we can rewrite it as

$$G = \frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)} \frac{(1-\beta)}{\beta} = \frac{U'(w_2-z+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)} \frac{(1-\beta)}{\beta} = 0.$$ By the Implicit Function Theorem

$$\frac{\partial I^*}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial I^*} = \frac{I^*\left[U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)-U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\right] - \frac{1}{(1-\gamma)^2} \frac{(1-\gamma)}{\beta} \left[U'(w_2-\gamma I^*)\right]^2}{-\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)+U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

Rearranging and using $\frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} = \frac{(1-\gamma)}{\beta} \Rightarrow$

$$\frac{\partial I^*}{\partial \gamma} \geq 0 \Leftrightarrow I^* \gamma \left(1-\gamma\right) \left[\frac{1}{r(w_2-z+(1-\gamma)I^*)} - \frac{1}{w_2-\gamma I^*)}\right] \geq 1$$

Recall that $I^*$ satisfies condition (M). By the Implicit Function Theorem and assumption 3,

$$\frac{\partial \beta^*}{\partial \gamma} = -\frac{\partial M/\partial \gamma}{\partial M/\partial \beta^*}$$
\[
= -u'(w_2-z+(1-\gamma)I)(-1)+u'(w_2-\gamma I)I \iff \frac{\partial \beta_*^*}{\partial \gamma} \geq 0 \implies \left[ u'(w_2-\gamma I) - u'(w_2 - z + (1-\gamma)I) \right] I \geq 0
\]
\[
\Leftrightarrow \begin{cases}
\text{if } I > 0 & \frac{\partial \beta_*^*}{\partial \gamma} \geq 0 \implies I \geq z \\
\text{if } I < 0 & \frac{\partial \beta_*^*}{\partial \gamma} < 0 \implies 0 \implies I < z
\end{cases}
\]

Thus:

\[
\begin{array}{ccc}
\text{DARA} & \text{CARA} & \text{IARA} \\
I < 0 & \frac{\partial \beta_*^*}{\partial \gamma} > 0, \frac{\partial \beta_*^*}{\partial \gamma} < 0 & 0, \frac{\partial \beta_*^*}{\partial \gamma} > 0, \frac{\partial \beta_*^*}{\partial \gamma} < 0 \\
0 < I < z & \frac{\partial \beta_*^*}{\partial \gamma} < 0, \frac{\partial \beta_*^*}{\partial \gamma} < 0 & \frac{\partial \beta_*^*}{\partial \gamma} < 0, \frac{\partial \beta_*^*}{\partial \gamma} < 0 \\
I = z & \frac{\partial \beta_*^*}{\partial \gamma} < 0, \frac{\partial \beta_*^*}{\partial \gamma} = 0 & \frac{\partial \beta_*^*}{\partial \gamma} < 0, \frac{\partial \beta_*^*}{\partial \gamma} = 0 \\
I > z & \frac{\partial \beta_*^*}{\partial \gamma} > 0, \frac{\partial \beta_*^*}{\partial \gamma} > 0 & \frac{\partial \beta_*^*}{\partial \gamma} > 0, \frac{\partial \beta_*^*}{\partial \gamma} > 0
\end{array}
\]

Recall \( \Pi(\beta; \beta_0) = I^*(\beta) - \tilde{I}(\beta; \beta_0) \). \( \Pi(\beta^*; \beta_0) > 0 \) and \( \Pi(\beta^*; \beta_0) < 0 \) and equilibria of this game is where \( \Pi(\beta; \beta_0) = 0 \).

Using Lemma 1 in Milgrom and Roberts (1994) one can conclude the following for any equilibria of type P (see proof of Proposition 7 above for Lemma 1 and an argument why this holds for any type-P equilibrium):

\[
\begin{array}{ccc}
\text{DARA} & \text{CARA} & \text{IARA} \\
I < 0 & \Delta \Pi \geq 0, \Delta \Pi \leq 0 & \Delta \Pi \leq 0 \\
0 < I < z & \Delta \Pi < 0 & \Delta \Pi < 0 \\
I = z & \Delta \Pi < 0 & \Delta \Pi < 0 \\
I > z & \Delta \Pi > 0 & \Delta \Pi > 0
\end{array}
\]

Note that if the NE is of type R, then the result is exactly the opposite.
Notes

1. The mixed fanning hypothesis argues that indifference curves fan out for less favorable lotteries, while they fan in for more favorable ones.

2. Weight assigned to a given state of the world according to a given function of the true probability.

3. The probability weight in the rank dependent expected utility (RDEU) is determined by its ranking in the distribution of possible outcomes (see Lopes, 1995; Camerer, 1995). As long as the ranking is maintained, the weight function according to RDEU is unchanged.

4. Choice stands for a result of personal motives or goals and does not mean a deliberate, fully conscious act.

5. One can normalize the mental cost at the base rate to be zero.

6. The psychology literature suggests a distinction between perceived personal risk and perceived societal risk. These two are fundamentally different. One implication is, as Tyler and Cook’s (1984) research indicates, that factors influencing perceived societal risk do not necessarily influence perceived personal risk. More specifically they show that mass media information about risk will influence risk judgment on societal level but not on a personal level.

7. Anchoring is the tendency of people to stick to an anchor rate in an ambiguous situation.

8. Although experiments indicate that subjects are anchored to motivated beliefs and then adjust to the provided base rate, in reality the anchor rate is endogenous. Hence, one could argue that agents are anchored to the base rate and then adjust towards their motivational goals.

9. All qualitative results are maintained for the case of \( \lim_{w \to 0} U'(w) = \infty, \lim_{w \to \infty} U'(w) = 0. \)

10. Gain can be either positive or negative.

11. For a low level of insurance, the difference in utilities in the two future states of the world is relatively large. Thus, changing beliefs will have a relatively large mental gain. As insurance increases, the same change in beliefs will have lower impact on the mental gain since the differences in the state contingent utilities is smaller.

12. Note that the function

\[ f(\beta; \beta_0) = \ln \left( \frac{1}{(\beta - \beta_0)(\beta - \beta)} \right) - \beta k \]

satisfies all of the above assumptions. \( \beta_0 \) is the minimum of this function.

13. This is for a strictly concave utility functions such that \( \beta^*(I) \) is convex in \( I \) for some range and then becomes concave (see Corollary 3 for exact conditions).

14. \( I^*(\beta) \) is defined over \((0, 1)\). However, since all Nash equilibria are within \( \beta \in (\underline{\beta}, \overline{\beta}) \), then we can restrict attention to this interval.

15. Note that \( \dot{h}(I) = h(I) - I \), a nash equilibria is where \( \dot{h}(I) = 0 \) and it is stable point of the function \( h \) iff \( \frac{\partial h}{\partial I} = \frac{\partial h(I)}{\partial I^{NE}} - 1 < 0 \) at the Nash equilibrium point. This is of course equivalent to \( \dot{h}(I) > 0 \) for \( I < I^{NE} \) and \( \dot{h}(I) < 0 \) for \( I > I^{NE} \).

16. In which case, by the boundary conditions, it is of type P.

17. This is the analysis for type P equilibria as by the above argument we capture the equilibria where \( \dot{h}(t) > 0 \) for \( I < I^{NE} \) and \( \dot{h}(t) < 0 \) for \( I > I^{NE} \).

18. An example is \( w_1 = 25, w_2 = 50, f(\beta; \beta_0 \approx 0.66) = \ln \frac{1}{(0.9 - \beta)(\beta - 0.1)} - 0.3\beta \), and a utility function of the form \(- \exp^{-0.01w} \).
Unless, of course, there is a unique equilibrium, or that there exists one P-type equilibrium between the two post manipulation points.

Think of a cost function $cf(\beta, \beta_0)$ instead of $f(\beta, \beta_0)$. As $c$ increases above a certain threshold, we will have a unique equilibrium. That is, no framing (attentional) effects. Note that as $c \to \infty$, equilibrium gets arbitrary close to the traditional outcome.

Indirectly through age-adjusted mortal probability.
References


